

Electromechanical Analogies

$F(t) = F \cos(\omega t) \rightarrow x(t) = X \cos \omega t$

Equation of Motion:

$$m_{eq} \ddot{x} + c_{eq} \dot{x} + k_{eq} x = F(t)$$

\Rightarrow using phasor concept:

$$F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} \dot{x} + c_{eq} x$$

\Rightarrow by analogy:

$$\begin{aligned} F &\rightarrow N \\ x &\rightarrow i \\ k_{eq} &\rightarrow \frac{1}{C_x} \\ c_{eq} &\rightarrow r_x \end{aligned}$$

[Parameter Relationships in the Current Analogy]

EE C245: Introduction to MEMS Design LecM 13 C. Nguyen 11/18/08 4

Force-to-Velocity Relationship

- The relationship between input voltage v_1 and force F_{d1} :

$$F_{d1} \approx -V_P \frac{\partial C_1}{\partial x} v_1$$

- When displacement x is the mechanical output variable:

$$\frac{X(s)}{F_{d1}(s)} = \frac{1}{k} \frac{\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

- When velocity v is the mechanical output variable:

$$\frac{v(s)}{F_{d1}(s)} = \frac{sX(s)}{F_{d1}(s)} = \frac{1}{k} \frac{\omega_o^2 s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

EE C245: Introduction to MEMS Design LecM 13 C. Nguyen 11/18/08 6

Bandpass Biquad Transfer Function

$F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} \dot{x} + c_{eq} x$

\Rightarrow converting to full phasor form:

$$F = (j\omega)(j\omega X) m_{eq} + \frac{k_{eq}}{j\omega} (j\omega X) + c_{eq} (j\omega X)$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}} \left[-\omega^2 \frac{m_{eq}}{k_{eq}} + 1 + j \frac{c_{eq}\omega}{k_{eq}} \right]^{-1} = \frac{1}{k_{eq}} \left[\left(\frac{\omega}{\omega_0} \right)^2 + 1 + j \frac{\omega}{Q\omega_0} \right]^{-1}$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2, Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}\omega_0}{\omega_0 c_{eq}} \Rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0 \right]$$

EE C245: Introduction to MEMS Design LecM 13 C. Nguyen 11/18/08 5

Force-to-Velocity Equiv. Ckt.

- Combine the previous lumped LCR mechanical equivalent circuit with a circuit modeling the capacitive transducer \rightarrow circuit model for voltage-to-velocity

Voltage

Current

Linear Two-Port Element

Electrical Mechanical

Force

EE C245: Introduction to MEMS Design LecM 13 C. Nguyen 11/18/08 7

Lecture 23: Comb Drive

Equiv. Circuit for a Linear Transducer

The diagram illustrates the equivalent circuit for a linear transducer. It features a central 'Linear Two-Port Element' box. On the left side, under the 'Electrical' heading, there is a voltage source labeled V with its positive terminal at the top. A current I flows through the element from the top terminal to the bottom terminal. On the right side, under the 'Mechanical' heading, there is a velocity source labeled $U = -\dot{x}$ with its positive terminal at the top. A force F acts on the element from the top terminal to the bottom terminal.

• A transducer ...

- ↳ converts energy from one domain (e.g., electrical) to another (e.g., mechanical)
- ↳ has at least two ports
- ↳ is not generally linear, but is virtually linear when operated with small signals (i.e., small displacements)

EE C245: Introduction to MEMS Design LecM 13 C. Nguyen 11/18/08 8

Equiv. Circuit for a Linear Transducer

The diagram illustrates the equivalent circuit for a linear transducer. It features a central 'Linear Two-Port Element' box. On the left side, under the 'Electrical' heading, there is a voltage source labeled V with its positive terminal at the top. A current I flows through the element from the top terminal to the bottom terminal. On the right side, under the 'Mechanical' heading, there is a velocity source labeled $U = -\dot{x}$ with its positive terminal at the top. A force F acts on the element from the top terminal to the bottom terminal.

• For physical consistency, use a transformer equivalent circuit to model the energy conversion from the electrical domain to mechanical domain

The diagram shows a transformer model for the energy conversion. On the left, there is a primary winding with an effort e_1 and a secondary winding with a flow f_1 . The turns ratio is indicated as $1:\eta$. On the right, there is a primary winding with a flow f_2 and a secondary winding with an effort e_2 . The turns ratio is also $1:\eta$. To the right of the transformer, the 'Describing Matrix' is given as:

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

EE C245: Introduction to MEMS Design LecM 13 C. Nguyen 11/18/08 9