

### MEMS-Based Tuning Fork Gyroscope

- Drive and sense axes must be stable or at least track one another to avoid output drift

**Problem:** if drive frequency changes relative to sense frequency, output changes  $\Rightarrow$  bias drift

**Need:** small or matched drive and sense axis temperature coefficients to suppress drift

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### Mode Matching for Higher Resolution

- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification

**Problem:** mismatch between drive and sense frequencies  $\Rightarrow$  even larger drift!

**Need:** small or matched drive and sense axis temperature coefficients to make this work

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### Issue: Zero Rate Bias Error

- Imbalances in the system can lead to zero rate bias error

**Mass imbalance  $\Rightarrow$  off-axis motion of the proof mass**

**Drive imbalance  $\Rightarrow$  off-axis motion of the proof mass**

**Output signal in phase with the Coriolis acceleration**

**Quadrature output signal that can be confused with the Coriolis acceleration**

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### Nuclear Magnetic Res. Gyroscope

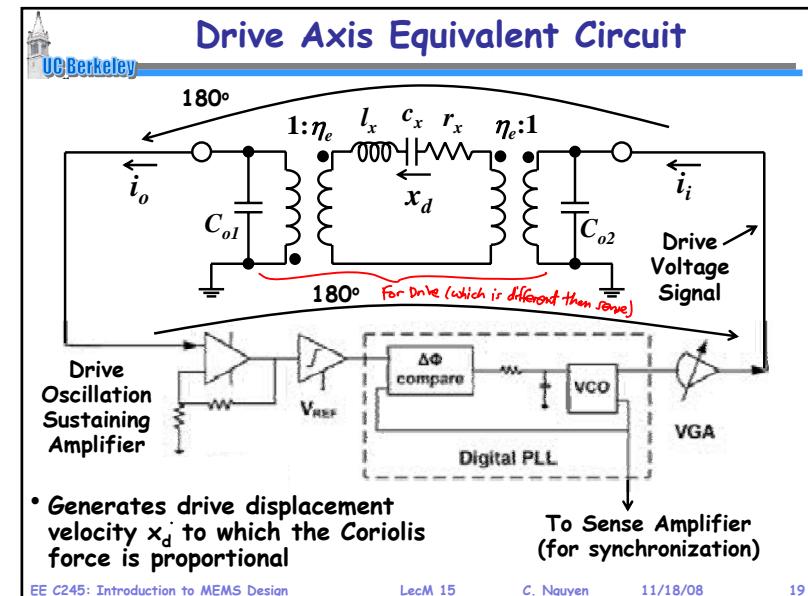
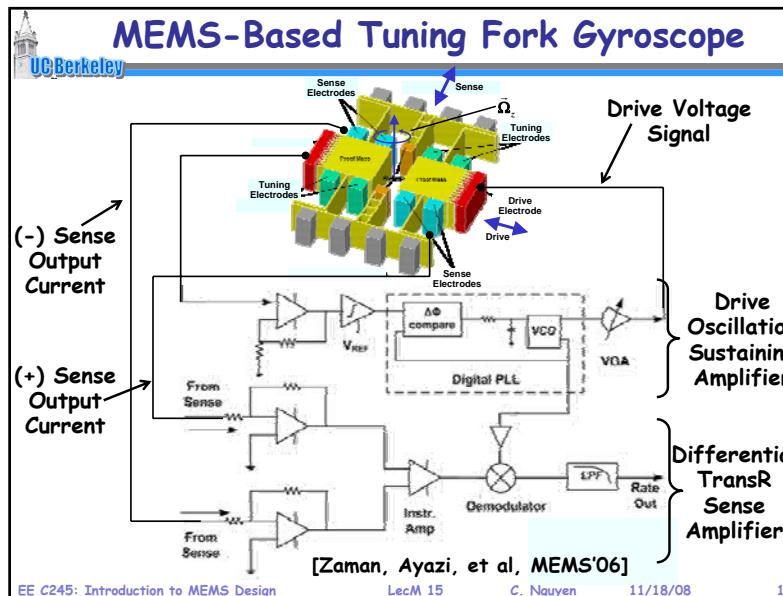
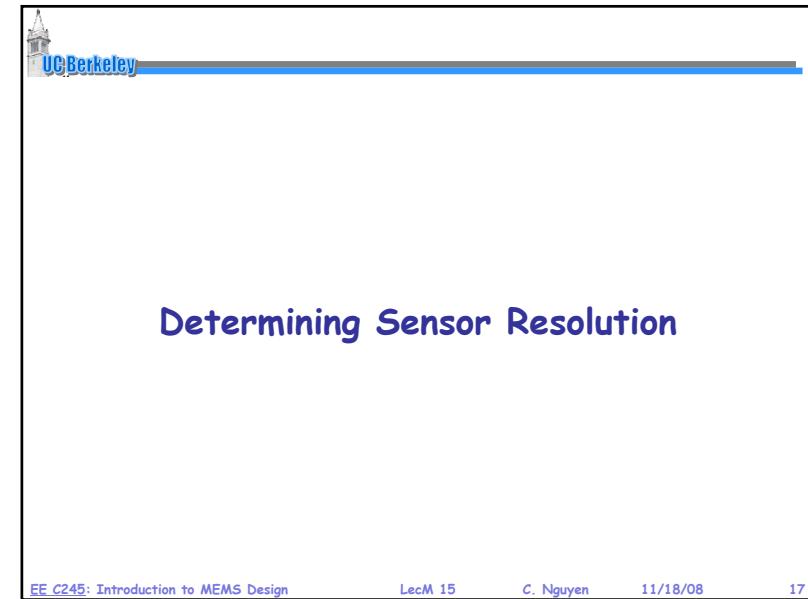
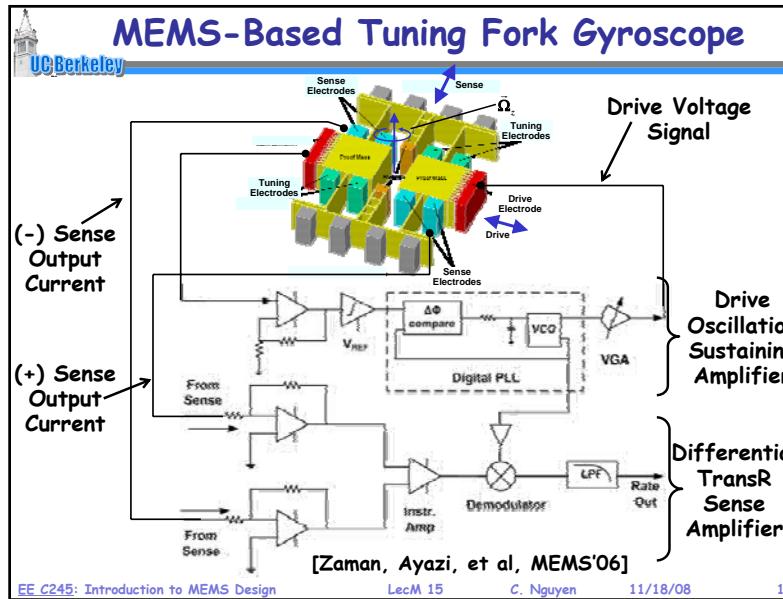
- The ultimate in miniaturized spinning gyroscopes?
- from CSAC, we may now have the technology to do this

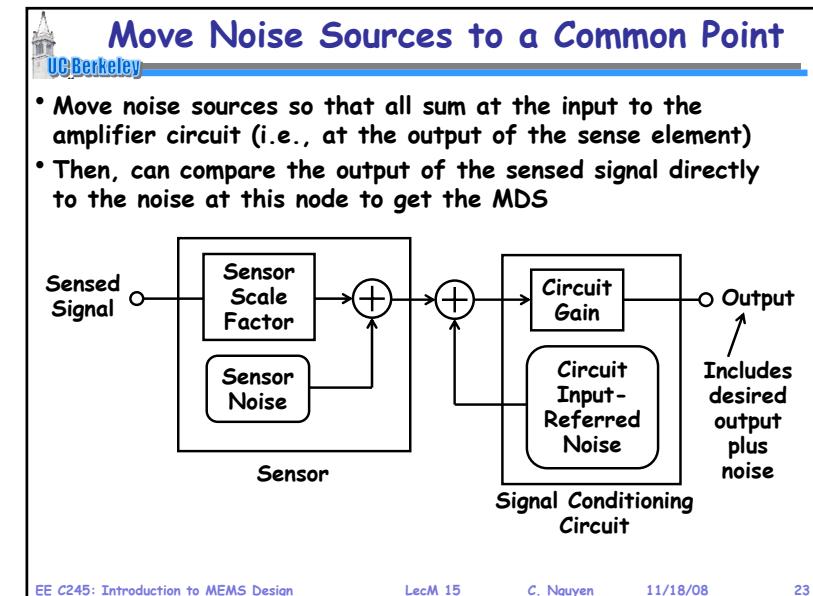
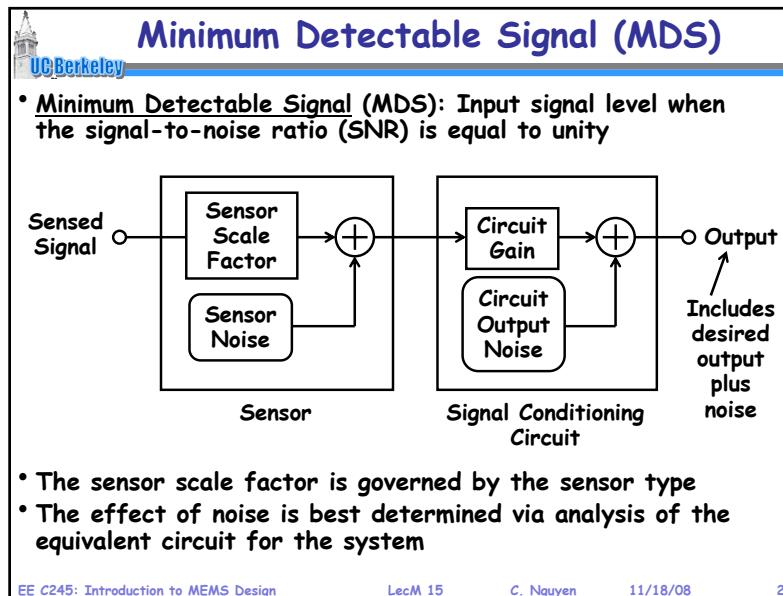
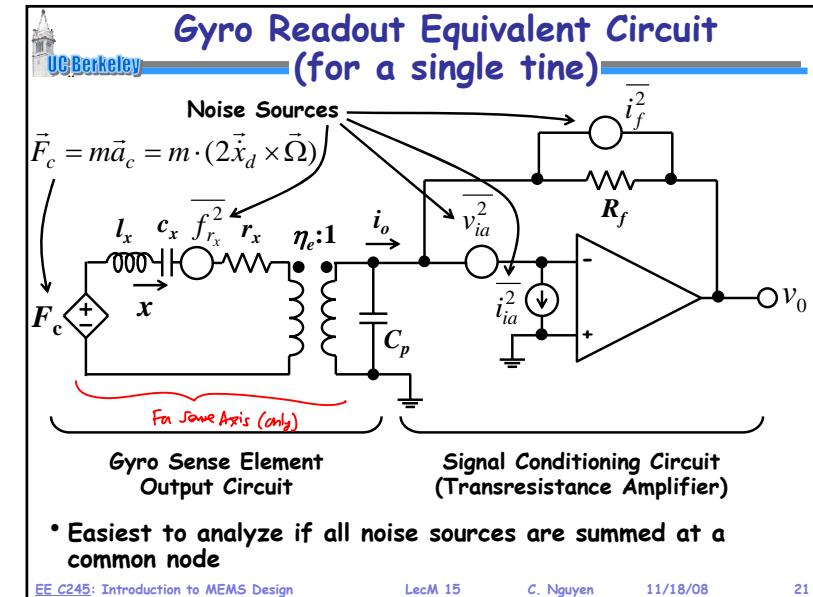
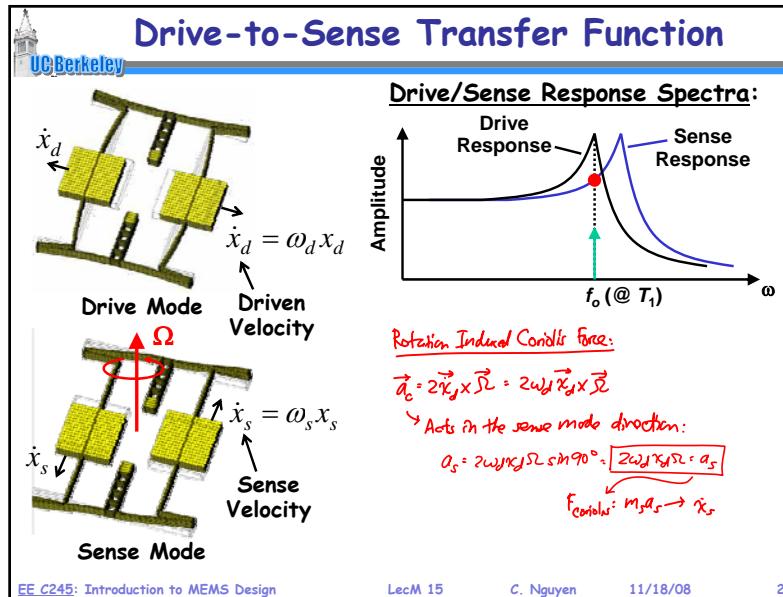
Better if this is a noble gas nucleus (rather than e-), since nuclei are heavier  $\Rightarrow$  less susceptible to B field

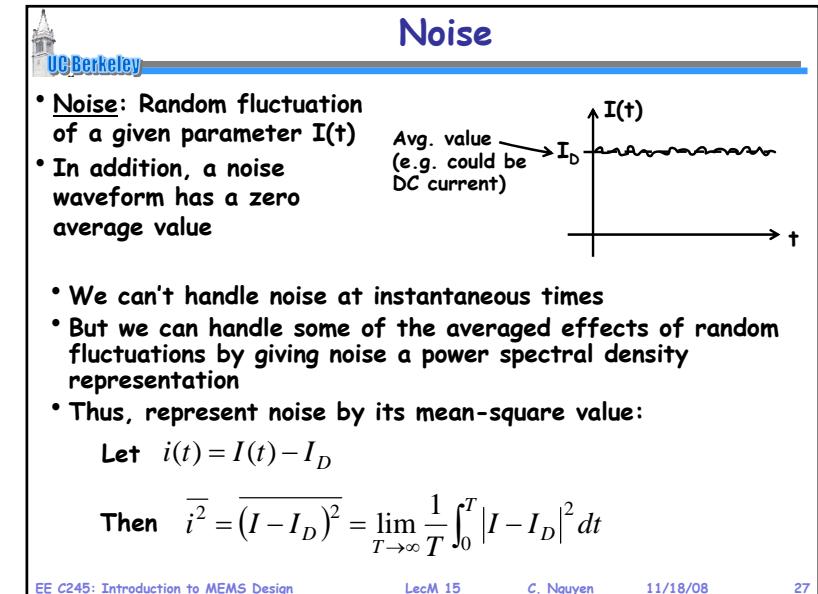
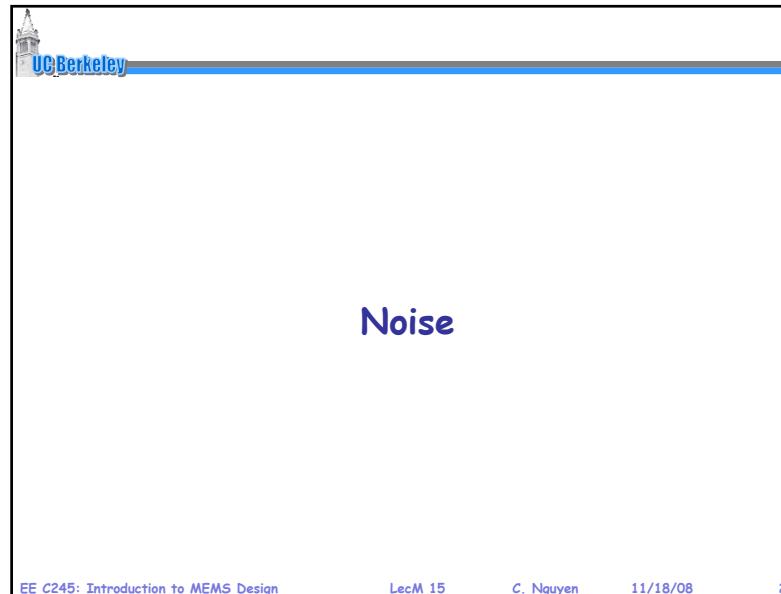
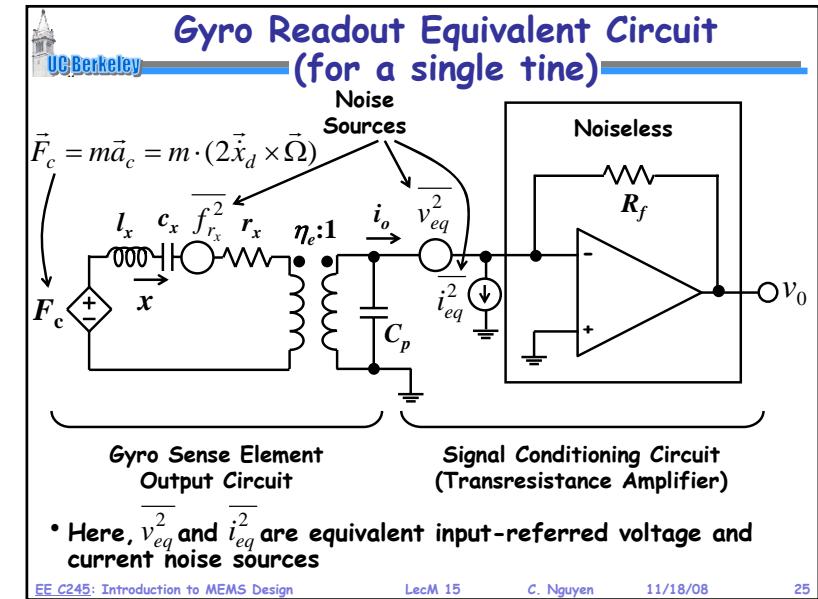
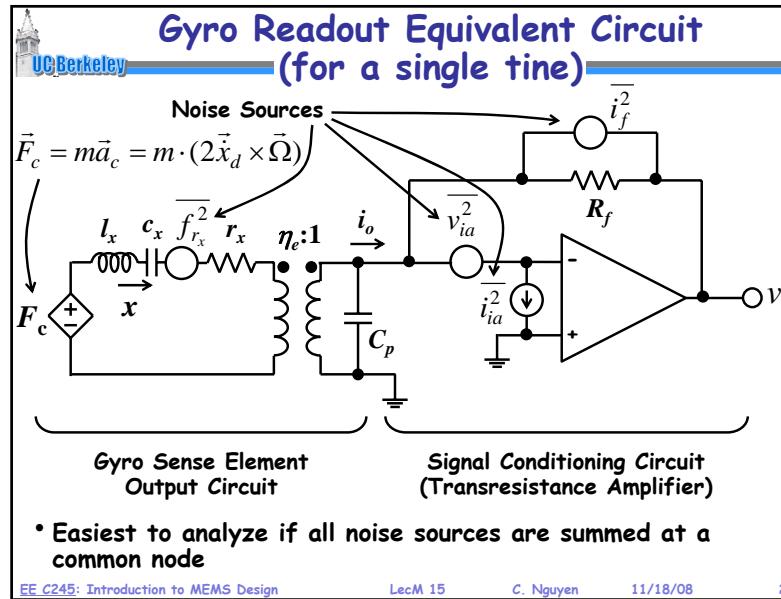
Soln: Spin polarize  $Xe^{129}$  nuclei by first polarizing e- of  $Rb^{87}$  (a la CSAC), then allowing spin exchange

Challenge: suppressing the effects of B field

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## Noise Spectral Density

- We can plot the spectral density of this mean-square value:

The graph shows a blue curve representing the two-sided spectral density  $\overline{i^2} / \Delta f$  [units<sup>2</sup>/Hz]. A shaded area under the curve between frequencies  $\omega_1$  and  $\omega_2$  is labeled "One-sided spectral density → used in circuits → measured by spectrum analyzers". Another shaded area under the curve from  $\omega_1$  to infinity is labeled "Two-sided spectral density (1/2 the one-sided) Often used in systems courses". A label indicates  $\overline{i^2}$  = integrated mean-square noise spectral density over all frequencies (area under the curve).

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## Circuit Noise Calculations

The diagram shows a Linear Time-Invariant System (LTIS) with inputs  $v_i(j\omega)$  and  $S_i(\omega)$  and outputs  $v_o(j\omega)$  and  $S_o(\omega)$ . The system is labeled "Deterministic" and "Random". The output voltage  $v_o(t)$  is shown as a sinusoidal wave, and its mean-square spectral density  $S_o(j\omega)$  is shown as a red curve. The text "No, j → noise has random phase, so j is pointers!" is written near the output.

- Deterministic:  $v_o(j\omega) = H(j\omega)v_i(j\omega)$
- Random:  $S_o(\omega) = [H(j\omega)H^*(j\omega)]S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$   
 $\sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)}$  — How is it we can do this?

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## Handling Noise Deterministically

- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

The graph shows a red curve  $\frac{v_{nl}^2}{\Delta f} = S_1(f) \rightarrow v_{nl} = \sqrt{S_1(f) \cdot B}$  approximated by a sinusoidal voltage generator with amplitude  $A|\cos \omega_o t|$  and period  $\tau \sim \frac{1}{B}$ .

The circuit diagram shows a noise source  $S_n(j\omega)$  at frequency  $\omega_o$  connected to an op-amp. The op-amp output is fed into a noise source  $S_i$  with bandwidth  $B$ , which is then connected to a load. The output voltage  $v_o(t)$  is a sinusoidal wave.

[This is actually the principle by which oscillators work → oscillators are just noise going through a tiny bandwidth filter] Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period  $1/B$ .

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## Systematic Noise Calculation Procedure

The diagram shows a general circuit with several noise sources:  $v_{n1}^2$ ,  $i_{n1}^2$ ,  $v_{n2}^2$ ,  $i_{n2}^2$ ,  $v_{n3}^2$ ,  $i_{n3}^2$ ,  $v_{n4}^2$ ,  $i_{n4}^2$ ,  $v_{n5}^2$ ,  $i_{n5}^2$ , and  $v_{on}$ . The output voltage is  $v_o^2$  and the transfer function is  $H_1(j\omega)$ .

- Assume noise sources are uncorrelated
- For  $i_{n1}^2$  replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{i_{n1}^2}{\Delta f} \cdot (1 \text{ Hz})}$$

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### Systematic Noise Calculation Procedure

2. Calculate  $v_{on1}(\omega) = i_{n1}(\omega)H(j\omega)$  (treating it like a deterministic signal)
3. Determine  $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H(j\omega)|^2$
4. Repeat for each noise source:  $\overline{i_{n1}^2}, \overline{v_{n2}^2}, \overline{v_{n3}^2}$
5. Add noise power (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots}$$

Total rms value

### Determining Sensor Resolution

### Example: Gyro MDS Calculation

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{x}_d \times \vec{\Omega})$

- The gyro sense presents a large effective source impedance
  - Currents are the important variable; voltages are "opened" out
- Must compare  $i_o$  with the total current noise  $i_{eqTOT}$  going into the amplifier circuit

### Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{x}_d \times \vec{\Omega})$

- First, find the rotation to  $i_o$  transfer function:

$$\dot{x}_s = \frac{w_s Q}{k_s} \Theta_s(j\omega_d) F_s = \frac{w_s Q}{k_s} \cdot 2\omega_d x_d \Sigma m \cdot \Theta(j\omega_d)$$

$F_s = F_c = 2\omega_d x_d \Sigma m$

$$\dot{x}_s = 2 \frac{\omega_d}{w_s} Q x_d \Theta(j\omega_d) \cdot \Sigma$$

### Example: Gyro MDS Calculation (cont)

$i_o = \eta_e \dot{x}_s = 2 \frac{w_d}{w_s} Q \chi_d \eta_e \Theta(j\omega_d) \cdot \dot{x}_s \rightarrow i_o = A \dot{x}_s$   
 $A \triangleq \text{scale factor}$

When  $\dot{x}_s = \dot{x}_{s,\min} \triangleq \text{MDS}$ ,  $i_o = i_{eq,TOT}$  input-referred noise current entering the sense amplifier  $\rightarrow$  in  $\mu\text{A}/\sqrt{\text{Hz}}$

$$\therefore i_{eq,TOT} = A \dot{x}_{s,\min} \rightarrow \dot{x}_{s,\min} = \frac{i_{eq,TOT}}{A} \left( \frac{3600\pi}{\text{hr}} \right) \left( \frac{180^\circ}{\pi} \right) \left[ (\%/\text{hr}) / \sqrt{\text{Hz}} \right]$$

Angle Random Walk: ARW =  $\frac{1}{60} \dot{x}_{s,\min} [\%/\text{hr}]$

Earlier to determine directional error as a function of elapsed time.

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### Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m \vec{a}_c = m \cdot (2 \vec{x}_d \times \vec{\Omega})$

$R_s$ : large  $\therefore \overline{i_{eq}^2}$  "opened" out

- Now, find the  $i_{eq,TOT}$  entering the amplifier input:

$$i_{eq,TOT} = i_s + i_{eq} \rightarrow \overline{i_{eq,TOT}^2} = \overline{i_s^2} + \overline{i_{eq}^2} + \overline{i_{ia}^2} + \frac{\overline{N_{ia}^2}}{R_f^2} \quad \overline{f_{rx}^2} = 4kT r_x$$

Brownian motion noise of the sense element  $\rightarrow$  determined entirely by the noise in  $r_x \rightarrow \overline{f_{rx}^2}$

easiest to convert to an all electrical equiv ckt.

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### Example: Gyro MDS Calculation (cont)

where  $L_{rx} = \frac{r_x}{\eta_e^2}$ ,  $C_{rx} = \eta_e^2 C_x$ ,  $R_{rx} = \frac{r_x}{\eta_e^2}$

$$\therefore i_s = N_{Rx} \left( \frac{1}{R_{rx}} \right) \Theta(j\omega_d) \rightarrow \frac{\overline{i_s^2}}{\Delta f} = 4kT R_x \left( \frac{1}{R_{rx}^2} \right) |\Theta(j\omega_d)|^2$$

$$\Rightarrow \frac{\overline{i_s^2}}{\Delta f} = \frac{4kT}{R_{rx}^2} |\Theta(j\omega_d)|^2$$

Thus:

$$\frac{\overline{i_{eq,TOT}^2}}{\Delta f} = \frac{4kT}{R_{rx}^2} |\Theta(j\omega_d)|^2 + \frac{4kT}{R_f^2} + \frac{i_{ia}^2}{\Delta f} + \frac{N_{ia}^2}{\Delta f} \left( \frac{1}{R_f^2} \right)$$

Learn to get these from EE240.  
 or just get them from a data sheet ...

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### LF356 Op Amp Data Sheet

**LF155/LF156/LF256/LF257/LF355/LF356/LF357 JFET Input Operational Amplifiers**

**General Description**

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (Bi-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

**Features**

**Advantages**

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

**Uncommon Features**

	LF155/ LF355	LF156/ LF356	LF257/ LF357	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	μs
Fast slew rate	5	12	50	V/μs
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	nV/√Hz

$\frac{i_{ia}^2}{\Delta f} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$

$\frac{N_{ia}^2}{\Delta f} = 12 \text{ nV}/\sqrt{\text{Hz}}$

### Example ARW Calculation

**UC Berkeley**

- Example Design:**
  - Sensor Element:**
 $m = (100\mu\text{m})(100\mu\text{m})(20\mu\text{m})(2300\text{kg/m}^3) = 4.6 \times 10^{-10}\text{kg}$ 
 $\omega_s = 2\pi(15\text{kHz})$ 
 $\omega_d = 2\pi(10\text{kHz})$ 
 $k_s = \omega_s^2 m = 4.09 \text{ N/m}$ 
 $x_d = 20 \mu\text{m}$ 
 $Q_s = 50,000$ 
 $V_p = 5\text{V}$ 
 $h = 20 \mu\text{m}$ 
 $d = 1 \mu\text{m}$
  - Sensing Circuitry:**
 $R_f = 100\text{k}\Omega$ 
 $i_{ia} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$ 
 $v_{ia} = 12 \text{ nV}/\sqrt{\text{Hz}}$

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### Example ARW Calculation (cont)

**UC Berkeley**

Get rotation rate to output current scale factor:

$$A = 2 \frac{\omega_d}{\omega_s} Q_s \chi_d \eta_e |\Theta(j\omega_d)| = 2 \left( \frac{10\text{K}}{15\text{K}} \right) (50\mu\text{})(20\mu\text{})(5)(2000\epsilon_0)(0.00024) = 2.83 \times 10^{-12} \text{C}$$

$$\Theta(j\omega_d) = \frac{(j\omega_d)(\omega_s/\omega_s)}{-\omega_d^2 + j\omega_d\omega_s + \omega_s^2} = \frac{j(10\text{K})(15\text{K})/(50\text{K})}{(15\text{K})^2 - (10\text{K})^2 + j(10\text{K})(10\text{K})/50\text{K}} = \frac{j(3\text{K})}{1.25 \times 10^8 + j(3\text{K})}$$

$$\Rightarrow |\Theta(j\omega_d)| = \frac{3\text{K}}{\sqrt{(1.25 \times 10^8)^2 + (3\text{K})^2}} = 0.000024 \quad 8.854 \times 10^{-8} \text{F/m}$$

$$\left[ \frac{\partial C}{\partial x} = \frac{C_0}{d} = \frac{\epsilon_0 h V_p}{d} = \frac{\epsilon_0 (20\mu\text{})(100\mu\text{})}{(1\mu\text{})^2} = 2000\epsilon_0 \rightarrow \eta_e = V_p \frac{\partial C}{\partial x} = 5(2000\epsilon_0) \quad 8.854 \times 10^{-12} \text{F/m} \right]$$

Assume electrode covers the whole sidewall.

Then, get noise:

$$\frac{i_{eq,TOT}^2}{\Delta f} = \frac{4kT}{R_f} |\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{\Delta f} + \frac{V_{ia}^2}{\Delta f} \left( \frac{1}{R_f^2} \right)$$

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### Example ARW Calculation (cont)

**UC Berkeley**

$$R_{eq} = \frac{w_s m}{Q_s \eta_e^2} = \frac{2\pi(15\text{K})(4.6 \times 10^{-10})}{(50\text{K})(8.854 \times 10^{-8})^2} = 110.6 \text{k}\Omega$$

$$\frac{i_{eq,TOT}^2}{\Delta f} = \frac{(1.66 \times 10^{-29})}{(110.6\text{K})} (0.000024)^2 + \frac{(1.66 \times 10^{-29})}{1\text{M}} + (0.01\text{p})^2 + \frac{(12\text{n})^2}{(1\text{M})^2}$$

$$\xrightarrow{\text{Sensor element noise insignificant}} 8.64 \times 10^{-25} \text{A}^2/\text{Hz} \quad \xrightarrow{\text{Noise from } R_f \text{ dominates!}} 1.66 \times 10^{-26} \text{A}^2/\text{Hz} \quad \xrightarrow{\text{Noise from } R_f \text{ dominates!}} 1.44 \times 10^{-28} \text{A}^2/\text{Hz}$$

$$\therefore \frac{i_{eq,TOT}^2}{\Delta f} = 1.68 \times 10^{-26} \text{A}^2/\text{Hz} \rightarrow i_{eq,TOT} = \sqrt{\frac{i_{eq,TOT}^2}{\Delta f}} = 1.30 \times 10^{-13} \text{A}/\sqrt{\text{Hz}}$$

$$\therefore \sigma_{min} = \frac{i_{eq,TOT}}{A} \left( \frac{3600\pi}{\text{hr}} \right) \left( \frac{180^\circ}{\pi} \right) = \frac{1.30 \times 10^{-13}}{2.83 \times 10^{-12}} (3600) \left( \frac{180}{\pi} \right) = 9448 (\%/\text{hr})/\sqrt{\text{Hz}}$$

And finally:

$$\text{ARW} = \frac{1}{60} \sigma_{min} = \frac{1}{60} (9448) = 157 \%/\text{hr} = \text{ARW} \quad \Rightarrow \text{Almost turned around in 1 hour!}$$

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### What if $\omega_d = \omega_s$ ?

**UC Berkeley**

If  $\omega_d = \omega_s = 15\text{kHz}$ , then  $|\Theta(j\omega_d)| = 1$  and

$$A = 2 \frac{\omega_d}{\omega_s} Q_s \chi_d \eta_e = 2 Q_s \chi_d \eta_e = 2(50\mu\text{})(20\mu\text{})(5)(2000\epsilon_0) = 1.77 \times 10^{-7} \text{C}$$

$$\frac{i_{eq,TOT}^2}{\Delta f} = \frac{(1.66 \times 10^{-29})}{(110.6\text{K})} (1)^2 + \frac{(1.66 \times 10^{-29})}{1\text{M}} + (0.01\text{p})^2 + \frac{(12\text{n})^2}{(1\text{M})^2}$$

$$\xrightarrow{\text{Now, the sensor element dominates!}} 1.51 \times 10^{-25} \text{A}^2/\text{Hz} \quad 1.66 \times 10^{-26} \text{A}^2/\text{Hz} \quad 1 \times 10^{-28} \text{A}^2/\text{Hz} \quad 1.44 \times 10^{-28} \text{A}^2/\text{Hz}$$

$$\therefore \frac{i_{eq,TOT}^2}{\Delta f} = 1.67 \times 10^{-25} \text{A}^2/\text{Hz} \rightarrow i_{eq,TOT} = \sqrt{\frac{i_{eq,TOT}^2}{\Delta f}} = 4.08 \times 10^{-13} \text{A}/\sqrt{\text{Hz}}$$

$$\therefore \sigma_{min} = \frac{i_{eq,TOT}}{A} \left( \frac{3600\pi}{\text{hr}} \right) \left( \frac{180^\circ}{\pi} \right) = \frac{4.08 \times 10^{-13}}{1.77 \times 10^{-7}} (3600) \left( \frac{180}{\pi} \right) = 0.476 (\%/\text{hr})/\sqrt{\text{Hz}}$$

And finally:

$$\text{ARW} = \frac{1}{60} \sigma_{min} = \frac{1}{60} (0.476) = 0.0079 \%/\text{hr} = \text{ARW} \quad \Rightarrow \text{Navigation grade!}$$

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