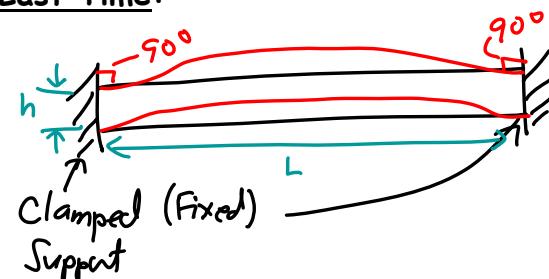


Lecture 3: Benefits of Scaling II

- Announcements:
- Discussion Time Change?
↳ 5-6: still pending
- Lecture room change: inquired
- -----

Today:

- Reading: Senturia, Chapter 1
- Lecture Topics:
 - ↳ Benefits of Miniaturization
 - ↳ Examples
 - GHz micromechanical resonators
 - Chip-scale atomic clock
 - Micro gas chromatograph
- -----

Last Time:

\Rightarrow Eq. for Resonance Freq.:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 1.03 \sqrt{\frac{E}{\rho}} \frac{h}{L^2} \quad (1)$$

where $E \triangleq$ Young's modulus [GPa]

$\rho \triangleq$ density [kg/m^3]

$h \triangleq$ thickness [m] $L \triangleq$ length [m]

Example: $L = 40 \mu\text{m}$, $h = 2 \mu\text{m}$

polysi $\rightarrow E = 150 \text{ GPa}$, $\rho = 2300 \text{ kg/m}^3$

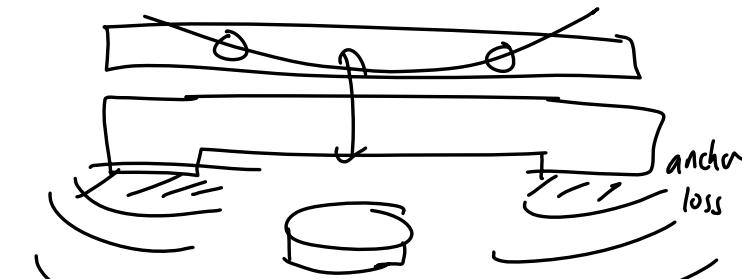
$$\therefore f_0 = (1.03) \sqrt{\frac{150 \text{ G}}{2300}} \frac{2 \mu\text{m}}{(40 \mu\text{m})^2} \rightarrow f_0 = 10.4 \text{ MHz}$$

acoustic velocity = $3,076 \text{ m/s}$

Get half a freq.

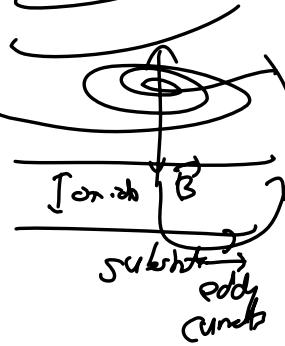
$$\Rightarrow \text{scale } L \rightarrow L = 4 \mu\text{m} \rightarrow f_0 = 1.04 \text{ GHz!}$$

But see remark!



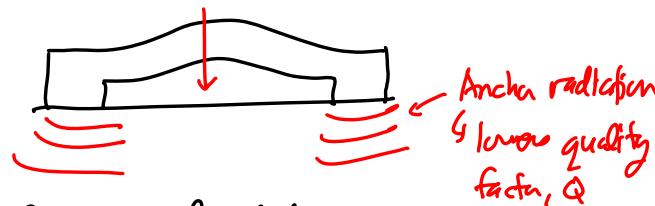
Anchor loss \rightarrow Bad!

- Solv.
- ① Δ the anchors
 - ② Δ the geometry
 - ③ Scale all dimensions!



Remarks.

- ① Eq. (1) not accurate when $L \approx h$. (must take into account rotary inertias)
- ② When $L \approx h$ (or when it isn't more than $10 \times h$), anchor losses become an issue



- ③ Solution: scale all dimensions

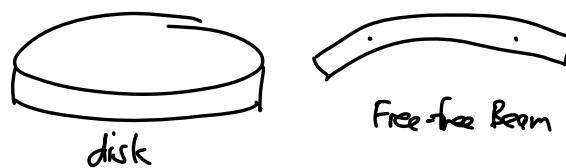
$$(i) \text{ Scale only } L \text{ by } S: f_0 \sim \frac{1}{S^2}$$

(ii) scale all dimensions (L and h):

$$f_0 \sim \frac{h}{L^2} \sim \frac{S}{S^2} \sim \frac{1}{S}$$

Stiffness increases w/ smaller dims, although more slowly

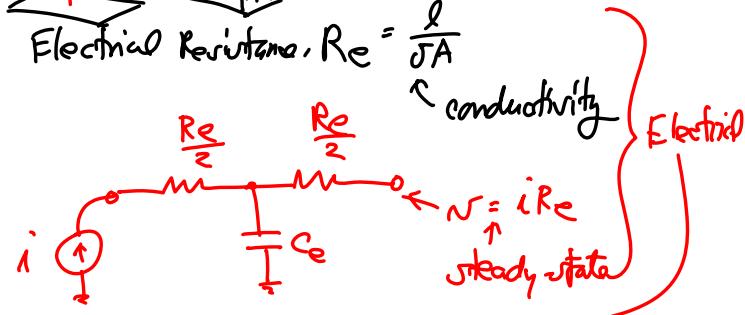
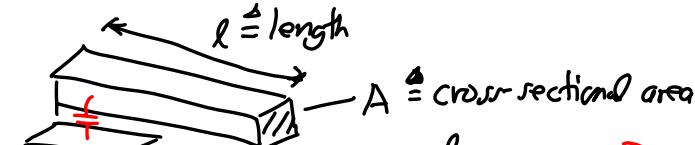
- ④ Better Solution: other geometries



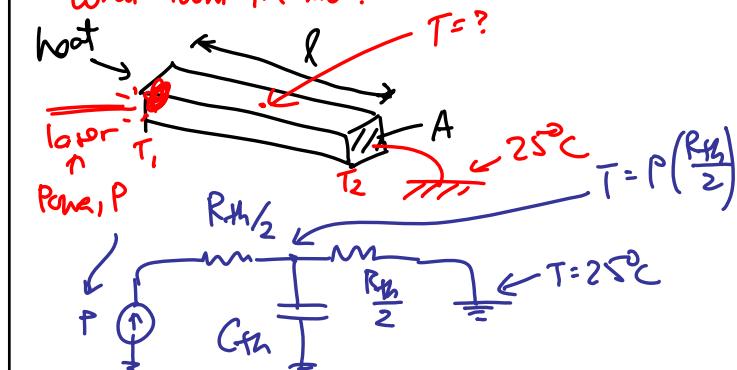
- HW#1 distributed today

- Go through Lecture Module 2, slides 6-26

Thermal Ckt. Modeling



What about thermal?



Thermal Resistance, R_{th} :

$$R_{th} = \frac{l}{kA} \leftarrow \begin{array}{l} \text{length} \\ \text{cross-sectional area} \\ \text{thermal conductivity} \leftarrow \text{associated w/ the material} \end{array}$$

