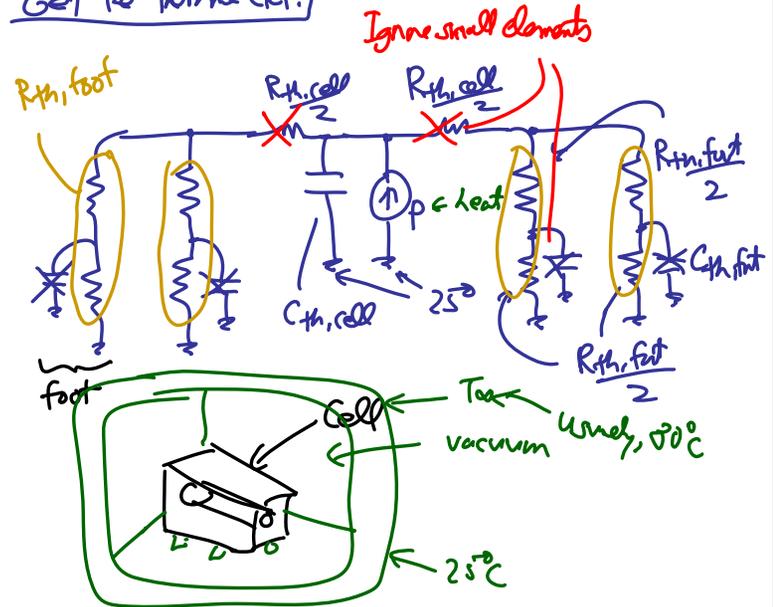
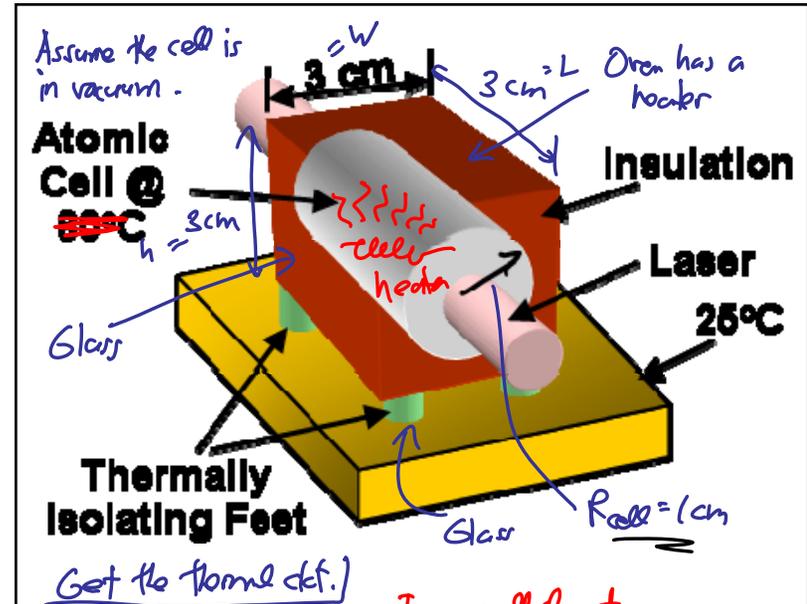
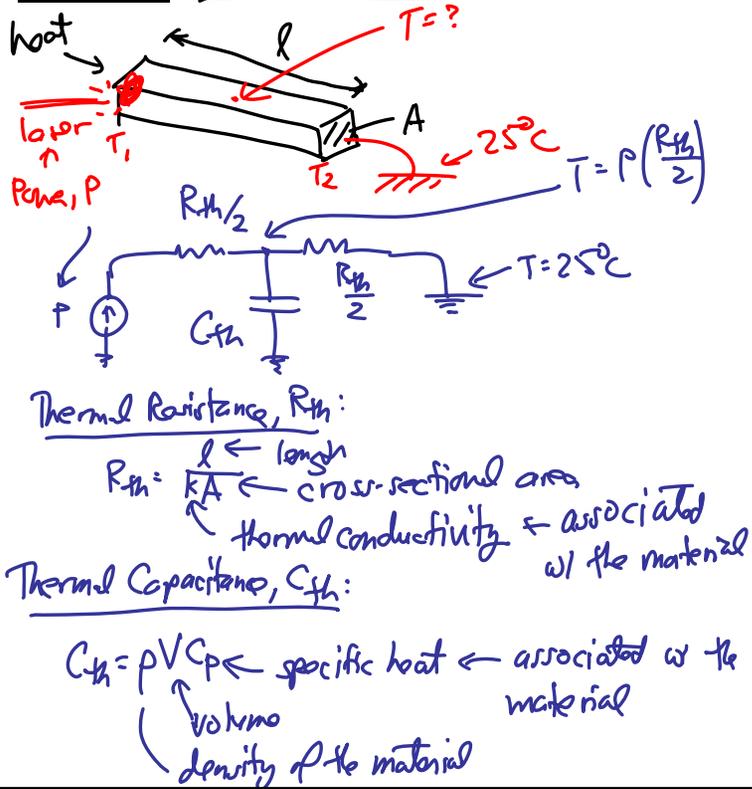


- **Announcements:**
- **Discussion Time Change?**
 ↪ 5-6: still pending

-
- **Today:**
 - **Reading:** Senturia, Chapter S11.1-11.6
 - **Lecture Topics:**
 ↪ Benefits of Miniaturization
 ↪ Thermal Circuits

• **Last Time:** Thermal Ckt. Modeling



Reduce the thermal ckt. further:

@ $t = \infty$: $\Delta T = T_1 - T_0 = P \left(\frac{R_{th,foot}}{4} \right)$

⇒ When power is switched on:

$\Delta T = \Delta T_{\infty} (1 - e^{-\frac{(t-t_0)}{\tau}})$

$\tau = \left(\frac{R_{th,foot}}{4} \right) C_{th,cell}$

Same How Thermal Ckt. Examp.:

laser → deliver power (heat) to the plate

Example 2)

$P_1 = I_1^2 R_{e1}$
 $P_3 = I_3^2 R_{e3}$

Draw the thermal ckt.

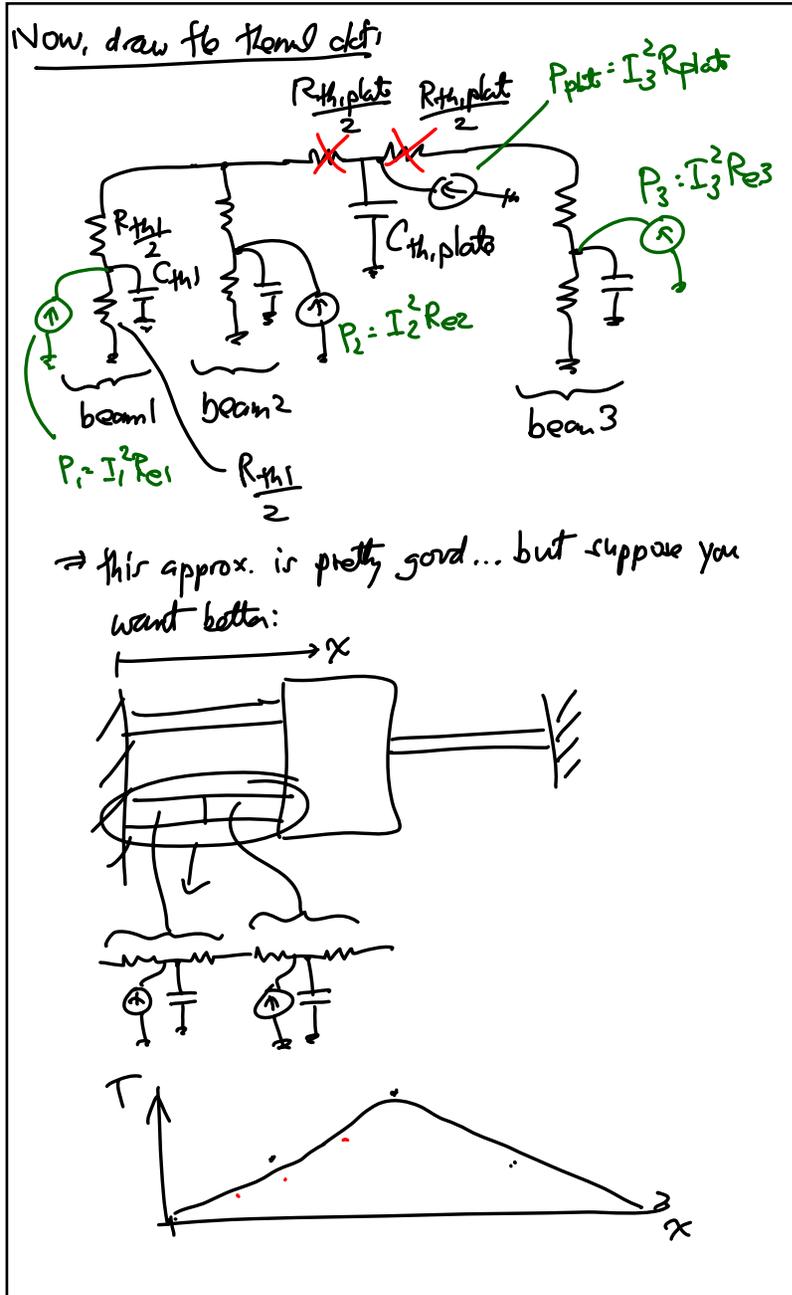
Electrical Gnd

resistivity \downarrow
 $\text{electrical resistance} = \frac{l_2}{\sigma A_2} = \frac{l_2}{\sigma W_2 h_2} = \frac{\rho}{h_2} \frac{l_2}{W_2}$
 width \uparrow thickness \uparrow

$R_D = \text{sheet resistance}$

Squares flow are $\frac{l_2}{W_2}$ of them

$R_{e2} = \left(\frac{l_2}{W_2} \right) (R_D)$



Plug in numbers:

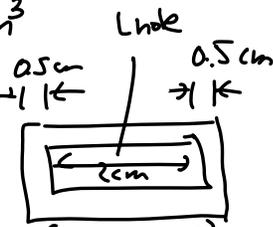
Assumptions: (for the atomic cell)

① Whole thing is glass \rightarrow feed it cell

$$C_{p,glass} = 0.5 \text{ J/(g}\cdot\text{K)}$$

$$\rho_{glass} = 2500 \text{ kg/m}^3$$

$$k_{glass} = 1.05 \frac{\text{W}}{\text{m}\cdot\text{K}}$$



Find $C_{th,cell}$:

\Rightarrow Find the volume of the cell:

$$V_{cell} = hWL - \pi R_{cell}^2 L_{hole}$$

$$= (3 \text{ cm})(3 \text{ cm})(3 \text{ cm}) - \pi (1 \text{ cm})^2 (2 \text{ cm})$$

$$= 20.7 \text{ cm}^3$$

(again, ignore the $R_{th,cell}$)

$$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$$

$$= (2500 \frac{\text{kg}}{\text{m}^3}) (1000 \frac{\text{J}}{\text{kg}}) (\frac{1}{100^3} \frac{\text{m}^3}{\text{cm}^3})$$

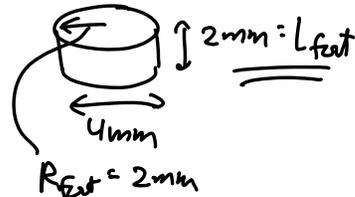
$$\times (20.7 \text{ cm}^3) (0.5 \frac{\text{J}}{\text{g}\cdot\text{K}})$$

$$\Rightarrow C_{th,cell} = \underline{\underline{25.7 \text{ J/K}}}$$

Find $R_{th,foot}$:

⇒ foot dimensions:

$$A_{foot} = \pi R_{foot}^2$$



$$R_{th,foot} = \frac{L_{foot}}{k_{glass} \cdot A_{foot}} = \frac{2\text{mm}}{(1.05 \frac{\text{W}}{\text{m}\cdot\text{K}}) \pi (2\text{mm})^2}$$

$$\Rightarrow R_{th,foot} = 151.6 \text{ K/W}$$

⇒ power required to maintain T₀₀ in steady state:

$$P = \frac{T_{00} - T_0}{(R_{th,foot}/4)} = \frac{(80 - 25)}{37.9} = 1.45 \text{ W}$$

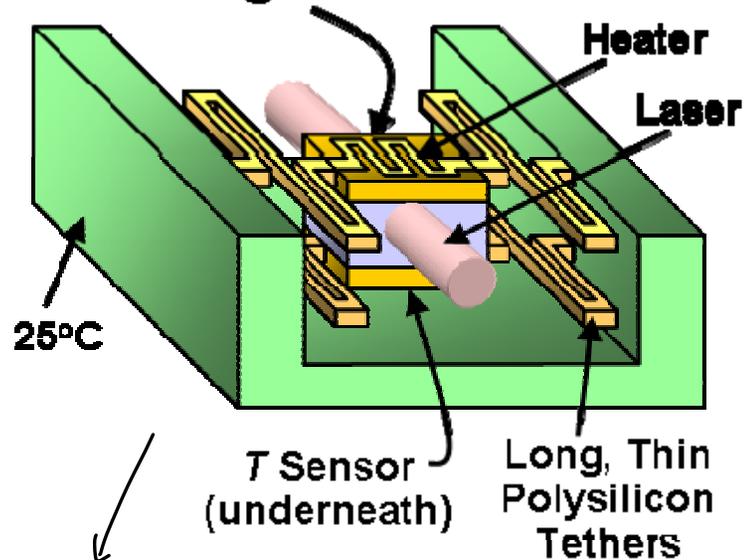
⇒ Find time constant:

$$\tau = \left(\frac{R_{th,foot}}{4} \right) \cdot C_{th,cell} = 16.4 \text{ min}$$

Time req'd to warm up & stabilize will be ~3X this!
∴ must wait ~45 min. before using this clock!

↓
Fix w MEMS!

300x300x300 μm³
Atomic Cell @ 80°C



After the same analysis:

$$C_{th,cell} = 6.31 \times 10^{-6} \text{ J/K} \leftarrow 4 \text{ million X smaller than macro case}$$

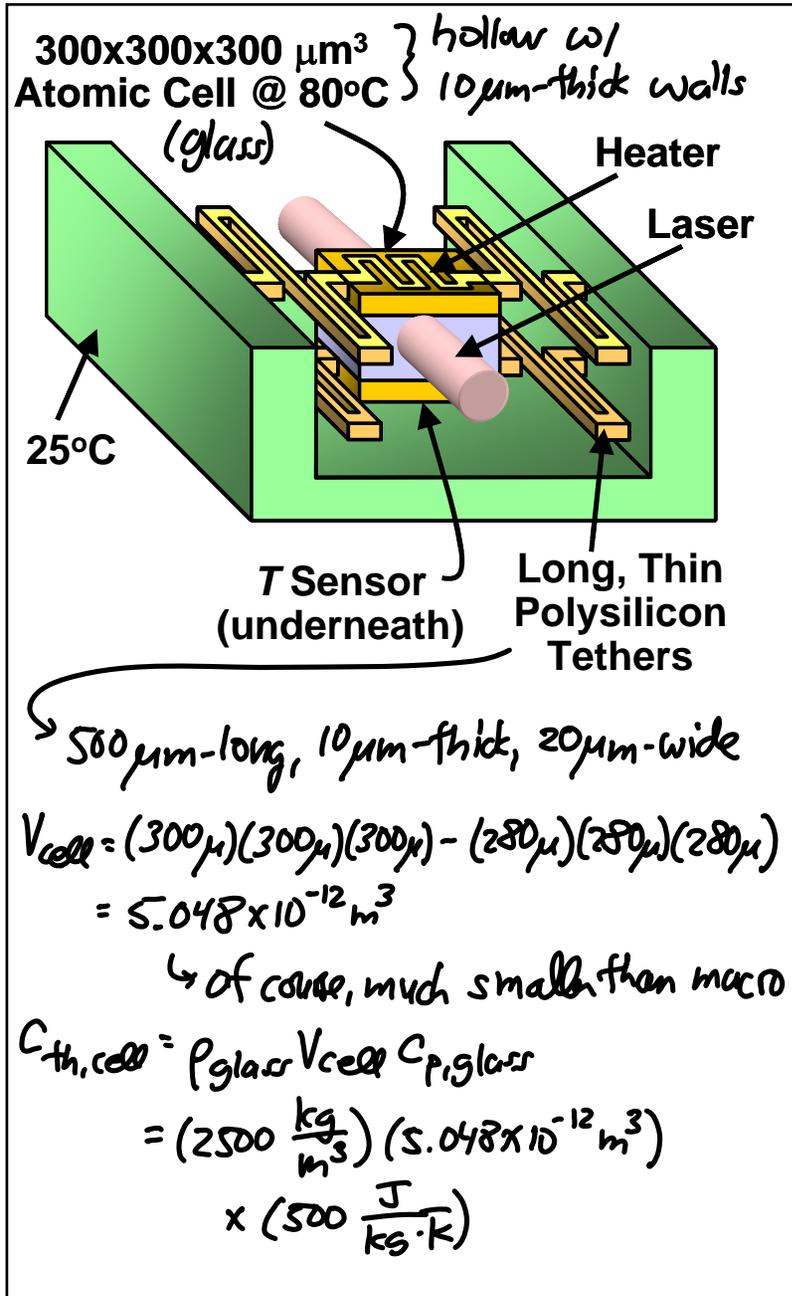
$$R_{th,supp} = 23,333 \text{ K/W} \leftarrow 500X longer than macro$$

$$P = \frac{(80 - 25)}{23.3\text{K}} = 2.64 \text{ mW}$$

$$\tau = 0.13 \text{ s}$$

What makes this possible? → scaling!!!

- ① Scaling reduces the $C_{th} \sim l^3 \sim s^3$ ← scaling factor
- ② Scaling allows the use of long, thin tethers!



$$\Rightarrow C_{\text{th, cell}} = \underline{\underline{6.31 \times 10^{-6} \frac{\text{J}}{\text{K}}}}$$

↳ 4 million x smaller than macro!

$$R_{\text{th, supp}} = \frac{L_{\text{supp}}}{k_{\text{poly}} \cdot W_{\text{supp}} \cdot h_{\text{supp}}}$$

$$= \frac{500\mu}{(30 \frac{\text{W}}{\text{m} \cdot \text{K}})(20\mu)(10\mu)} = \underline{\underline{83,333 \text{ K/W}}}$$

↳ 548x larger

and...

$$P = \frac{(80-25)}{83,333} = \underline{\underline{2.64 \text{ mW}}} \leftarrow 548x \text{ smaller!}$$

$$\tau = \underline{\underline{0.135}} \leftarrow 7300x \text{ faster!}$$

All due to scaling!