



**EE C245 - ME C218**  
**Introduction to MEMS Design**  
**Fall 2009**

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**Lecture Module 15: Gyros, Noise, & MDS**

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**Lecture Outline**

- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - ↳ Gyroscopes
  - ↳ Gyro Circuit Modeling
  - ↳ Minimum Detectable Signal (MDS)
    - Noise
    - Angle Random Walk (ARW)

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## Gyrosopes

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### Classic Spinning Gyroscope

- A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
- Principle of operation based on conservation of momentum
- Example: classic spinning gyroscope

Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbaled chassis

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## Vibratory Gyroscopes

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- Generate momentum by vibrating structures
- Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
- Example: vibrating mass in a rotating frame

Mass at rest

Driven into vibration along the y-axis

y-displaced mass

$C(t)$

$C(t_1)$

$C(t_2)$

Get an  $x'$  component of motion

Rotate 30°

Capacitance between mass and frame = constant

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## Basic Vibratory Gyroscope Operation

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### Principle of Operation

- Tuning Fork Gyroscope:

Side View:  $\dot{x} = \dot{x}$

not free air

Support = 0

very little added dissipation

Top View: Past Present

Input Rotation  $\vec{\Omega}$

Driven Vibration @  $f_o$

Coriolis (Sense) Response  $\vec{a}_c$

Coriolis Torque

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## Basic Vibratory Gyroscope Operation

**Principle of Operation**

- Tuning Fork Gyroscope:

Input Rotation  $\vec{\Omega}$

Driven Vibration @  $f_o$

Coriolis (Sense) Response

Coriolis Force

Coriolis Acceleration

Coriolis Torque

Beam Mass

Beam Stiffness

Sense Frequency (in sense direction)

Driven Velocity

Rotation Rate

Coriolis Displacement

$\vec{a}_c = 2\vec{v} \times \vec{\Omega}$

$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$

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**Drive/Sense Response Spectra:**

Amplitude

Drive Response

Sense Response

$f_o (@ T_1)$

## Vibratory Gyroscope Performance

**Principle of Operation**

- Tuning Fork Gyroscope:

Input Rotation  $\vec{\Omega}$

Driven Vibration @  $f_o$

Coriolis (Sense) Response

Coriolis Force

Coriolis Acceleration

Coriolis Torque

Beam Mass

Beam Stiffness

Sense Frequency

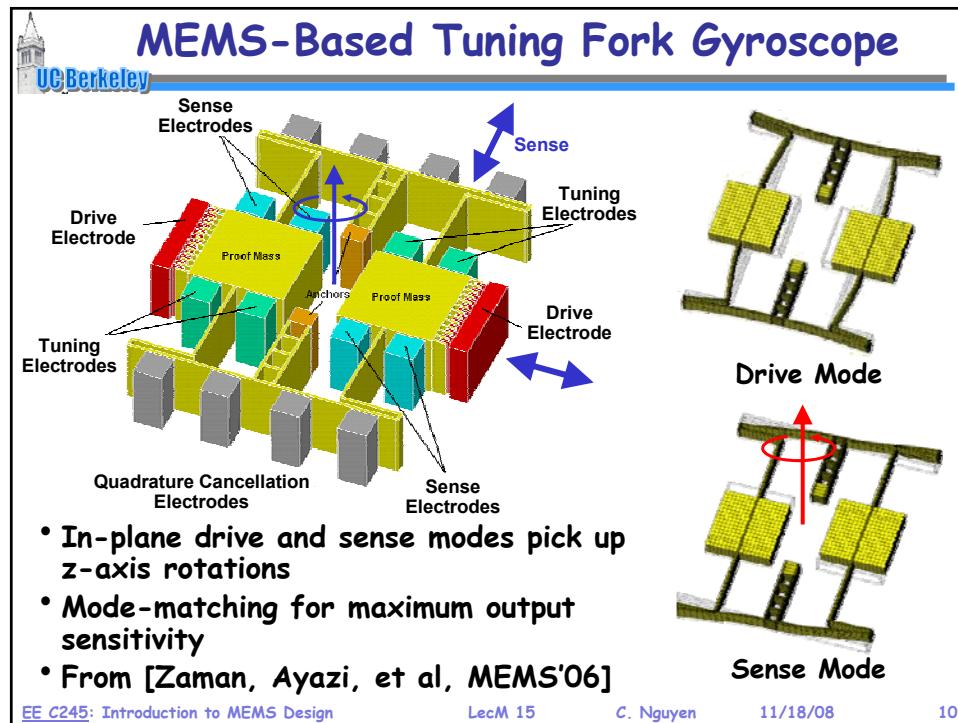
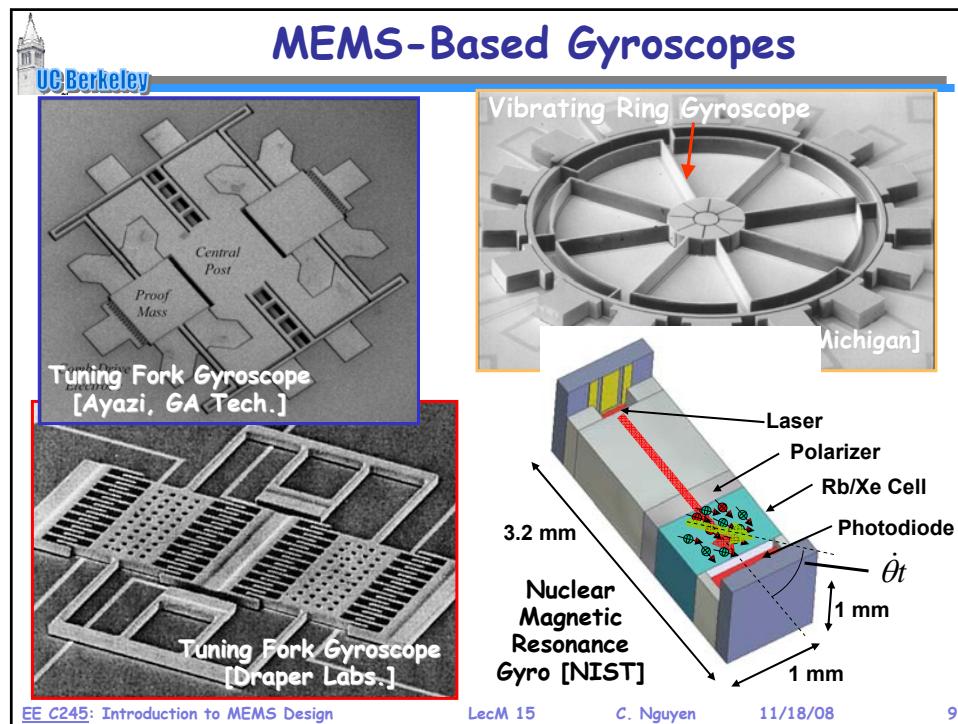
Driven Velocity

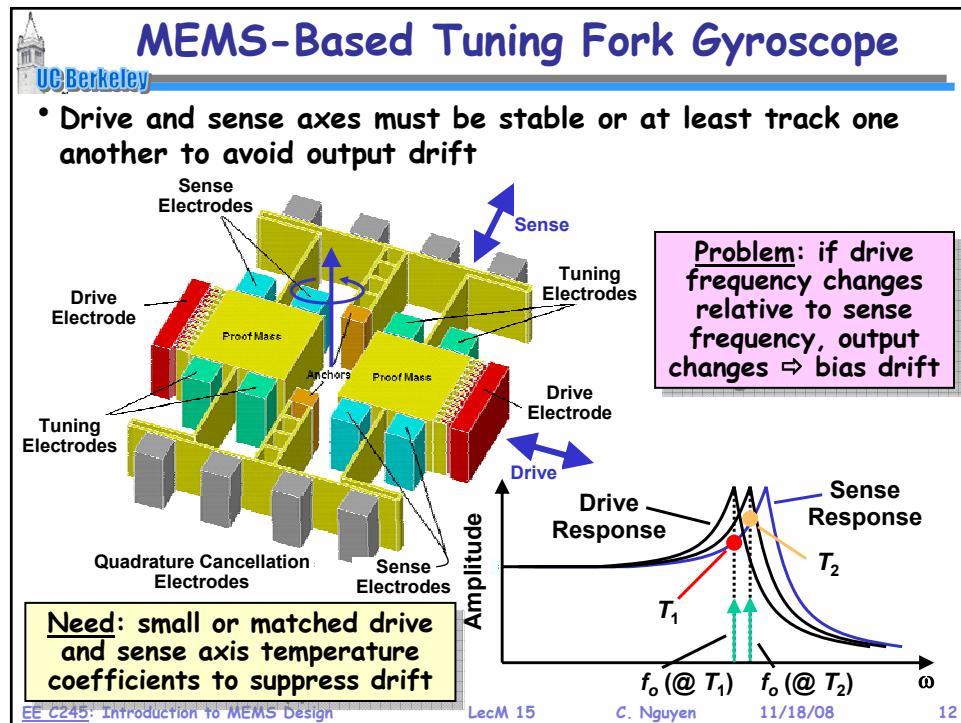
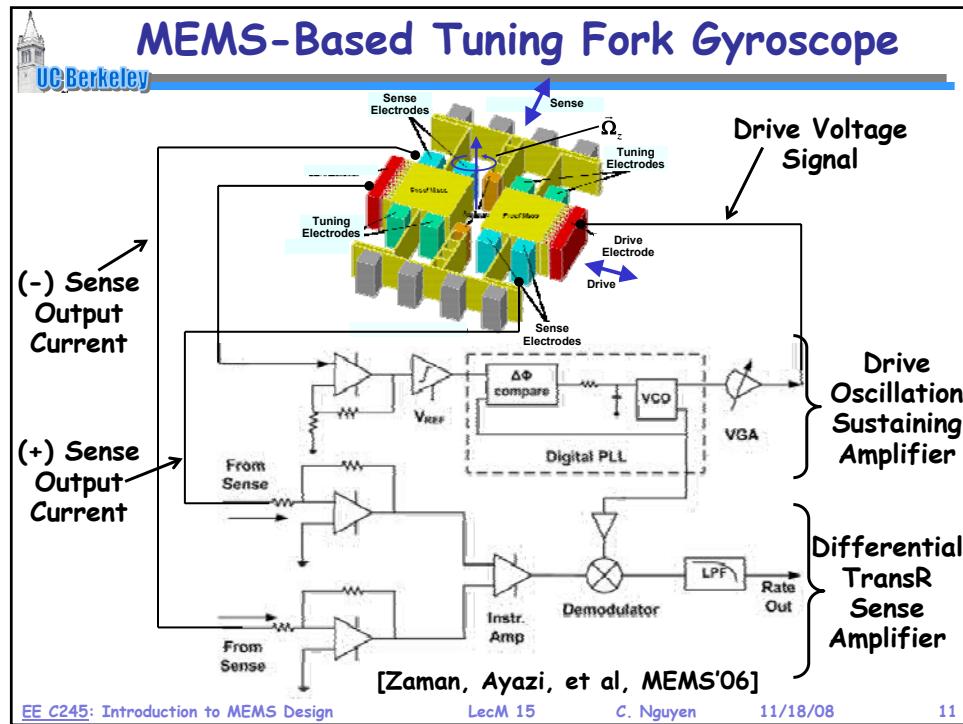
$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$

$\vec{a}_c = 2\vec{v} \times \vec{\Omega}$

- To maximize the output signal  $x$ , need:
  - Large sense-axis mass
  - Small sense-axis stiffness
  - (Above together mean low resonance frequency)
  - Large drive amplitude for large driven velocity (so use comb-drive)
  - If can match drive freq. to sense freq., then can amplify output by Q times

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## Mode Matching for Higher Resolution

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- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification

**Problem:** mismatch between drive and sense frequencies  $\Rightarrow$  even larger drift!

**Need:** small or matched drive and sense axis temperature coefficients to make this work

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## Issue: Zero Rate Bias Error

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- Imbalances in the system can lead to zero rate bias error

**Drive imbalance  $\Rightarrow$  off-axis motion of the proof mass**

**Sense Electrodes**

**Quadrature Cancellation Electrodes**

**Tuning Electrodes**

**Proof Mass**

**Proof Mass**

**Drive Electrode**

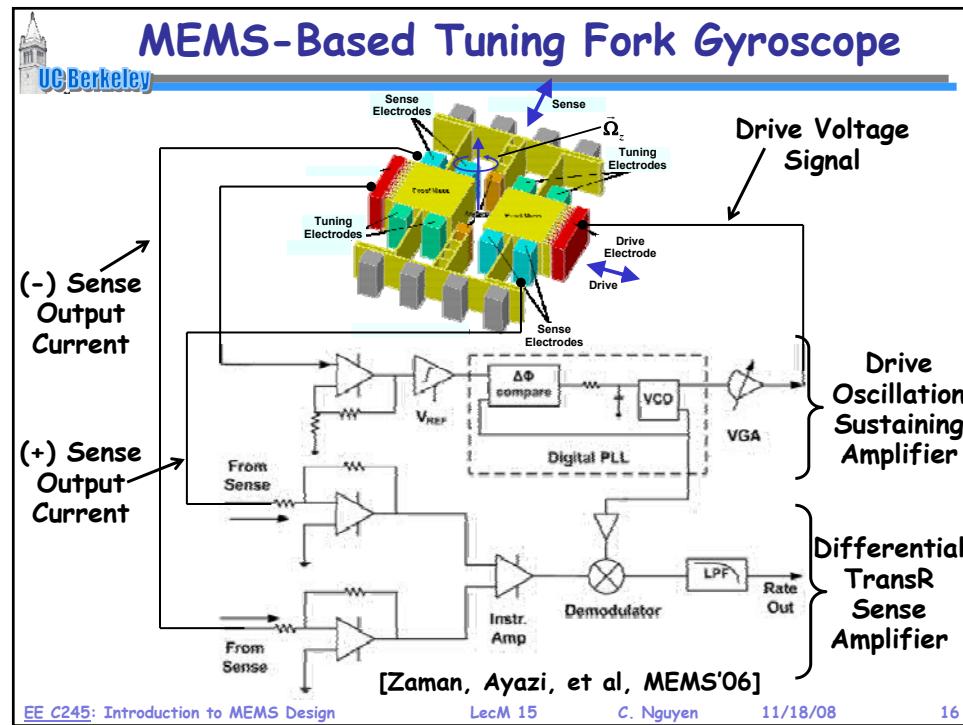
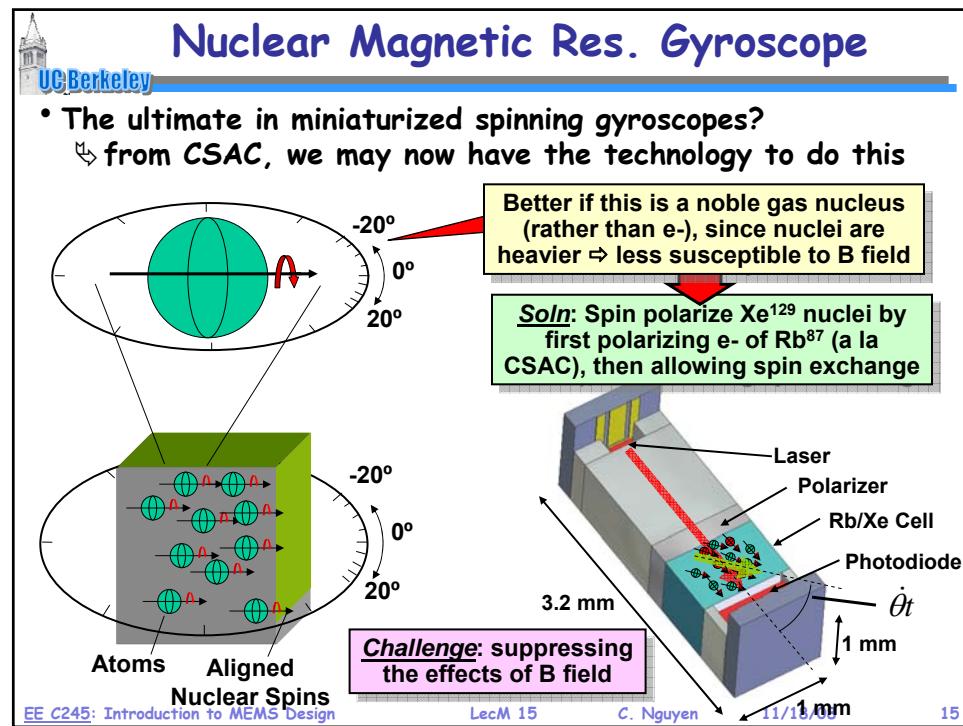
**Quadrature Cancellation Electrodes**

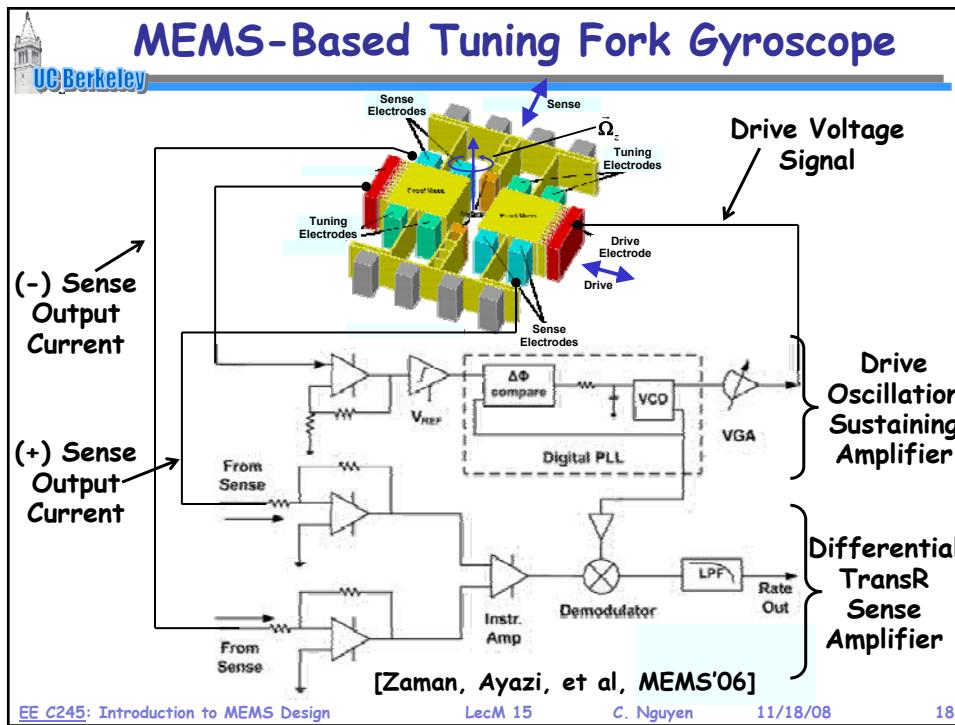
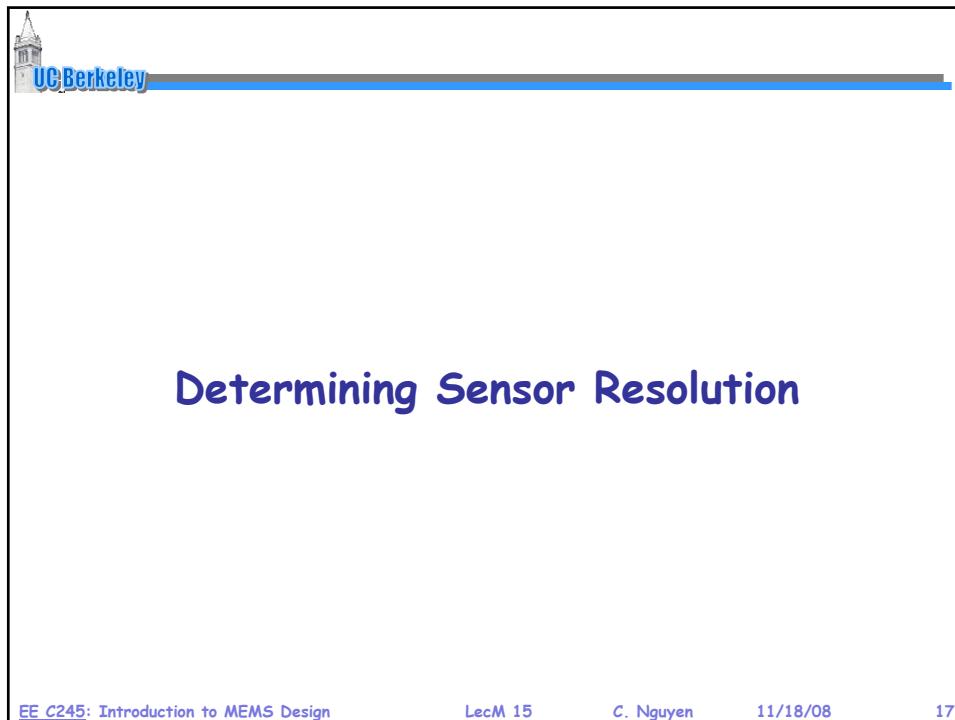
**Sense Electrodes**

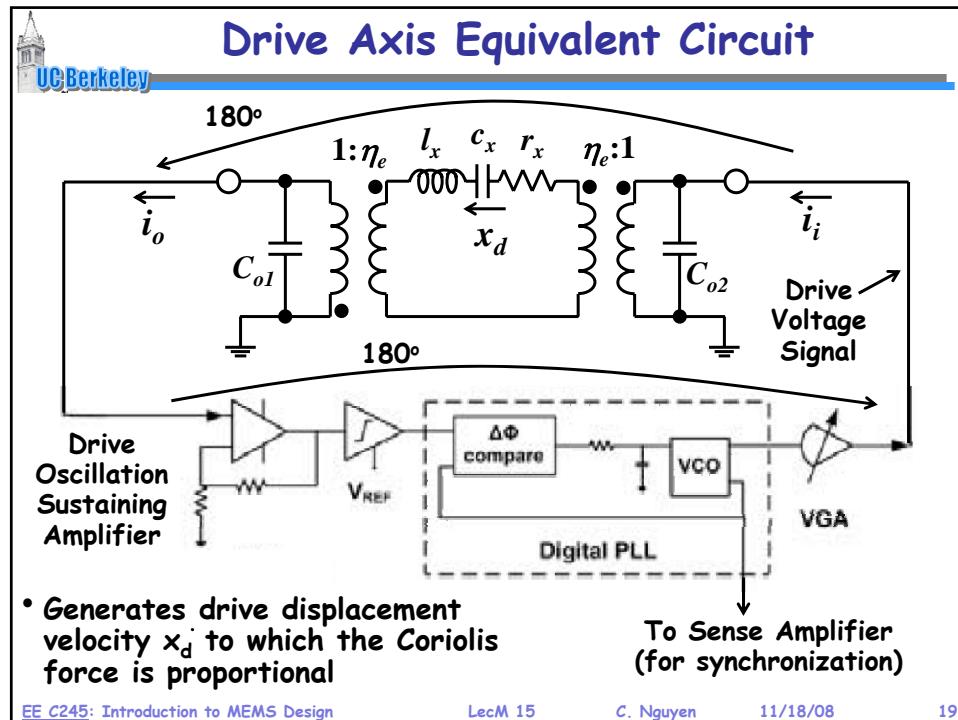
**Mass imbalance  $\Rightarrow$  off-axis motion of the proof mass**

**Quadrature output signal that can be confused with the Coriolis acceleration**

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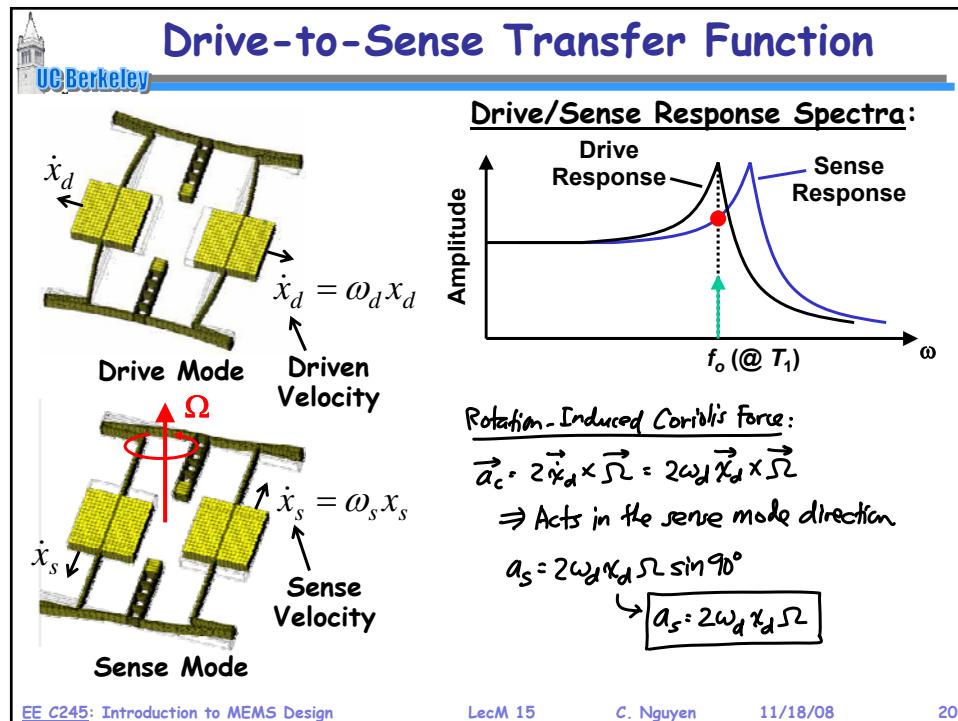
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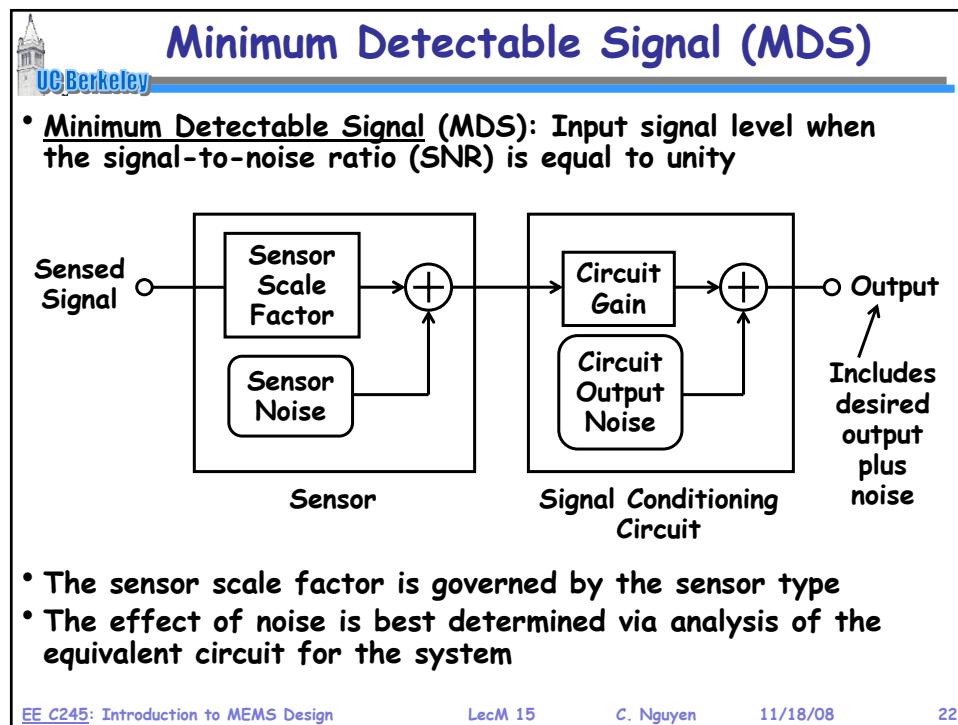
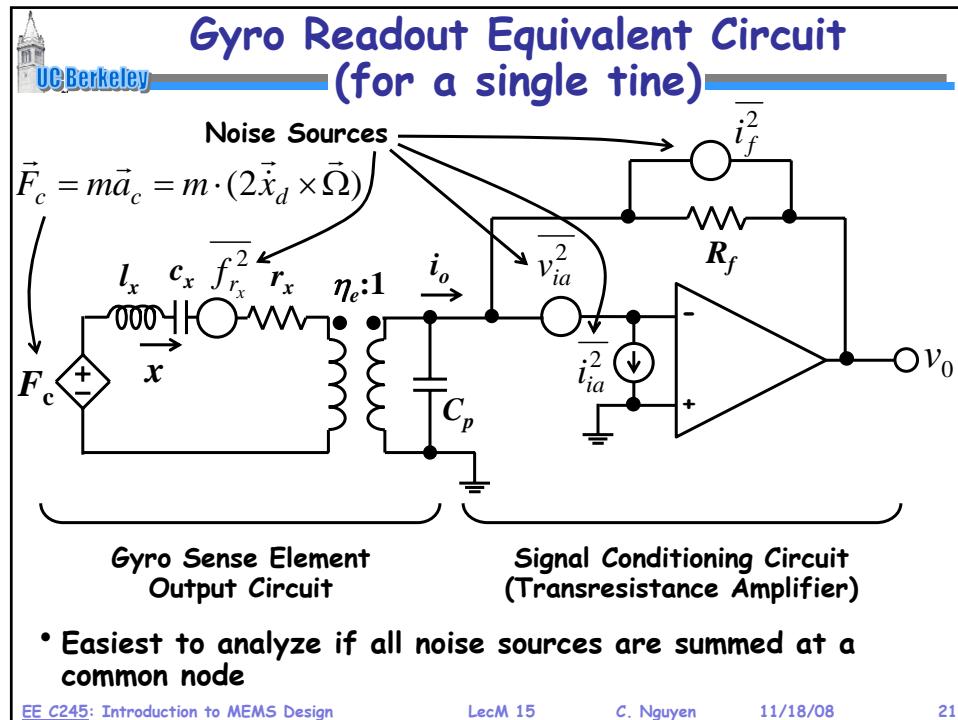
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## Move Noise Sources to a Common Point

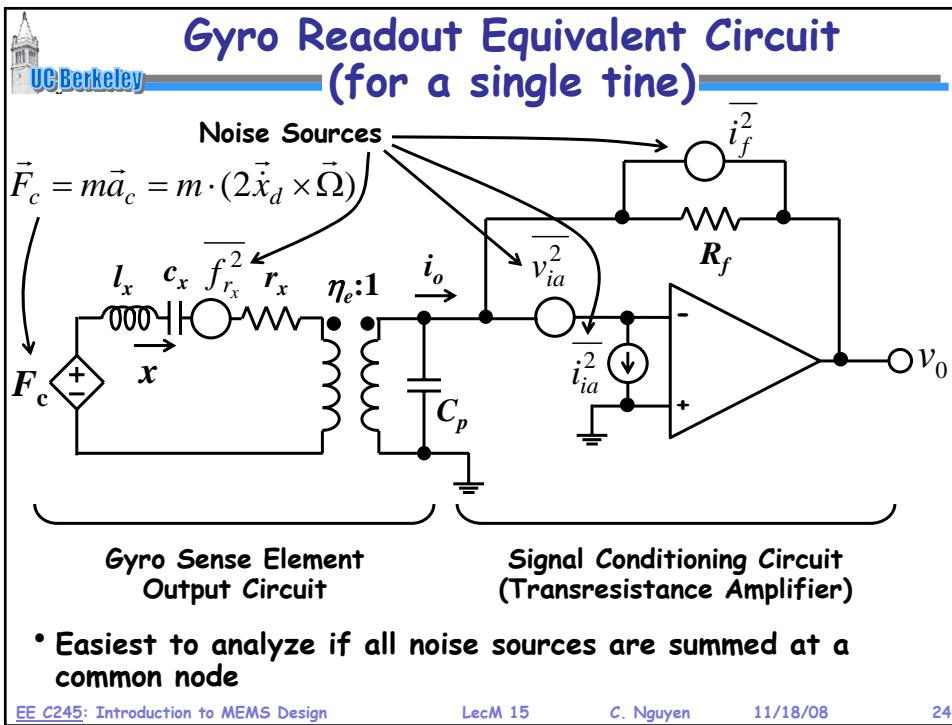
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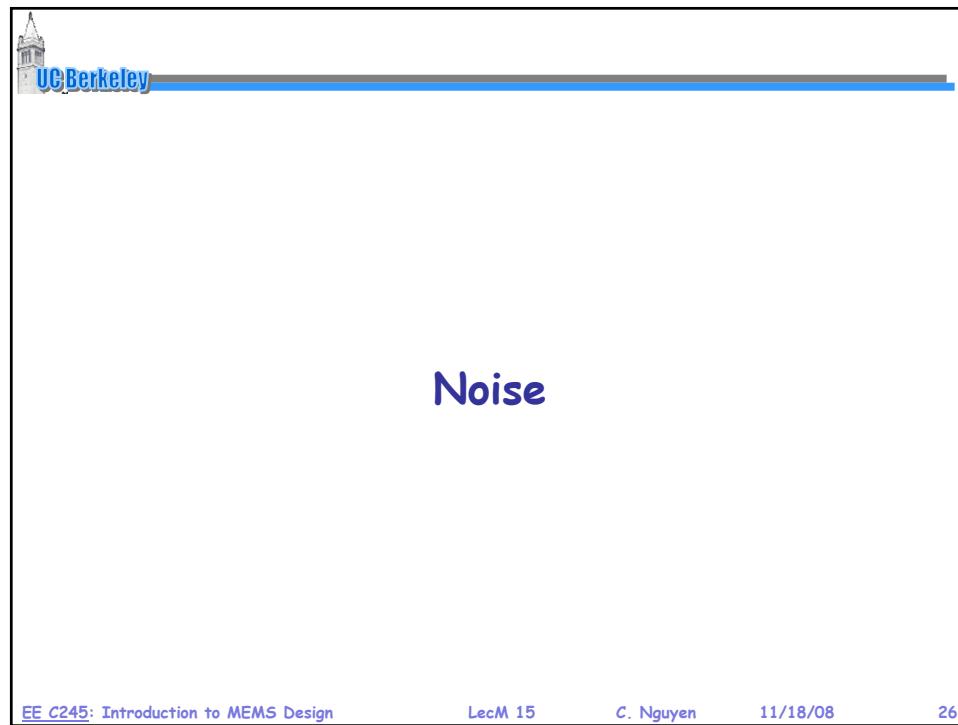
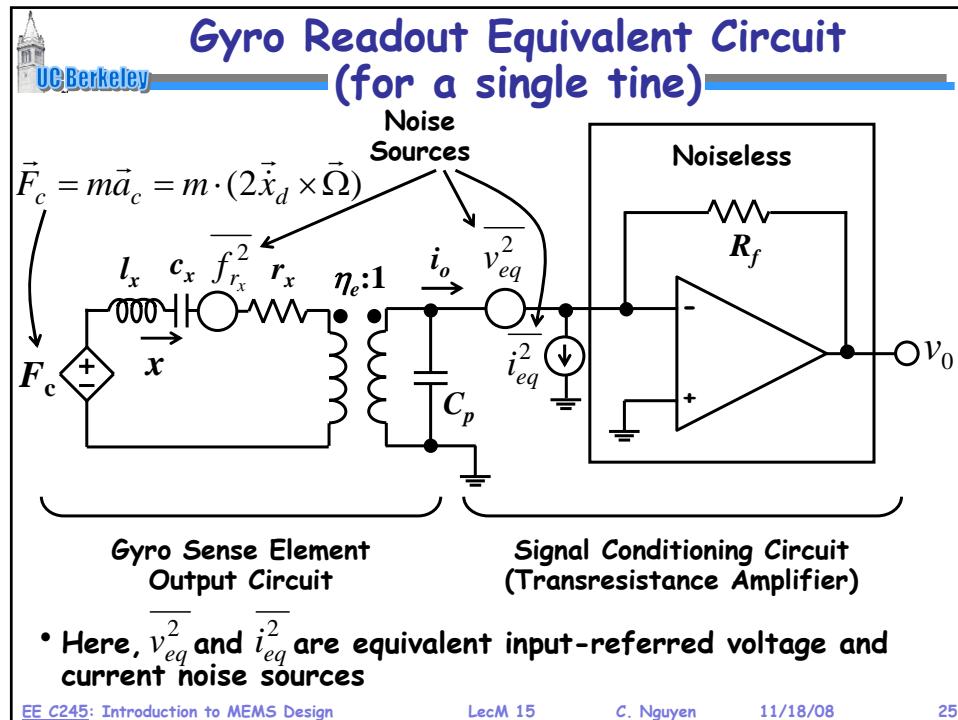
- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

Sensed Signal → Sensor → Signal Conditioning Circuit → Output

Includes desired output plus noise

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## Noise

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- **Noise:** Random fluctuation of a given parameter  $I(t)$
- In addition, a noise waveform has a zero average value

Avg. value (e.g. could be DC current)  $\rightarrow I_D$

Let  $i(t) = I(t) - I_D$

Then  $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |I - I_D|^2 dt$

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## Noise Spectral Density

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- We can plot the spectral density of this mean-square value:

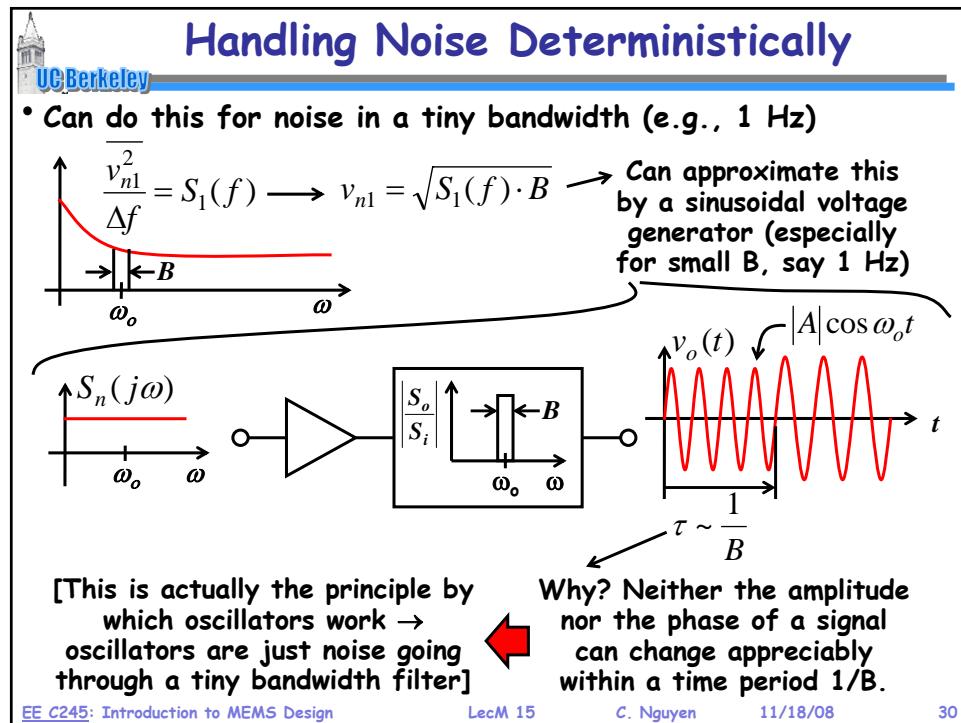
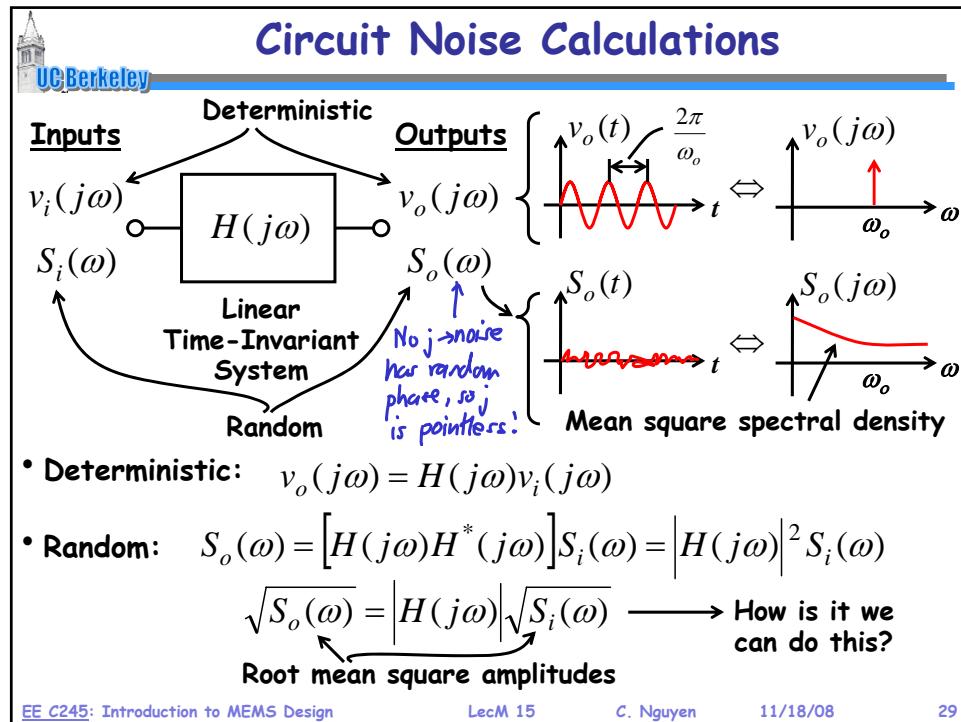
Two-sided spectral density (1/2 the one-sided)  
Often used in systems courses

One-sided spectral density  
→ used in circuits  
→ measured by spectrum analyzers

$\overline{i^2} / \Delta f$  [units<sup>2</sup>/Hz]

$\overline{i^2}$  = integrated mean-square noise spectral density over all frequencies (area under the curve)

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## Systematic Noise Calculation Procedure

**General Circuit With Several Noise Sources**

• Assume noise sources are uncorrelated

1. For  $\overline{i_{n1}^2}$  replace w/ a deterministic source of value

$$i_{n1} = \sqrt{\frac{\overline{i_{n1}^2}}{\Delta f}} \cdot (1 \text{ Hz})$$

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## Systematic Noise Calculation Procedure

2. Calculate  $v_{on1}(\omega) = i_{n1}(\omega)H(j\omega)$  (treating it like a deterministic signal)
3. Determine  $\overline{v_{on1}^2} = \overline{i_{n1}^2} \cdot |H(j\omega)|^2$
4. Repeat for each noise source:  $\overline{i_{n1}^2}, \overline{v_{n2}^2}, \overline{v_{n3}^2}$
5. Add noise power (mean square values)

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots$$

$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \dots}$$

Total rms value

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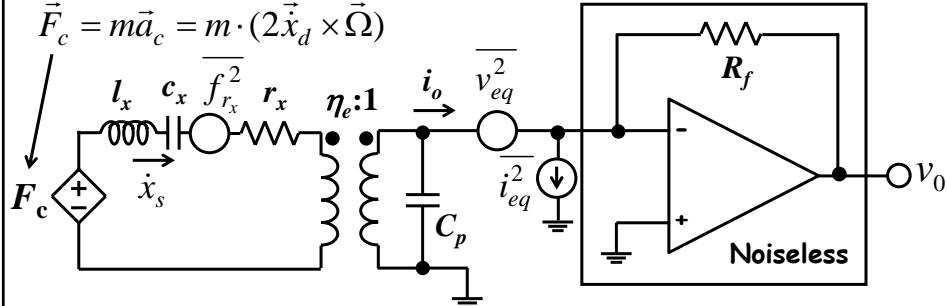


## Determining Sensor Resolution

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### Example: Gyro MDS Calculation



$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{x}_d \times \vec{\Omega})$   
 $F_c$        $I_x$        $c_x$        $\frac{f_r^2}{r_x}$        $r_x$        $n_e:1$        $i_o$        $\overline{v_{eq}^2}$   
 $\dot{x}_s$        $\eta_e:1$        $i_{eq}^2$        $R_f$        $v_0$   
 $C_p$       Noiseless

- The gyro sense presents a large effective source impedance
  - Currents are the important variable; voltages are "opened" out
  - Must compare  $i_o$  with the total current noise  $i_{eqTOT}$  going into the amplifier circuit

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**Example: Gyro MDS Calculation (cont)**

$$\vec{F}_c = m\vec{a}_c = m \cdot (2\vec{x}_d \times \vec{\Omega})$$

$$F_c = \frac{\omega_s Q}{k_s} \dot{x}_s$$

$$i_o = \eta_e \dot{x}_s$$

$$v_{eq} = \frac{i_o^2}{R_f}$$

$$v_0 = \text{Noiseless Op-Amp Output}$$

• First, find the rotation to  $i_o$  transfer function:

$$i_o = \frac{\omega_s Q}{k_s} H_s(j\omega_d) F_s = \frac{\omega_s Q}{k_s} \cdot 2\omega_d \chi_d \zeta \omega_m H(j\omega_d) \frac{1}{\omega_c^2}$$

$$[F_s = F_c = 2\omega_d \chi_d \zeta \omega_m]$$

$$\dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q \chi_d H(j\omega_d) \cdot \zeta \omega$$

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**Example: Gyro MDS Calculation (cont)**

$$i_o = \eta_e \dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q \chi_d \eta_e H(j\omega_d) \cdot \zeta \omega \rightarrow i_o = A \zeta \omega$$

$$A \triangleq \text{scale factor}$$

$$\text{where } A = 2 \frac{\omega_d}{\omega_s} Q \chi_d \eta_e H(j\omega_d)$$

When  $\zeta \omega = \zeta_{min} \triangleq MDS$ ,  $i_o = i_{eq,TOT}$   $\leftarrow$  input-referred noise current entering the sense amplifier  $\rightarrow$  in  $pA/\sqrt{Hz}$

$$\therefore i_{eq,TOT} = A \zeta_{min} \rightarrow \zeta_{min} = \frac{i_{eq,TOT}}{A} \left( \frac{3600s}{hr} \right) \left( \frac{180^\circ}{\pi} \right) \left[ (\%) / \sqrt{Hz} \right]$$

$$\text{Angle Random Walk} = ARW = \frac{1}{60} \zeta_{min} [\%/\text{hr}]$$

→ Easier to determine directional error as a function of elapsed time.

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### Example: Gyro MDS Calculation (cont)

The circuit diagram illustrates the calculation of Gyro MDS. It shows a sense element with inductance  $L_x$ , capacitance  $C_x$ , and resistance  $R_x$ . A noise source  $\eta_e : 1$  is connected to the sense element. The output current  $i_s$  is given by  $i_s = m \cdot (2 \vec{x}_d \times \vec{\Omega})$ . This current is converted to a voltage  $v_{eq}$  through a resistor  $R_f$ . The noise voltage  $v_{eq}^2$  is calculated as  $v_{eq}^2 = i_{eq}^2 R_f^2$ . The noise current  $i_{eq}$  is the sum of Brownian motion noise  $i_s$  and  $i_{eq}$ . The noise current  $i_{eq}$  is determined by the noise in  $R_x$  as  $i_{eq}^2 = 4kT R_x / \Delta f$ . The noise voltage  $v_0$  is the output of the amplifier.

- Now, find the  $i_{eqTOT}$  entering the amplifier input:

$$i_{eqTOT}^2 = i_s^2 + i_{eq}^2 \rightarrow i_{eqTOT}^2 = \overline{i_s^2} + \overline{i_f^2} + \overline{i_{ia}^2} + \frac{\overline{N_{ia}^2}}{R_f^2} \quad \frac{\overline{f_{rx}^2}}{\Delta f} = 4kT r_x$$

Brownian motion noise of the sense element → determined entirely by the noise in  $r_x \rightarrow \overline{f_{rx}^2}$   
 easiest to convert to an all electrical equiv. ckt.

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### Example: Gyro MDS Calculation (cont)

The handwritten derivation shows the calculation of  $i_{eqTOT}^2$  for the gyro MDS. It starts with the noise voltage  $N_{Rx}^2 = 4kT R_x$  and the noise current  $i_s^2 = N_{Rx}^2 / R_x$ . The total noise current  $i_{eqTOT}^2$  is the sum of  $i_s^2$ ,  $i_{ia}^2$ ,  $i_f^2$ , and  $N_{ia}^2 / R_f^2$ . The noise current  $i_s$  is given by  $i_s = N_{Rx} \left( \frac{1}{R_x} \right) \Theta(j\omega_d)$ . The noise current  $i_s$  is approximated as  $i_s = \frac{4kT}{R_x} \left| \Theta(j\omega_d) \right|^2$ . The noise current  $i_{eqTOT}$  is then given by  $\frac{i_{eqTOT}^2}{\Delta f} = \frac{4kT}{R_x} \left| \Theta(j\omega_d) \right|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{\Delta f} + \frac{N_{ia}^2}{R_f^2} \left( \frac{1}{R_x^2} \right)$ .

Learn to get these from EE240.  
 or just get them from a data sheet ...

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## LF356 Op Amp Data Sheet

**LF155/LF156/LF256/LF257/LF355/LF356/LF357**

### JFET Input Operational Amplifiers

#### General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (Bi-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

#### Features

**Advantages**

- Replace expensive hybrid and module FET op amps
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems
- Internal compensation and large differential input voltage capability

**Applications**

- Precision high speed integrators
- Fast D/A and A/D converters
- High impedance buffers
- Wideband, low noise, low drift amplifiers

**Common Features**

- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High input impedance:  $10^{12}\Omega$
- Low input noise current:  $0.01\text{ pA}/\sqrt{\text{Hz}}$
- High common-mode rejection ratio: 100 dB
- Large dc voltage gain: 106 dB

**Uncommon Features**

	LF155/ LF355	LF156/ LF256	LF257/ LF357	Units
Extremely fast settling time to 0.01%	4	1.5	1.5	$\mu\text{s}$
Fast slew rate	5	12	50	$\text{V}/\mu\text{s}$
Wide gain bandwidth	2.5	5	20	MHz
Low input noise voltage	20	12	12	$\text{nV}/\sqrt{\text{Hz}}$

*Handwritten notes:*

- $i_{ia} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$
- $v_{ia} = 12 \text{ nV}/\sqrt{\text{Hz}}$

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## Example ARW Calculation

**• Example Design:**

**↳ Sensor Element:**

$$m = (100\mu\text{m})(100\mu\text{m})(20\mu\text{m})(2300\text{kg/m}^3) = 4.6 \times 10^{-10}\text{kg}$$

$$\omega_s = 2\pi(15\text{kHz})$$

$$\omega_d = 2\pi(10\text{kHz})$$

$$k_s = \omega_s^2 m = 4.09 \text{ N/m}$$

$$x_d = 20 \mu\text{m}$$

$$Q_s = 50,000$$

$$V_p = 5\text{V}$$

$$h = 20 \mu\text{m}$$

$$d = 1 \mu\text{m}$$

**↳ Sensing Circuitry:**

$$R_f = 100\text{k}\Omega$$

$$i_{ia} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$$

$$v_{ia} = 12 \text{ nV}/\sqrt{\text{Hz}}$$

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**Example ARW Calculation (cont)**

Get rotation rate to output current scale factor:

$$A = 2 \frac{w_d}{w_s} Q_s \chi_d \eta_e |\Theta(j\omega_d)| = 2 \left( \frac{10k}{15k} \right) \left( 50k \right) \left( 20\mu \right) \left( 5 \right) \left( 2000 \epsilon_0 \right) \left( 0.000024 \right) = 2.83 \times 10^{-12} C$$

$$\left[ \begin{aligned} \Theta(j\omega_d) &= \frac{(j\omega_d)(w_s/Q_s)}{-\omega_d^2 + j\omega_d w_s + w_s^2} = \frac{j(10k)(15k)/(50k)}{(15k)^2 - (10k)^2 + j(10k)(15k)/50k} = \frac{j(3k)}{1.25 \times 10^8 + j(3k)} \\ &\rightarrow |\Theta(j\omega_d)| = \frac{3k}{\sqrt{(1.25 \times 10^8)^2 + (3k)^2}} = 0.000024 \end{aligned} \right]$$

$$\left[ \begin{aligned} \frac{\partial C}{\partial x} &= \frac{C_0}{d} = \frac{\epsilon_0 h w_p}{d} = \frac{\epsilon_0 (20\mu)(100\mu)}{(1\mu)^2} = 2000 \epsilon_0 \rightarrow \eta_e = V_p \frac{\partial C}{\partial x} = \frac{5(2000 \epsilon_0)}{8.854 \times 10^{-12} F/m} \\ &\text{Assume electrode covers the whole sidewall.} \end{aligned} \right]$$

Then, get noise:

$$\frac{i_{eq}^2}{\Delta f} = \frac{4kT}{R_x} |\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{\Delta f} + \frac{N_{ia}^2}{\Delta f} \left( \frac{1}{R_f^2} \right)$$

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**Example ARW Calculation (cont)**

$R_X = \frac{w_s m}{Q_s \eta_e} = \frac{2\pi(15k)(4.6 \times 10^{-10})}{(50k)(8.854 \times 10^{-12})^2} = 110.6 k\Omega$

$$\frac{i_{eq,TOT}^2}{\Delta f} = \underbrace{\frac{(1.66 \times 10^{-29})}{(110.6k)} (0.000024)^2}_{8.64 \times 10^{-25} A^2/Hz} + \underbrace{\frac{(1.66 \times 10^{-29})}{1M}}_{1.66 \times 10^{-26} A^2/Hz} + \underbrace{(0.01\rho)^2}_{1 \times 10^{-28} A^2/Hz} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44 \times 10^{-28} A^2/Hz}$$

sensor element noise      noise from  $R_f$  dominates!

$$\therefore \frac{i_{eq,TOT}^2}{\Delta f} = 1.68 \times 10^{-26} A^2/Hz \rightarrow i_{eq,TOT} = \sqrt{\frac{i_{eq,TOT}^2}{\Delta f}} = 1.30 \times 10^{-13} A/\sqrt{Hz}$$

$$\therefore S2_{min} = \frac{i_{eq,TOT}}{A} \left( \frac{3600\pi}{hr} \right) \left( \frac{180}{\pi} \right) = \frac{1.30 \times 10^{-13}}{2.83 \times 10^{-12}} (3600) \left( \frac{180}{\pi} \right) = 9448 (\%hr)/\sqrt{Hz}$$

And finally:

$$ARW = \frac{1}{60} S2_{min} = \frac{1}{60} (9448) = 157 \%/\sqrt{hr} = ARW$$

→ Almost turned around in 1 hour!

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**What if  $\omega_d = \omega_s$ ?**

If  $\omega_d = \omega_s = 15\text{kHz}$ , then  $|H(j\omega_d)| = 1$  and

$$A = 2 \frac{\omega_d}{\omega_s} Q_s \chi_d \eta_e |H(j\omega_d)| = 2 Q_s \chi_d \eta_e = 2(50k)(20\mu)(5)(2000\epsilon_0) = 1.77 \times 10^{-7} \text{C}$$

$$\frac{i_{eq,TOT}^2}{\Delta f} = \underbrace{\frac{(1.66 \times 10^{-29})}{(110.6k)} (1)^2}_{1.51 \times 10^{-25} \text{ A}^2/\text{Hz}} + \underbrace{\frac{(1.66 \times 10^{-29})}{1M}}_{1.66 \times 10^{-26} \text{ A}^2/\text{Hz}} + \underbrace{(0.01\rho)^2}_{1 \times 10^{-28} \text{ A}^2/\text{Hz}} + \underbrace{\frac{(12n)^2}{(1M)^2}}_{1.44 \times 10^{-28} \text{ A}^2/\text{Hz}}$$

Now, the sensor element dominates!

$$\therefore \frac{i_{eq,TOT}^2}{\Delta f} = 1.67 \times 10^{-25} \text{ A}^2/\text{Hz} \rightarrow i_{eq,TOT} = \sqrt{\frac{i_{eq,TOT}^2}{\Delta f}} = 4.08 \times 10^{-13} \text{ A}/\sqrt{\text{Hz}}$$

$$\therefore \Sigma_{min} = \frac{i_{eq,TOT}}{A} \left( \frac{3600\pi}{hr} \right) \left( \frac{180^\circ}{\pi} \right) = \frac{4.08 \times 10^{-13}}{1.77 \times 10^{-7}} (3600) \left( \frac{180}{\pi} \right) = 0.476 (\%hr)/\sqrt{\text{Hz}}$$

And finally:

$$ARW = \frac{1}{60} \Sigma_{min} = \frac{1}{60} (0.476) = 0.0079 \%/\sqrt{\text{hr}} = ARW \Rightarrow \text{Navigation grade!}$$

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