



EE C245 - ME C218 Introduction to MEMS Design Fall 2009

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Lecture Module 11: Equivalent Circuits I

EE C245: Introduction to MEMS Design LecM 11 C. Nguyen 11/6/08 1



Lecture Outline

- Reading: Senturia, Chpt. 5
- Lecture Topics:
 - ↳ Lumped Mass
 - ↳ Lumped Stiffness
 - ↳ Lumped Damping
 - ↳ Lumped Mechanical Equivalent Circuits
 - ↳ Electromechanical Analogies

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Lumped Parameter Mechanical Equivalent Circuit

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Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity

The diagram shows a beam of length L with a central node labeled A . The beam is divided into four segments by supports at $\frac{L}{4}$ and $\frac{3L}{4}$, each applying a force $F/4$. The beam's deflection is shown as a sinusoidal wave. A coordinate system (x, y, z) is defined at the center node A . A red arrow points to "Location x " on the right side of the beam. Below the beam, a formula for equivalent mass is derived:

$$\text{Equivalent Mass} = M_{eq,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^L V^2(x) dx}{\frac{1}{2}V_x^2}$$

Annotations include "Maximum Kinetic Energy" pointing to the integral term, "Density" pointing to ρA , and "Maximum Velocity Function" pointing to $V^2(x)$.

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Equivalent Dynamic Mass

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- For the folded-beam structure, we've already determined the maximum kinetic energy
- And in our resonance frequency analysis, we've already determined expressions for velocity

Location on the Truss:

$$M_{eq(truss)} = \frac{KE_{max}}{\frac{1}{2}V_{truss}^2} = \frac{\omega_0^2 V_0^2 \left(\frac{1}{2}\right) [M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]}{\frac{1}{2}(\frac{1}{4})\omega_0^2 x_0^2}$$

$$\therefore M_{eq(truss)} = 4 [M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]$$

Location on the Shuttle:

$$M_{eq(shuttle)} = \frac{KE_{max}}{\frac{1}{2}V_{shuttle}^2} = \frac{\omega_0^2 V_0^2 \left(\frac{1}{2}\right) [M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]}{\frac{1}{2}\omega_0^2 x_0^2}$$

$$\therefore M_{eq(shuttle)} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$$

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Equivalent Dynamic Stiffness & Damping

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- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_0^2 M_{eq}(x)$$

→ large equiv. mass \downarrow
large stiffness go hand-in-hand

- And damping also follows readily from knowledge of Q or other loss measurands

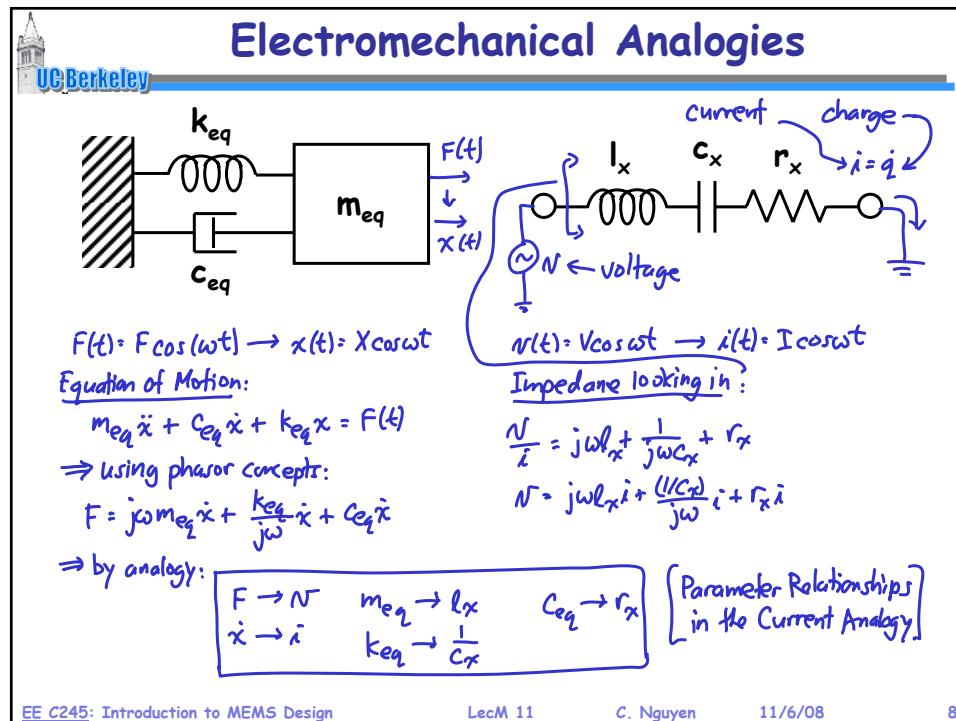
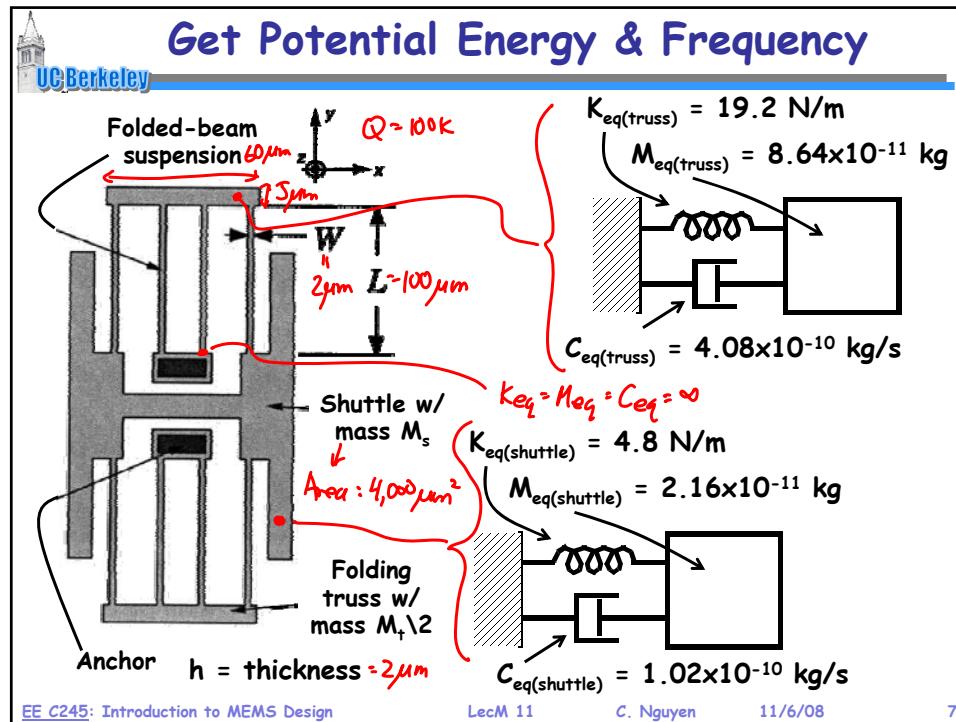
$$Q = \frac{\omega_0 M_{eq}(x)}{C_{eq}(x)}$$

$\underbrace{\quad}_{\text{damping}}$

$$\rightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \sqrt{\frac{K_{eq}(x) M_{eq}(x)}{Q}}$$

- With mass, stiffness, and damping \Rightarrow lumped parameter equivalent circuit

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Electromechanical Analogies (cont)

- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness $^{-1}$, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

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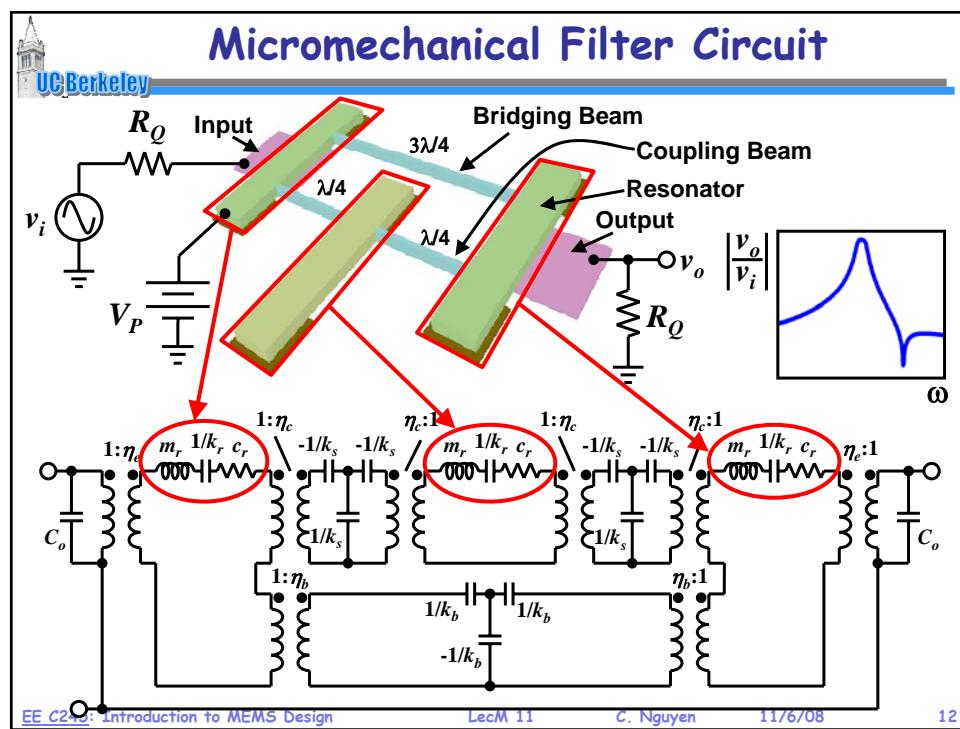
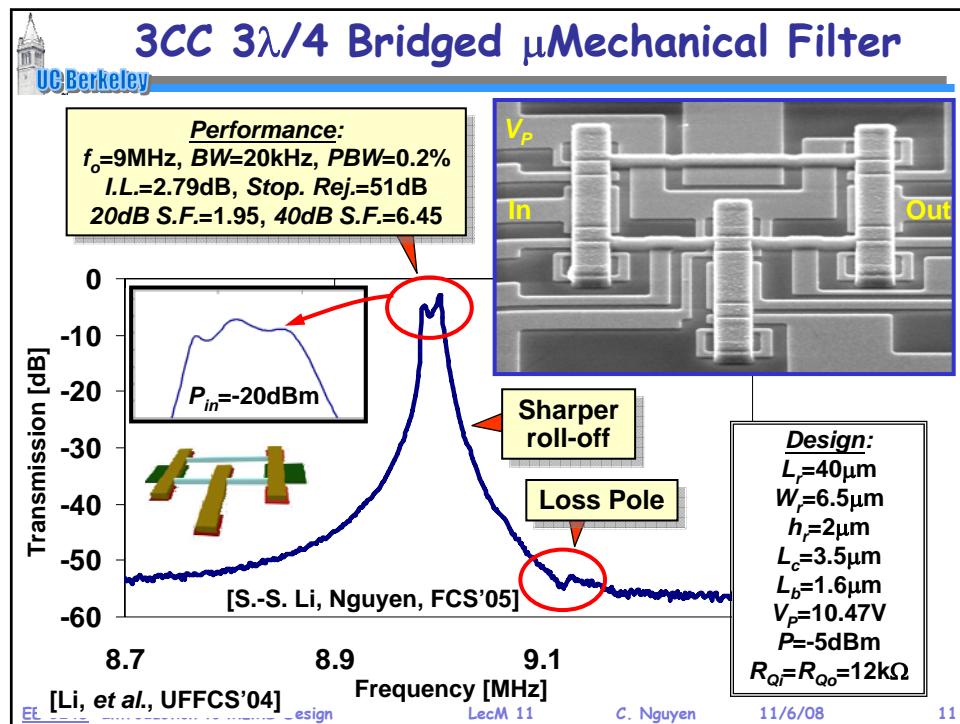
Bandpass Biquad Transfer Function

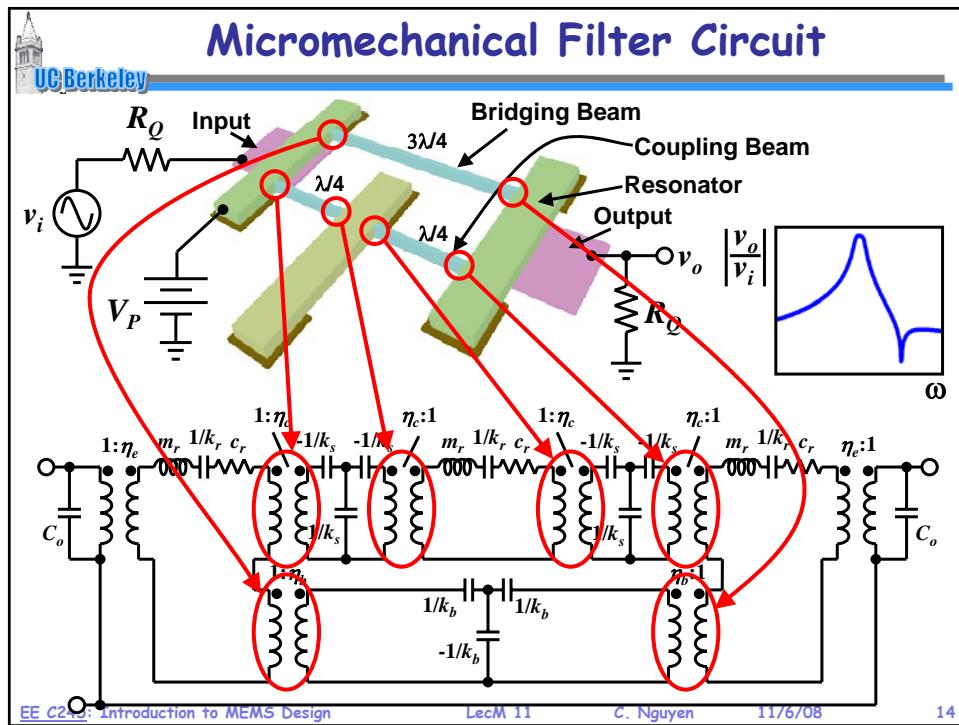
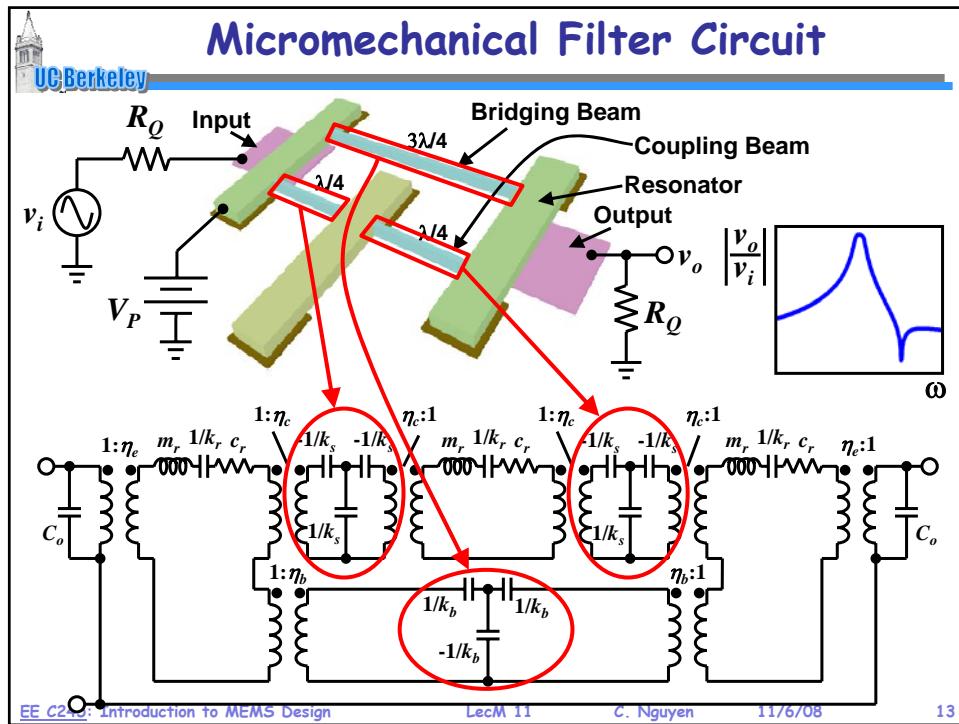
$F = j\omega m_{eq}\ddot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}x$
 \Rightarrow Converting to full phasor form:
 $F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{j\omega}(j\omega X) + c_{eq}(j\omega X)$

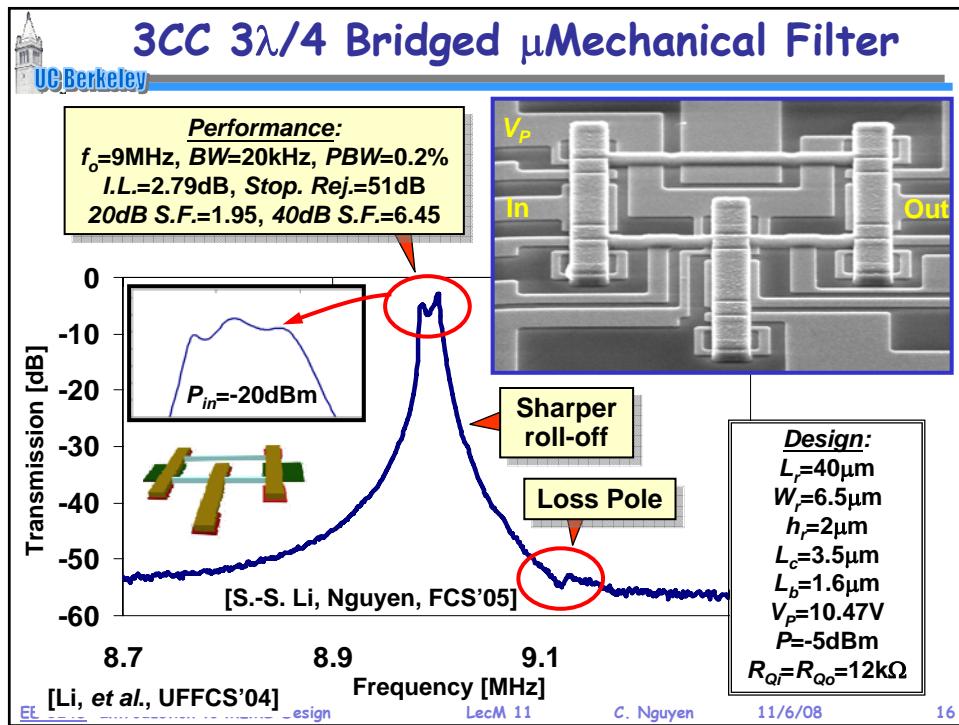
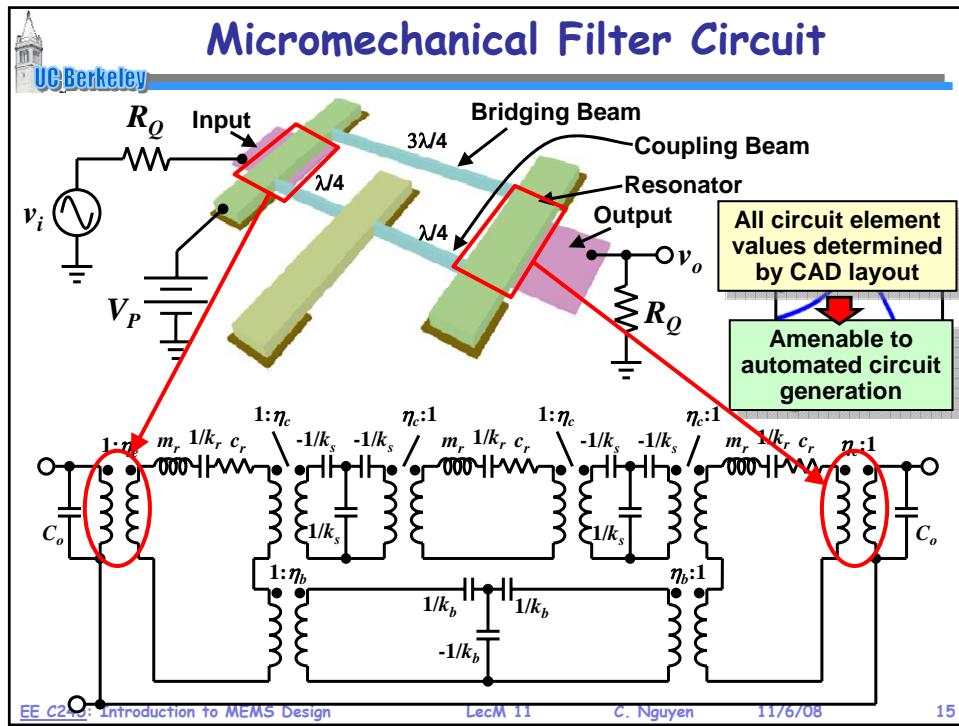
$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}} \left[-\omega^2 \frac{m_{eq}}{k_{eq}} + 1 + j \frac{c_{eq}\omega}{k_{eq}} \right]^{-1} = \frac{1}{k_{eq}} \left[\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j \frac{\omega}{Q\omega_0} \right]^{-1}$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2, Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0 \right]$$

EE C245: Introduction to MEMS Design LecM 11 C. Nguyen 11/6/08 10









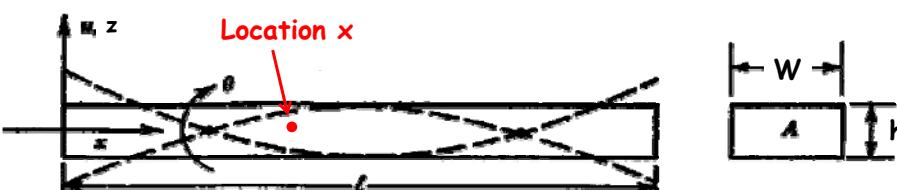
Beam Resonator Equivalent Circuits (Pretty Much the Same Stuff)

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Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy

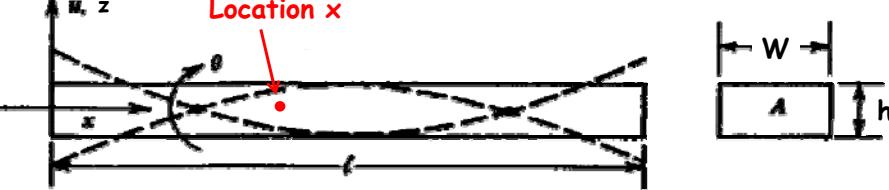
Equivalent Mass = $M_{eq,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^L V^2(x) dx}{\frac{1}{2}V_x^2}$

Maximum Velocity @ location x

Maximum Velocity Function

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Equivalent Dynamic Mass



Location x

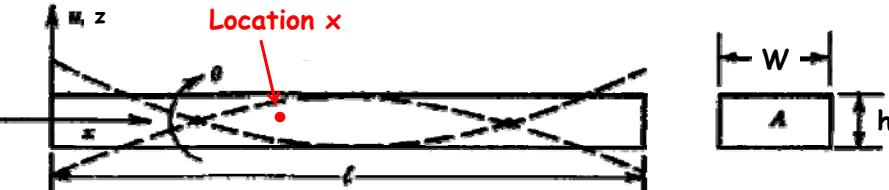
Displacement: $u(x) = B \left[S(\cosh kx + \cos kx) + (\sinh kx + \sin kx) \right]$, $S = \frac{A}{B}$

$[V(x) = \omega_0 u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{\frac{1}{2}\rho A \int_0^L \omega_0^2 [u(x')]^2 dx'}{\frac{1}{2}\omega_0^2 [u(x)]^2}$

$$M_{eq}(x) = \frac{\rho A \int_0^L B^2 [S(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')]^2 dx'}{B^2 [S(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}$$

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Equivalent Dynamic Stiffness & Damping



Location x

Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_0^2 M_{eq}(x)$$

And damping also follows readily

$$Q = \frac{\omega_0 M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$

C_{eq}(x) damping

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