# EE C245 – ME C218 Introduction to MEMS Design Fall 2009

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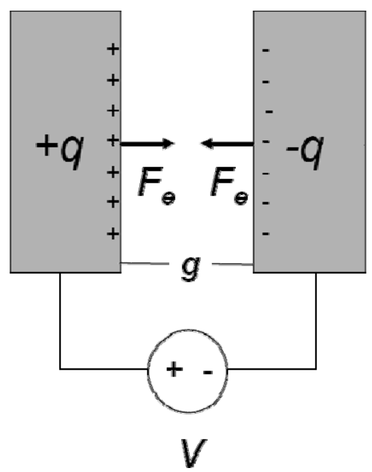
Lecture Module 12: Capacitive Transducers



## Lecture Outline

- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis

## Basic Physics of Electrostatic Actuation




- Goal: Determine gap spacing g as a function of input variables
- First, need to determine the energy of the system
- Two ways to change the energy:
  - Change the charge q
  - Change the separation g

$$\Delta W(q,g) = V\Delta q + F_e \Delta g$$

$$dW = Vdq + F_e dg$$

- Note: We assume that the plates are supported elastically, so they don't collapse

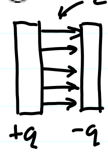


## Stored Energy

- Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap

$$W = 0 + \int_0^g F_e dg'$$

No change in charge: $dq=0$

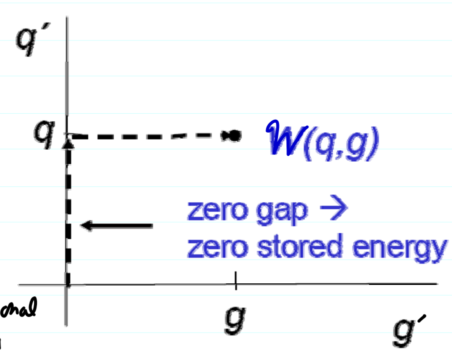



$\mathcal{E} = \frac{q}{\epsilon A}$ ← Cross-sectional Area

$F_e = \left(\frac{q}{2}\right)\mathcal{E} = \frac{1}{2}\frac{q^2}{\epsilon A}$ (independent of g)

$\therefore W = \int_0^g F_e dg' = F_e g'\Big|_0^g = F_e g$

$$\therefore \boxed{W(g) = \frac{1}{2}\frac{q^2}{\epsilon A} g}$$




For a capacitor C:

$q = CV \rightarrow V = \frac{q}{C}$

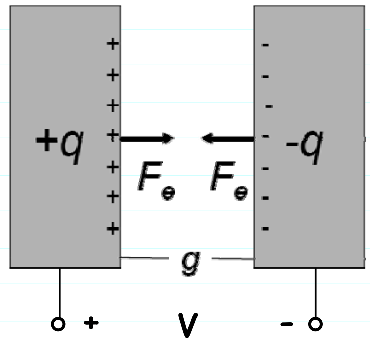
$\therefore W(q) = \int_0^q V dq = \int_0^q \left(\frac{q}{C}\right) dq$

$= \frac{1}{2}\frac{q^2}{C} = \boxed{\frac{1}{2}\frac{q^2 g}{\epsilon A} = W(q)}$

same answer when Work done to charge C to q at fixed gap

## Charge-Control Case

- Having found stored energy, we can now find the force acting on the plates and the voltage across them:




From $dW = Vdq + F_e dg$:

⇒ Force is given by:

$$F_e = \left.\frac{\partial W(q,g)}{\partial g}\right|_{q=\text{const.}} = \frac{\partial}{\partial g}\left(\frac{1}{2}\frac{q^2}{\epsilon A} g\right)$$

$$\therefore \boxed{F_e = \frac{1}{2}\frac{q^2}{\epsilon A}} \Rightarrow \text{independent of gap spacing!}$$

⇒ Voltage is given by:

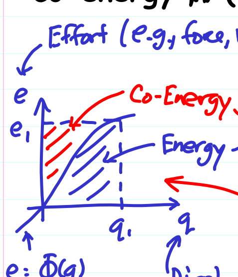
$$V = \left.\frac{\partial W(q,g)}{\partial q}\right|_{g=\text{const.}} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{\epsilon A} g\right) = \frac{qg}{\epsilon A} \quad \therefore \boxed{V = \frac{q}{C}}$$

⇒ consistent w/ what we already know ✓



## Voltage-Control Case

- Practical situation: We control V
  - Charge control on the typical sub-pF MEMS actuation capacitor is difficult
  - Need to find $F_e$ as a partial derivative of the stored energy $\mathcal{W} = \mathcal{W}(V,g)$ with respect to g with V held constant? But can't do this with present $\mathcal{W}(q,g)$ formula
  - <u>Solution</u>: Apply Legendre transformation and define the co-energy $\mathcal{W}'(V,g)$




$$W(q_1) = \int_0^{q_1} e\,dq = \int_0^{q_1} \Phi(q)\,dq$$

$$W'(e_1) = \int_0^{e_1} q\,de = \int_0^{e_1} \Phi^{-1}(e)\,de$$

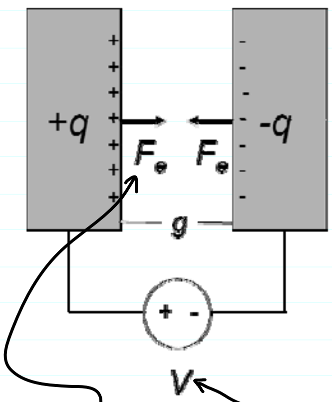
For a linear system, these will be equal.

⇒ Can define co-energy as: $W'(e) = eq - W(q)$ (from this plot)

## Co-Energy Formulation

- For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

$$\mathcal{W}'(V,g) = qV - \mathcal{W}$$




Differentially, this becomes:

$$d\mathcal{W}'(V,g) = (q\,dV + V\,dq) - d\mathcal{W}(q,g)$$

But $\left[d\mathcal{W}(q,g) = F_e\,dg + V\,dq\right]$

$$\boxed{d\mathcal{W}'(V,g) = q\,dV - F_e\,dg}$$ ← Working Co-Energy Expression

From which:

$$\text{Charge, } Q = \left.\frac{\partial \mathcal{W}'(V,g)}{\partial V}\right|_{g=\text{const.}}$$

$$\text{Force, } F_e = -\left.\frac{\partial \mathcal{W}'(V,g)}{\partial g}\right|_{V=\text{const.}}$$

⇒ this gives force as a function of applied voltage



## Electrostatic Force (Voltage Control)

- Find co-energy in terms of voltage

$$\mathcal{W}' = \int_0^V q(g,V')\,dV' = \int_0^V \left(\varepsilon \frac{A}{g}\right) V'\,dV' = \frac{1}{2}\left(\frac{\varepsilon A}{g}\right) V^2 = \frac{1}{2}CV^2$$

(as expected)

- Variation of co-energy with respect to gap yields electrostatic force:

$$F_e = -\left.\frac{\partial \mathcal{W}'(V,g)}{\partial g}\right|_V = -\frac{1}{2}\underbrace{\left(-\frac{\varepsilon A}{g^2}\right)}V^2 = \frac{1}{2}\frac{C}{g}V^2$$

*strong* function of gap!

- Variation of co-energy with respect to voltage yields charge:

$$q = \left.\frac{\partial \mathcal{W}'(V,g)}{\partial V}\right|_g = \left(\frac{\varepsilon A}{g}\right)V = CV$$ as expected

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# Spring-Suspended Capacitive Plate



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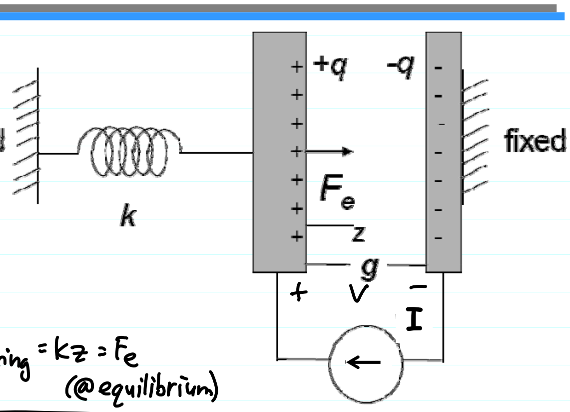
# Charge Control of a Spring-Suspended C




Force generated by charge q supplied by current I:

$$F_e = \frac{\partial W(q,g)}{\partial g}\bigg|_q = \frac{q^2}{2\epsilon A}$$

Restoring force of spring: $F_{spring} = kz = F_e$ (@ equilibrium)

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2}\frac{q^2}{\epsilon A}\frac{1}{k} = g}$$

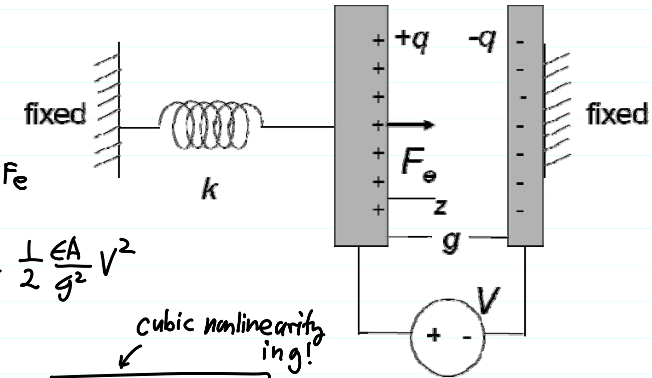
($g_0$ ↑ initial gap)

⇒ can increase q and drive $g \rightarrow 0$

$$V = \frac{q}{C} = \frac{q}{\epsilon A} g = \boxed{\frac{q}{\epsilon A}\left(g_0 - \frac{1}{2}\frac{q^2}{\epsilon A}\frac{1}{k}\right) = V}$$

⇒ V↓ as g↓

## Voltage Control of a Spring-Suspended C




Again, $F_{spring} = kz = F_e$

But now:

$$F_e = \frac{\partial W'(V,g)}{\partial g}\bigg|_q = \frac{1}{2}\frac{\epsilon A}{g^2}V^2$$

And the gap:

$$g = g_o - z = g_o - \frac{F_e}{k} = \boxed{g_o - \frac{1}{2}\frac{\epsilon A}{g^2}V^2 = g}$$

cubic nonlinearity ing! $\Rightarrow$ g shows up on both sides!

If $V\uparrow \rightarrow g\downarrow \rightarrow F_e\uparrow$ — Feedback!

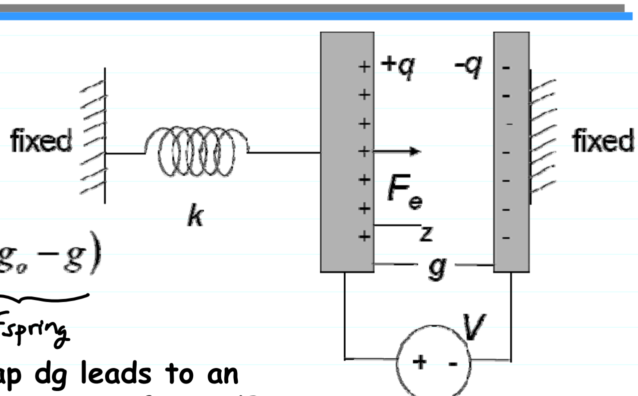
If loop gain > 1, then this will go unstable!

Charge: (for a stable gap)

$$q = \frac{\partial W'(V,g)}{\partial V}\bigg|_g = CV = \boxed{\frac{\epsilon A}{g}V = q}$$



## Stability Analysis




- Net attractive force on the plate:

$$F_{net} = \underbrace{\frac{\varepsilon A V^2}{2g^2}}_{F_e} - \underbrace{k(g_o - g)}_{F_{spring}}$$

- An increment in gap dg leads to an increment in net attractive force $dF_{net}$

$$dF_{net} = \frac{\partial F_{net}}{\partial g}dg = \left[-\frac{\epsilon A V^2}{g^3} + k\right]dg$$

Thus, need this = (+) $\Rightarrow$ $\boxed{k > \frac{\epsilon A V^2}{g^3}}$ (for a stable uncollapsed state)

$F_{net}\downarrow \rightarrow dF_{net} = (-)$

If $g\downarrow \rightarrow dg = (-)$, then for stability need $\rightarrow$ otherwise, plate collapses

## Pull-In Voltage $V_{PI}$

- $V_{PI}$ = voltage at which the plates collapse
- The plate goes unstable when

$$k = \frac{\varepsilon A V_{PI}^2}{g_{PI}^3} \quad (1) \quad \text{and} \quad F_{net} = 0 = \frac{\varepsilon A V_{PI}^2}{2 g_{PI}^2} - k(g_o - g_{PI}) \quad (2)$$

← pull-in gap

- Substituting (1) into (2):

$$0 = \frac{\varepsilon A V_{PI}^2}{2 g_{PI}^2} - \frac{\varepsilon A V_{PI}^2}{g_{PI}^3}(g_o - g_{PI})$$

$$\frac{g_o - g_{PI}}{g_{PI}} = \frac{1}{2} \rightarrow g_o = \frac{3}{2} g_{PI}$$

$$\therefore \boxed{g_{PI} = \frac{2}{3} g_o}$$

$$V_{PI} = \sqrt{\frac{k g_{PI}^3}{\epsilon A}}$$

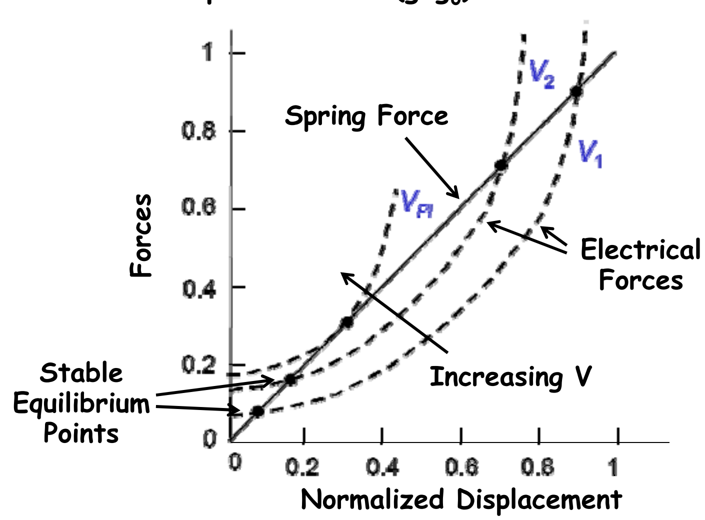
$$\therefore \boxed{V_{PI} = \sqrt{\frac{8}{27}\frac{k g_o^3}{\epsilon A}}}$$

**When a gap is driven by a voltage to (2/3) its original spacing, collapse will occur!**



## Voltage-Controlled Plate Stability Graph

- Below: Plot of normalized electrostatic and spring forces vs. normalized displacement $1-(g/g_o)$



## Advantages of Electrostatic Actuators

- Easy to manufacture in micromachining processes, since conductors and air gaps are all that's needed → low cost!
- Energy conserving → only parasitic energy loss through $I^2R$ losses in conductors and interconnects
- Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
- Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
- Same capacitive structures can be used for both drive and sense of velocity or displacement
- Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q's for resonant structures



## Problems With Electrostatic Actuators

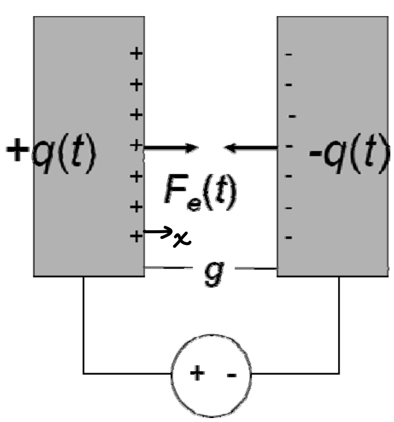
- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale

# Linearizing the Voltage-to-Force Transfer Function



## Linearizing the Voltage-to-Force T.F.

- Apply a DC bias (or polarization) voltage $V_P$ together with the intended input (or drive) voltage $v_i(t)$, where $V_P >> v_i(t)$




$v(t) = V_P + v_i(t)$

$$F_e(t) = \frac{\partial W}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2} C [v(t)]^2\right)$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}[v(t)]^2 = \frac{1}{2}(V_P + v_i(t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2}\left[V_P^2 + 2V_P v_i(t) + [v_i(t)]^2\right]\frac{\partial C}{\partial x}$$

$$[V_P >> v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2}V_P^2\frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_P\frac{\partial C}{\partial x}v_i(t)}_{\text{AC drive signal}}$$

$$C(x) = \frac{\epsilon A}{g_o - x} = C_o\left(1 - \frac{x}{g_o}\right)^{-1} \underset{[x << g_o]}{\approx} C_o\left(1 + \frac{x}{g_o}\right)$$

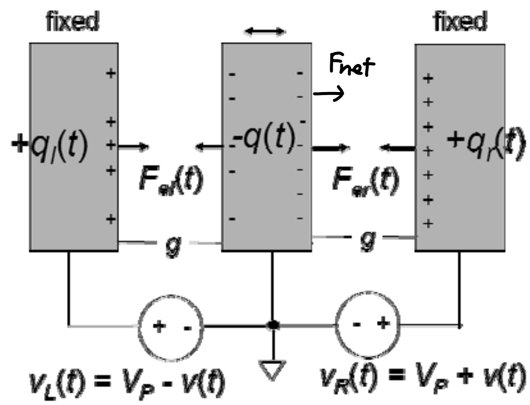
$$\therefore \frac{\partial C}{\partial x} \approx \frac{C_o}{g_o} \Rightarrow \boxed{F_e(t) = \frac{1}{2}\frac{C_o}{g_o}V_P^2 + V_P\frac{C_o}{g_o}v_i(t)}$$

~const. ∴ linear

# Differential Capacitive Transducer

- The net force on the suspended center electrode is

$$F_{net} = F_{er}(t) - F_{el}(t)$$




Do the math.

Assume matched gaps.

$$F_{net}(t) = \frac{1}{2}\frac{\partial C}{\partial x}\left\{[v_R(t)]^2 - [v_L(t)]^2\right\}$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{\left(V_P^2 + 2V_P v(t) + [v(t)]^2\right) - \left(V_P^2 - 2V_P v(t) + [v(t)]^2\right)\right\}$$

$$\therefore \boxed{F_{net}(t) = 2V_P\frac{\partial C}{\partial x}v(t) = 2V_P\frac{C_o}{g_o}v(t)}$$

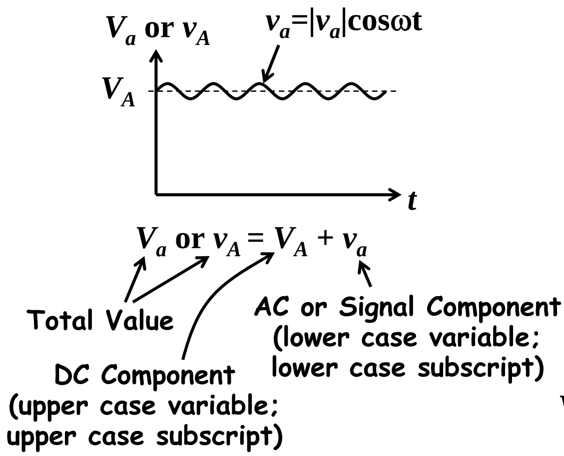
⇒ Linear w/ v(t)! (gap match limited)

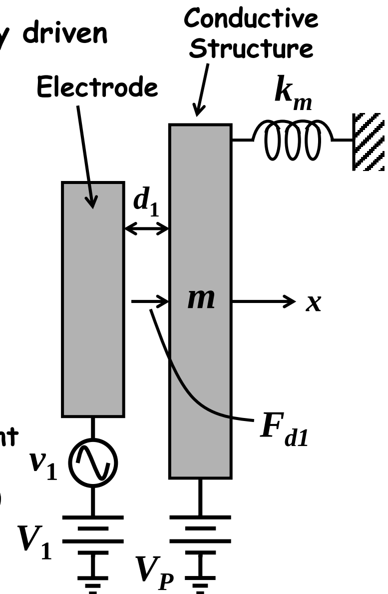


# Remaining Nonlinearity (Electrical Stiffness)

## Parallel-Plate Capacitive Nonlinearity

- Example: clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:






## Parallel-Plate Capacitive Nonlinearity

- Example: clamped-clamped laterally driven beam with balanced electrodes
- Expression for ∂C/∂x:

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_{o1}\left(1 + \frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_{o1}}{d_1}\left(1 + \frac{x}{d_1}\right)^{-2}$$

[Expand the Taylor series further]

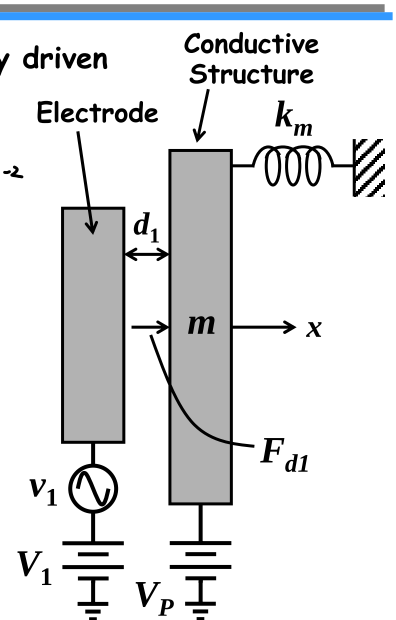
$$\frac{\partial C_1}{\partial x} = -\frac{C_{o1}}{d_1}\left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots\right)$$

where $A_1 = -\frac{2}{d_1}$

$A_2 = \frac{3}{d_1^2}$

$A_3 = -\frac{4}{d_1^3}$

⋮



## Parallel-Plate Capacitive Nonlinearity

- Thus, the expression for force from the left side becomes:

$$F_{d1} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - V_1 - v_i)^2 = \frac{1}{2}\frac{\partial C}{\partial x}(V_{P1} - v_i)^2$$

{small displacements: $x \ll d_1$}

$$F_{d1} = \frac{1}{2}\left(-\frac{C_{o1}}{d_1}\right)(1 + A_1 x)(V_{P1}^2 - 2V_{P1}v_i + v_i^2)$$

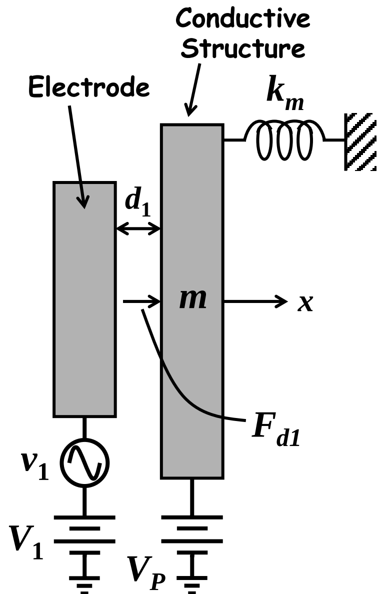
$$= \frac{1}{2}\left(-\frac{C_{o1}}{d_1}\right)\{V_{P1}^2 - 2V_{P1}v_i + v_i^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x v_i + A_1 x v_i^2\}$$

@resonance: $x = \frac{QF_{d1}}{jk} \approx \frac{Q}{jk}\frac{\partial C}{\partial x}V_{P1}v_i$

Thus:

$v_i = |v_i|\cos\omega_o t \rightarrow x = |x|\sin\omega_o t$

$x$ 90° phase-shifted from $v_i$






## Parallel-Plate Capacitive Nonlinearity

- Retaining only terms at the drive frequency:

$$F_{d1}\Big|_{\omega_o} = V_{P1}\frac{C_{o1}}{d_1}|v_1|\cos\omega_o t + V_{P1}^2\frac{C_{o1}}{d_1^2}|x|\sin\omega_o t$$

Drive force arising from the input excitation voltage at the frequency of this voltage

Proportional to displacement

90° phase-shifted from drive, so in phase with displacement

- These two together mean that this force acts against the spring restoring force!
  - A negative spring constant
  - Since it derives from $V_P$, we call it the electrical stiffness, given by:

$$k_e = V_{P1}^2\frac{C_{o1}}{d_1^2} = V_{P1}^2\frac{\varepsilon A}{d_1^3}$$

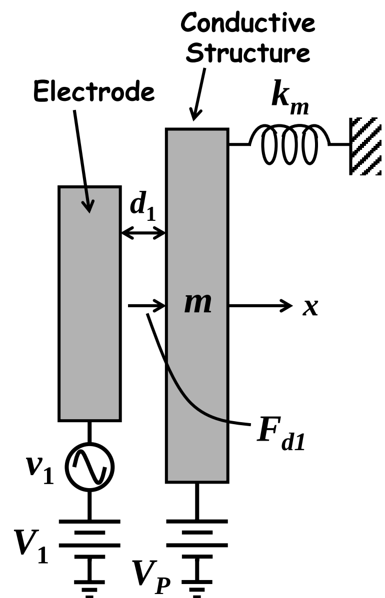
## Electrical Stiffness, $k_e$

- The electrical stiffness $k_e$ behaves like any other stiffness
- It affects resonance frequency:

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}} = \sqrt{\frac{k_m}{m}}\left(1-\frac{k_e}{k_m}\right)^{1/2}$$

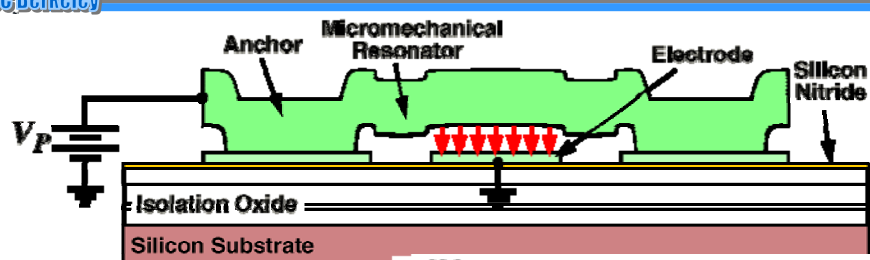
$$\boxed{\omega_o' = \omega_o\left(1-\frac{V_{P1}^2}{k_m}\frac{\varepsilon A}{d_1^3}\right)^{1/2}}$$

Frequency is now a function of dc-bias $V_{P1}$






## Voltage-Controllable Center Frequency

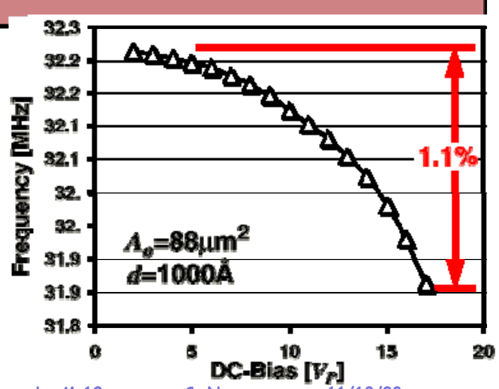



- Quadrature force ⇒ voltage-controllable electrical stiffness:

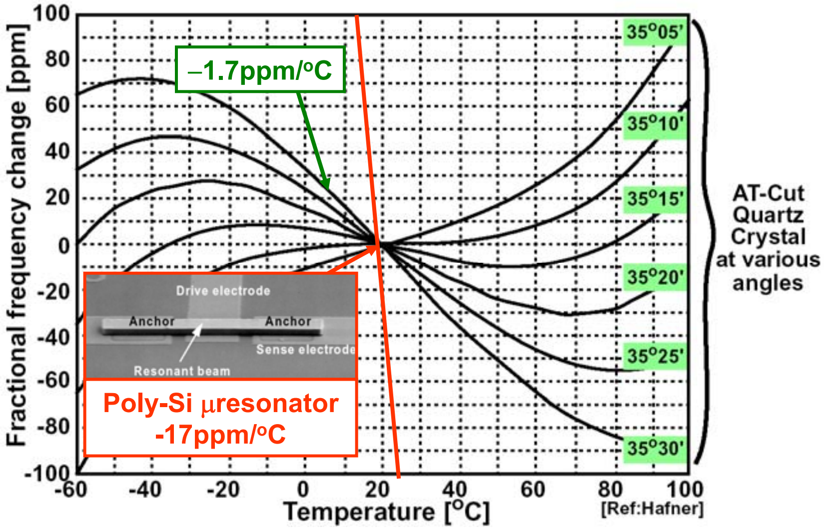
$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

Electrode Overlap Area ($A_o$); Gap ($d$)

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



## Microresonator Thermal Stability

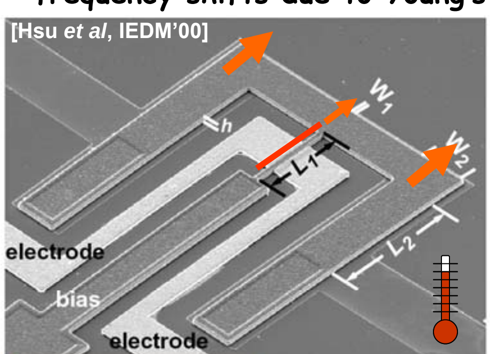


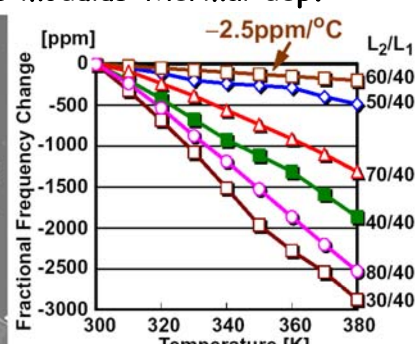

- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

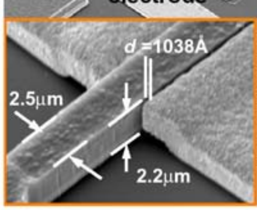


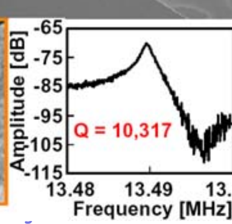
## Geometric-Stress Compensation

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.



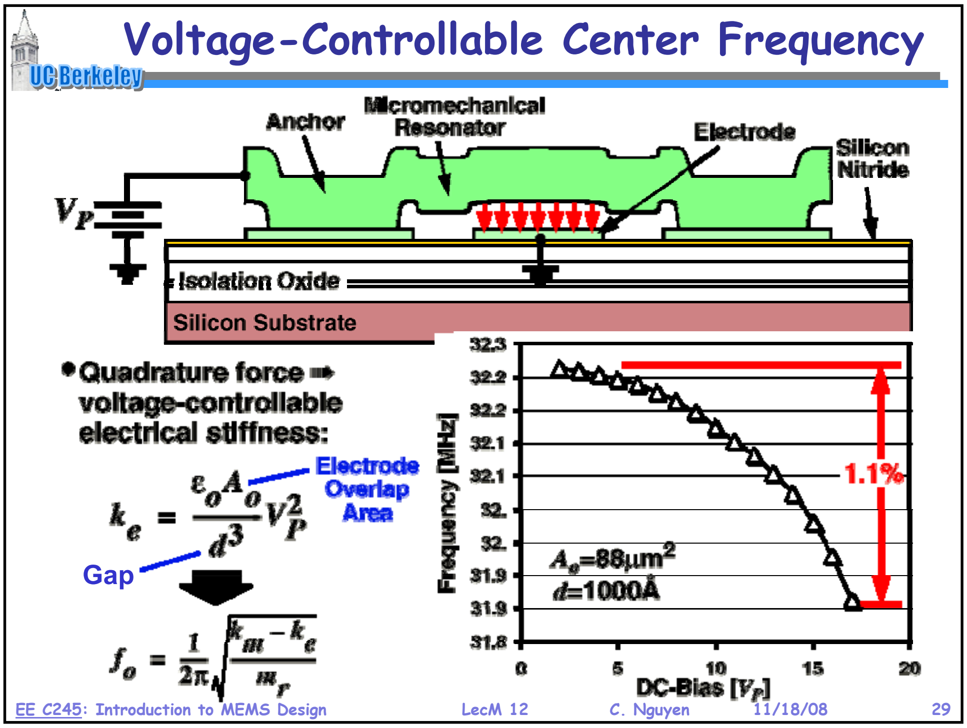




- ***Problems***:
  - stress relaxation
  - compromised design flexibility

[Hsu *et al* IEDM 2000]


Voltage-Controllable Center Frequency
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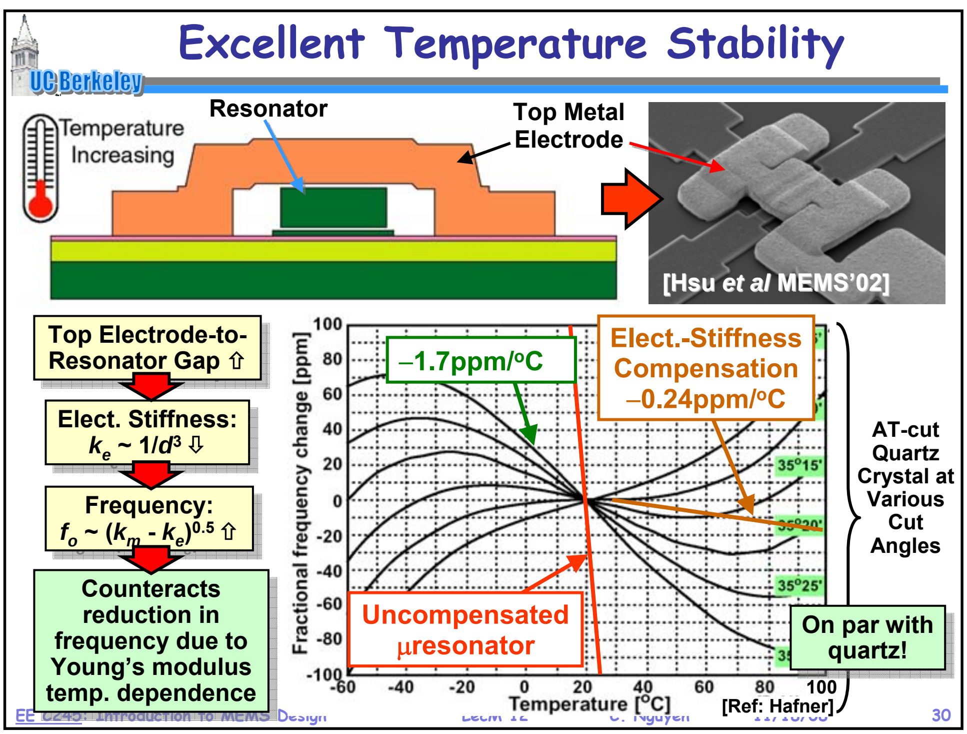
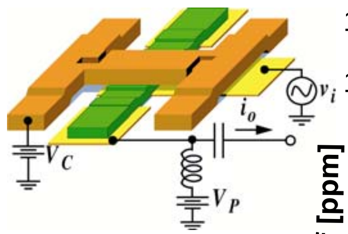
Anchor
Micromechanical Resonator
Electrode
Silicon Nitride
$V_P$
Isolation Oxide
Silicon Substrate
Quadrature force ⇒ voltage-controllable electrical stiffness:
$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$
Electrode Overlap Area
Gap
$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$
Frequency [MHz]
DC-Bias [$V_P$]
1.1%
$A_o$=88μm²
$d$=1000Å
EE C245: Introduction to MEMS Design
LecM 12
C. Nguyen
11/18/08
29

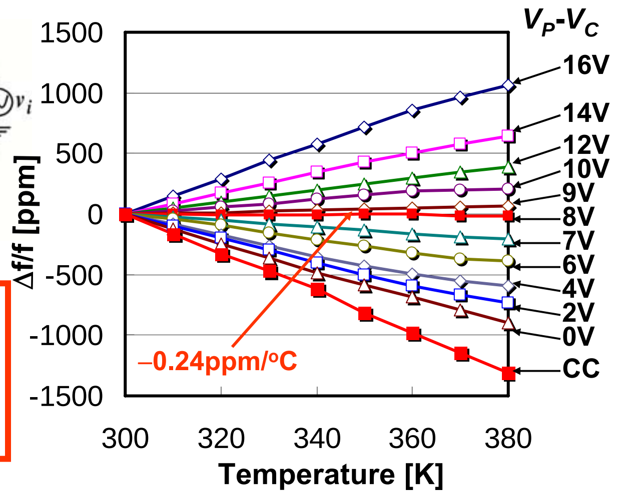

Excellent Temperature Stability
UC Berkeley
Temperature Increasing
Resonator
Top Metal Electrode
[Hsu et al MEMS'02]
Top Electrode-to-Resonator Gap ⇧
Elect. Stiffness: $k_e \sim 1/d^3$ ⇩
Frequency: $f_o \sim (k_m - k_e)^{0.5}$ ⇧
Counteracts reduction in frequency due to Young's modulus temp. dependence
–1.7ppm/°C
Elect.-Stiffness Compensation –0.24ppm/°C
Uncompensated μresonator
Fractional frequency change [ppm]
Temperature [°C]
35°15'
35°20'
35°25'
AT-cut Quartz Crystal at Various Cut Angles
On par with quartz!
[Ref: Hafner]
30

## Measured $\Delta f/f$ vs. $T$ for $k_e$-Compensated μResonators




**Design/Performance:**
$f_o$ =10MHz, $Q$=4,000
$V_P$=8V, $h_e$=4μm
$d_o$=1000Å, $h$=2μm
$W_r$=8μm, $L_r$=40μm

[Hsu *et al* MEMS'02]




- Slits help to release the stress generated by lateral thermal expansion → linear $TC_f$ curves → –0.24ppm/°C!!!



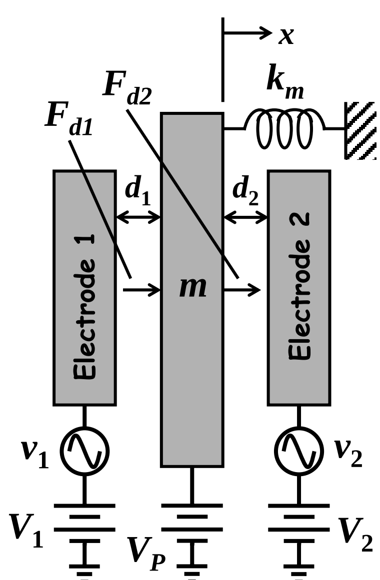
## Can One Cancel $k_e$ w/ Two Electrodes?

- What if we don't like the dependence of frequency on $V_P$?
- Can we cancel $k_e$ via a differential input electrode configuration?
- If we do a similar analysis for $F_{d2}$ at Electrode 2:

Subtracts from the $F_{d1}$ term, as expected

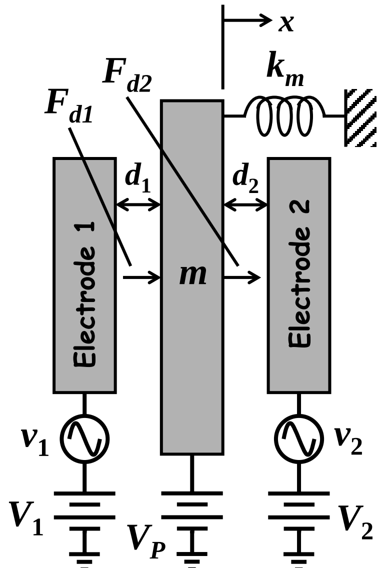
$$F_{d2}\Big|_{\omega_o} = -V_{P2}\frac{C_{o2}}{d_2}|v_2|\cos\omega_o t + V_{P2}^2\frac{C_{o2}}{d_2^2}|x|\sin\omega_o t$$

Adds to the quadrature term → $k_e$'s add, no matter the electrode configuration!



## Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage $V_P$)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches $d_o$)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap … but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

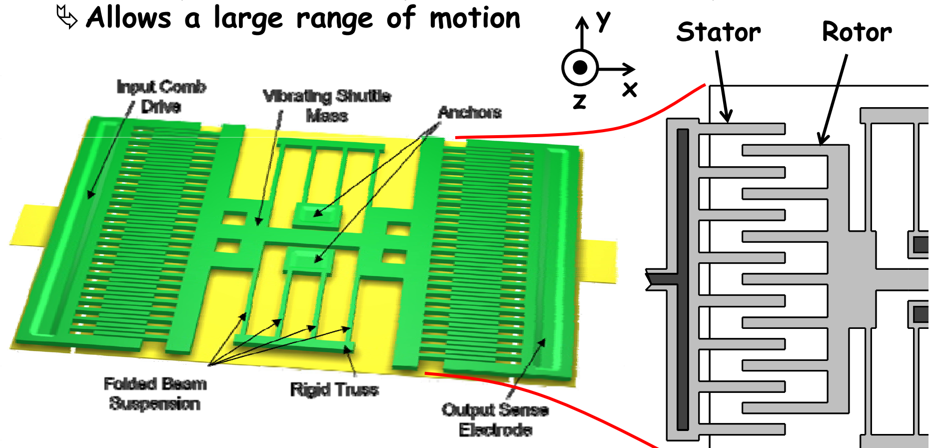





## Electrostatic Comb Drive

## Electrostatic Comb Drive

- Use of comb-capacitive tranducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion

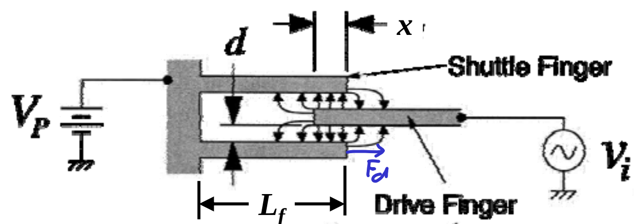


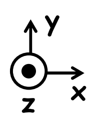

Comb-Driven Folded Beam Actuator



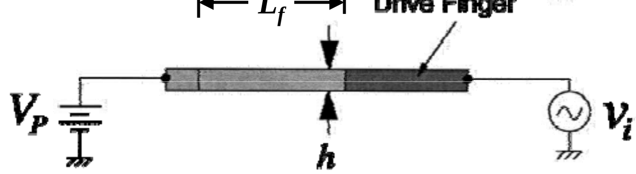
## Comb-Drive Force Equation (1st Pass)

Top View




Side View




$$C(x) = \frac{2\epsilon_o x h}{d} \rightarrow \left[\frac{\partial C}{\partial x} = \frac{2\epsilon_o h}{d}\right]$$

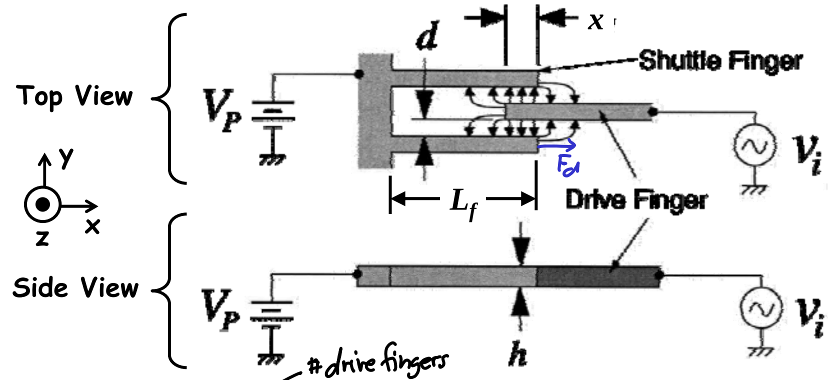
When $v_i = (+) \rightarrow F_d = (-)$ ✓

$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2 = \frac{2}{2}\frac{\epsilon_o h}{d}(V_P^2 - 2V_P v_i + v_i^2) \approx \boxed{-2V_P\frac{\epsilon_o h}{d}v_i = F_d}$$

But wait! This ignores other practical effects!

(No dependence on x! LINEAR!)

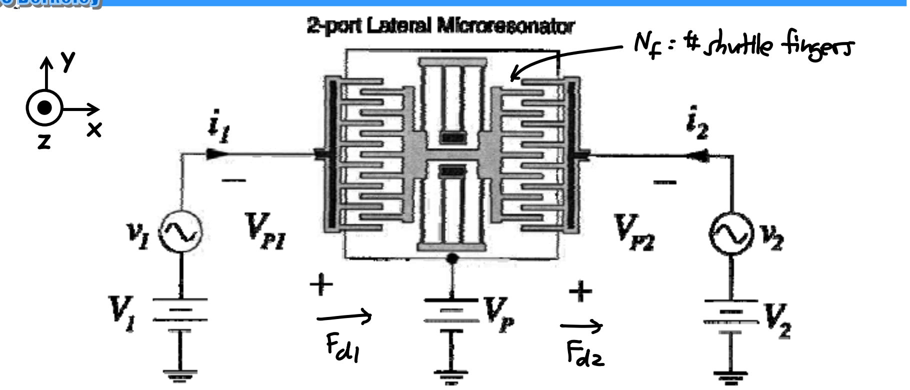
## Lateral Comb-Drive Electrical Stiffness




- Again: $C(x) = \frac{2N\varepsilon hx}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2N\varepsilon h}{d}$
- No ($\partial C/\partial x$) x-dependence → no electrical stiffness: $k_e = 0$!
- Frequency immune to changes in $V_P$ or gap spacing!



## Typical Drive & Sense Configuration




Simple Analysis:

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(v_i - V_{P1})^2 = \frac{1}{2}\left(-\frac{\epsilon_o h}{d_1}\right)(v_i^2 - 2V_{P1}v_i + V_{P1}^2)(2N_f)$$

$$F_{d2} = \frac{1}{2}\frac{\partial C_2}{\partial x}(v_2 - V_{P2})^2 = \frac{1}{2}\left(\frac{\epsilon_o h}{d_2}\right)(v_2^2 - 2V_{P2}v_2 + V_{P2}^2)(2N_f)$$

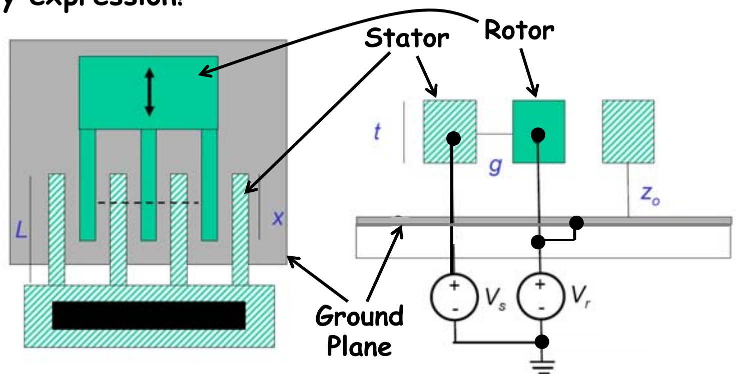
$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2}\left(\frac{\epsilon_o h}{d}\right)\left(v_2^2 - v_i^2 - 2(V_{P2}v_2 - V_{P1}v_i) + V_{P2}^2 - V_{P1}^2\right)(2N_f)$$

For $V_1 = V_2$, $v_1 = -v_2$

$$F_{net} = 2(2N_f)\left(\frac{\epsilon_o h}{d}\right)V_{P1}v_1$$

## Comb-Drive Force Equation (2nd Pass)

- In our 1st pass, we neglected
  - Fringing fields
  - Parallel-plate capacitance between stator and rotor
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!

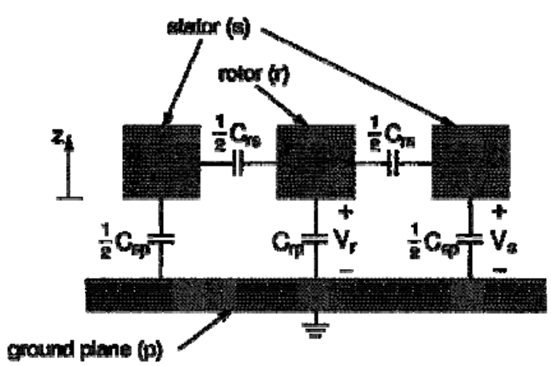





## Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

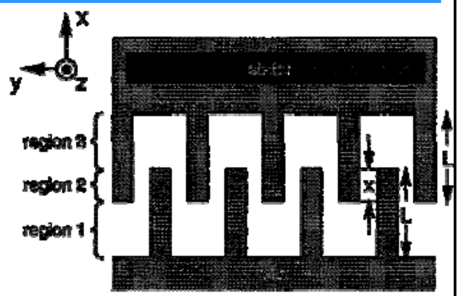
$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$




[Gary Fedder, Ph.D., UC Berkeley, 1994]

## Capacitance Expressions

- Case: $V_r = V_P = 0V$
- $C_{sp}$ depends on whether or not fingers are engaged

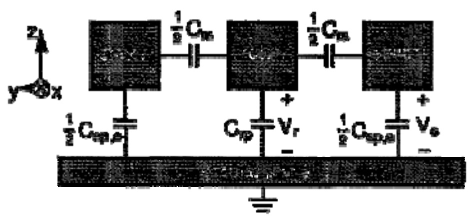



$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

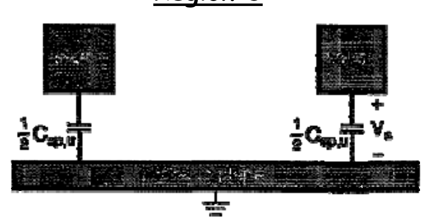
$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length

**Region 2**




**Region 3**




[Gary Fedder, Ph.D., UC Berkeley, 1994]



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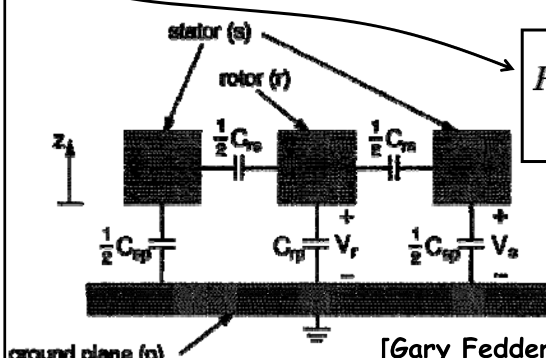


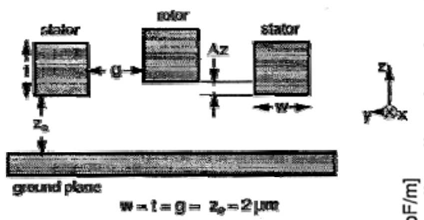

$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$

(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]

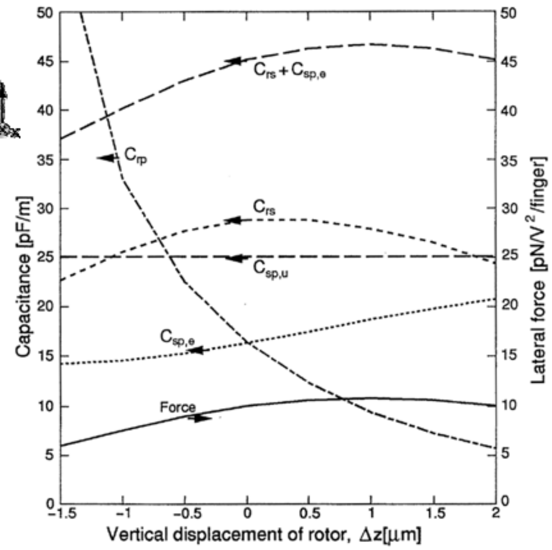
## Simulate to Get Capacitors → Force

- Below: 2D finite element simulation



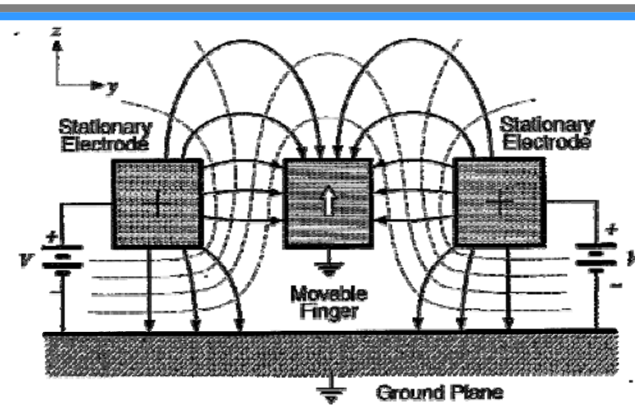

$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$

20-40% reduction of $F_{e,x}$






## Vertical Force (Levitation)




$$F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2}\frac{dC_{sp}}{dz}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dz}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dz}(V_s - V_r)^2$$

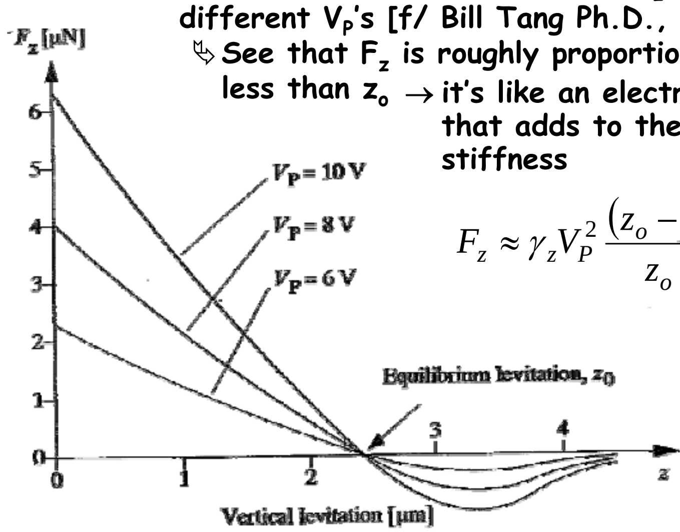
- For $V_r$ = 0V (as shown): $F_{e,z} = \frac{1}{2}Nx\left[\frac{d\left(C'_{sp,e} + C'_{rs}\right)}{dz}\right]V_s^2$

## Simulated Levitation Force

- Below: simulated vertical force $F_z$ vs. z at different $V_P$'s [f/ Bill Tang Ph.D., UCB, 1990]
  - See that $F_z$ is roughly proportional to –z for z less than $z_o$ → it's like an electrical stiffness that adds to the mechanical stiffness

$$F_z \approx \gamma_z V_P^2 \frac{(z_o - z)}{z_o} = k_e (z_o - z)$$

↑ Electrical Stiffness



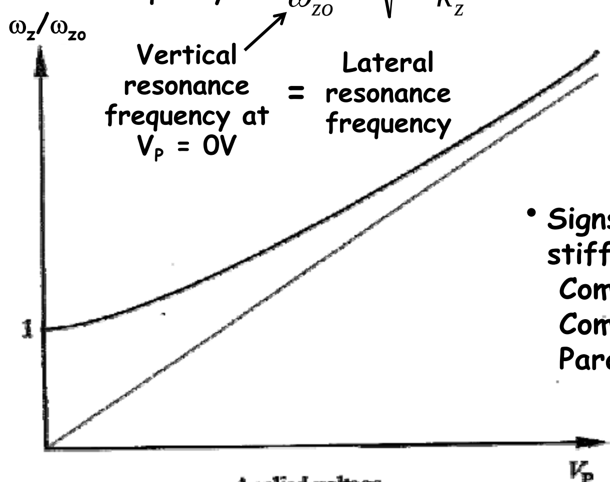



## Vertical Resonance Frequency

$$\frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}} \quad \text{where} \quad k_e = \left(\frac{\gamma_z}{z_o}\right) V^2$$

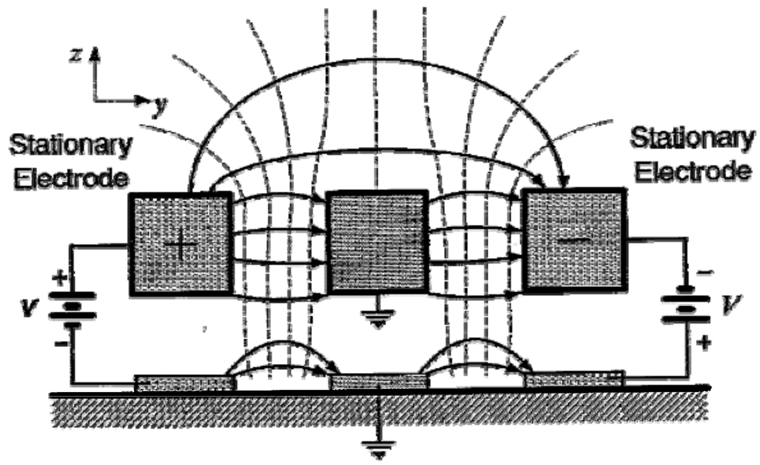
Vertical resonance frequency → $\omega_z$

Vertical resonance frequency at $V_P$ = 0V → $\omega_{zo}$ = Lateral resonance frequency




- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → $k_e$ = 0
  - Comb (z-axis) → $k_e$ > 0
  - Parallel Plate → $k_e$ < 0

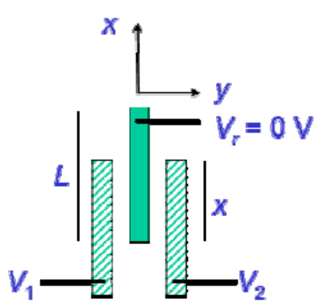
## Suppressing Levitation




- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced



## Force of Comb-Drive vs. Parallel-Plate




gap = $g$ = 1 μm,
thickness = $t$ = 2 μm
finger length = $L$ = 100 μm
overlap length $x$ = 75 μm

- Comb drive (x-direction)
  - $V_1 = V_2 = V_S = 1V$

$$F_{e,x} = \frac{\varepsilon_o t}{g} V_s^2$$

- Differential Parallel-Plate (y-direction)
  - $V_1 = 0V,\ V_2 = 1V$

$$F_{e,y} = \frac{1}{2}\frac{\varepsilon_o t x}{g^2} V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\varepsilon_o t x}{g^2} V_2^2}{\frac{2\varepsilon_o t}{g} V_s^2} = \frac{1}{2}\frac{x}{g}$$

Parallel-plate generates a much larger force; but at the cost of linearity