



EE C245 - ME C218
Introduction to MEMS Design
Fall 2009

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 14: Sensing Circuits

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 1



Lecture Outline

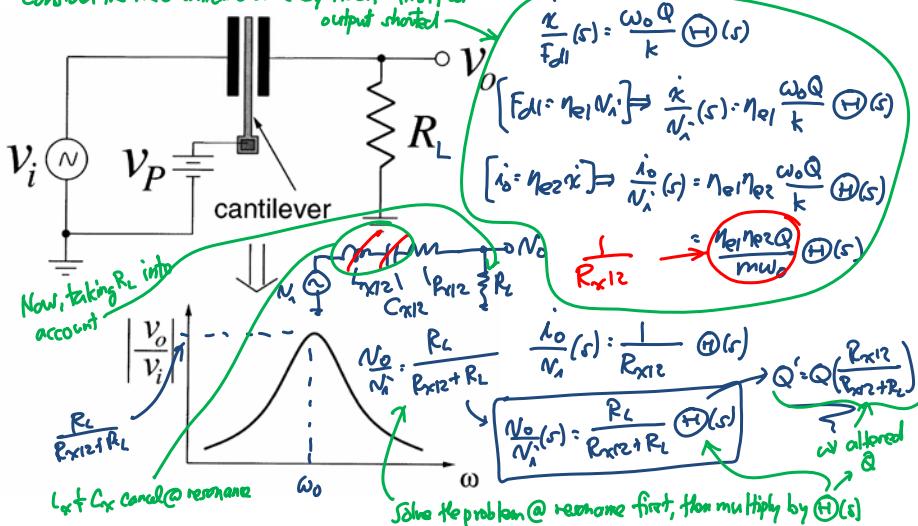
- Reading: Senturia, Chpt. 14
- Lecture Topics:
 - ↳ Detection Circuits
 - Velocity Sensing
 - Position Sensing

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 2

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, we



EE C245: Introduction to MEMS Design

LecM 14

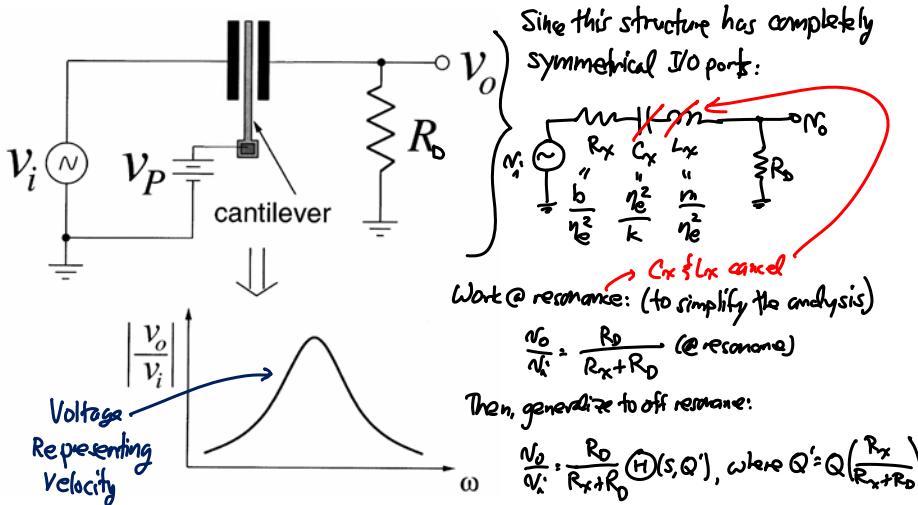
C. Nguyen

11/18/08

3

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



EE C245: Introduction to MEMS Design

LecM 14

C. Nguyen

11/18/08

4

Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$Q = \frac{\omega_0 L_x}{R_x} \rightarrow Q' = \frac{\omega_0 L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_0}{Q'}$$

Brute force approach:

$$\frac{N_0(s)}{V_i} = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_x C_x}{sR_x C_x + 1 + sL_x C_x + sR_D C_x} = \frac{s \frac{R_D}{L_x}}{s^2 + s \frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$$= \frac{R_D}{R_x + R_D} \frac{s \left(\frac{R_x + R_D}{L_x} \right)}{s^2 + s \left(\frac{R_x + R_D}{L_x} \right) + \frac{1}{L_x C_x}}$$

$$\Rightarrow \frac{N_0(s)}{V_i} = \frac{R_D}{R_x + R_D} \frac{s (\omega_0 / Q')}{s^2 + s (\omega_0 / Q') + \omega_0^2} = \frac{R_D}{R_x + R_D} H(s, Q')$$

$Q' = Q \left(\frac{R_x}{R_x + R_D} \right)$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 5

Position-to-Voltage Conversion

UC Berkeley

- To sense position (i.e., displacement), use a capacitive load

Again, here port-to-port I/O symmetry:

Brute force approach:

$$\frac{N_0(s)}{V_i} = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

$$\Rightarrow \frac{N_0(s)}{V_i} = \frac{\frac{1}{sC_D}}{sR_x C_x + 1 + s^2 L_x C_x + \frac{1}{sC_D}} = \frac{\frac{1}{sC_D}}{1 + \frac{sR_x C_x}{1 + C_x/C_D} + s^2 \frac{L_x C_x}{1 + C_x/C_D}}$$

$$\Rightarrow \frac{N_0(s)}{V_i} = \frac{\frac{1}{sC_D}}{1 + \frac{sR_x C_x}{1 + C_x/C_D} + s^2 \frac{L_x C_x}{1 + C_x/C_D}}$$

$$= \frac{C_x/C_D}{1 + C_x/C_D} \frac{\frac{1}{sC_D}}{s^2 + s \left(\frac{R_x}{L_x} \right) + \frac{(1 + C_x/C_D)}{L_x C_x}}$$

$$\left[\omega_0^2 = \frac{1}{L_x C_x} \rightarrow (\omega_0')^2 = \omega_0^2 (1 + C_x/C_D) \right]$$

$$\left[Q' = \frac{\omega_0' L_x}{R_x} = \frac{R_x}{L_x} \frac{\omega_0'}{Q'}, Q' = Q \sqrt{1 + C_x/C_D} \right] \text{ over}$$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 6

Position-to-Voltage Conversion

UC Berkeley

- To sense position (i.e., displacement), use a capacitive load

$$\frac{V_o}{V_i}(s) = \frac{C_x/C_0}{1 + C_x/C_0} \frac{(w_0')^2}{s^2 + (w_0')^2 + (w_s')^2}$$

DC Gain Term Low-Pass Biquad

To maximize gain $\rightarrow 1$, need $C_0 \ll C_x$.
 (must minimize C_0)

Note: Can we similar short-cut to the R case.

- ① Get DC response $\rightarrow C$'s dominate.
- ② Then:

$$\frac{V_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, w_0', Q') \cdot w_0' Q'$$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 7

Velocity Sensing Circuits

UC Berkeley

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 8

Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D}$$

Work @ resonance: (to simplify the analysis)

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D} \quad (@\text{resonance})$$

Then, generalize to off resonance:

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D} H(s, Q'), \text{ where } Q' = Q \left(\frac{R_x}{R_x + R_D} \right)$$

Cx & Lx cancel

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 9

Problems With Purely Resistive Sensing

UC Berkeley

Now, we get: (approximately)

$$\frac{V_o}{V_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot H(s, \omega_b, Q')$$

$\omega_p = \frac{1}{(R_x || R_D) C_p}$

Depend on both R_D & C_p .

Impact depends on where ω_p is relative to ω_b .

Not Good *Okay*

ω_c ω_b ω_D

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 10

Problems With Purely Resistive Sensing

UC Berkeley

- In general, the sensor output must be connected to the inputs of further signal conditioning circuits \rightarrow input R_i of these circuits can load R_D

These change w/ hook-up \rightarrow not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

Soln: use op amps.

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 11

The TransR Amplifier Advantage

UC Berkeley

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p and R_i
- The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground
 \Rightarrow No voltage across C_p
 $\hookrightarrow C_p$ effectively isn't there!

$V_0 = -\frac{R_2}{R_x} H(s) i_1$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 12

 UC Berkeley

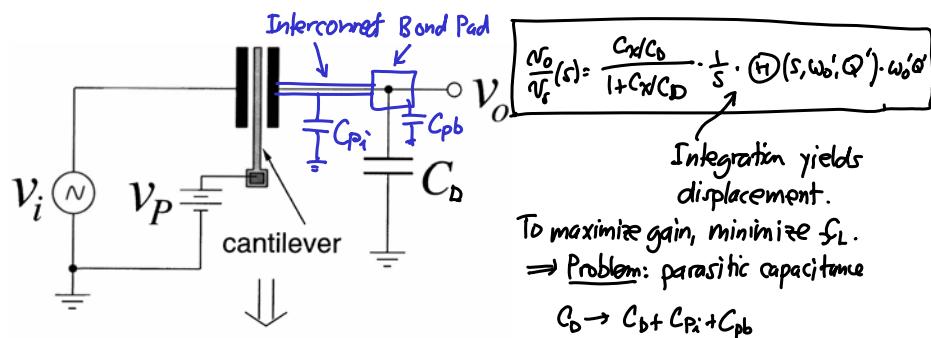
Position Sensing Circuits

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 13

 UC Berkeley

Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load

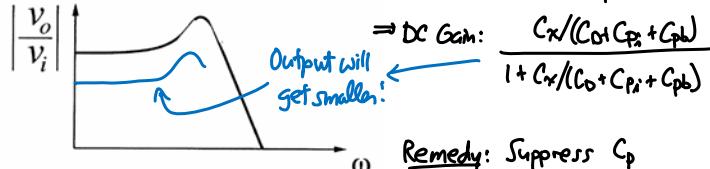


$$\frac{V_O}{V_i} = \frac{C_{pi}/C_b}{1 + C_x/C_b} \cdot \frac{1}{s} \cdot H(s, w_0, Q') \cdot w_0 \alpha$$

Integration yields displacement.

To maximize gain, minimize f_L .
 ⇒ Problem: parasitic capacitance

$C_b \rightarrow C_b + C_{pi} + C_{pb}$



$$\Rightarrow \text{DC Gain: } \frac{C_x/(C_b + C_{pi} + C_{pb})}{1 + C_x/(C_b + C_{pi} + C_{pb})}$$

Remedy: Suppress C_p via use of op amps.

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 14

The Op Amp Integrator Advantage

The diagram shows a MEMS mass-spring system. A mass m is attached to a spring with stiffness k , which is fixed to a wall. The mass is positioned between two electrodes, Electrode 1 and Electrode 2. The distance from the wall to the center of the mass is b . The distance from the wall to the top of Electrode 1 is d_1 , and the distance from the bottom of Electrode 2 to the center of the mass is d_2 . A force F_{dI} acts on Electrode 1. A voltage source v_1 is connected to Electrode 1, and a voltage V_P is connected to ground. The output current i_1 is measured at Electrode 1. A parasitic capacitance C_p is shown between the mass and ground.

The op-amp circuit consists of a non-inverting input terminal connected to the mass through a capacitor C_2 and a resistor R_2 . The inverting input terminal is connected to ground through a capacitor C_p and a resistor R_2 . The output voltage v_0 is taken from the non-inverting input terminal. The feedback path from the inverting input to the output is through a resistor $R_o = 0.5\Omega$.

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p

Handwritten notes:

$$N_o = -i_o \left(R_2 / (sC_2) \right)$$

$$\approx - \frac{V_i}{R_X} \frac{1}{sC_2} \text{ well defined! good!}$$

$$\frac{N_o}{N_s}(s) = - \frac{1}{R_X C_2} \frac{H(s)}{s}$$

(for biasing)

$R_2 >> \frac{1}{sC_2}$

$R_o = 0.5\Omega$

Can drive next stage's R_i w/o interference to transfer function!

Differential Position Sensing

EE C245: Introduction to MEMS Design

LecM 14

C. Nguyen

11/18/08

16

Differential Position Sensing

Example: ADXL-50

Tethers with fixed ends

Proof Mass

Sense Finger

Applied Acceleration

Fixed Electrodes

V_p

C_1

C_2

V_o

C_p

$V_b = -V_p + \frac{2V_p}{C_1+C_2}$

$$= -\frac{V_p C_1 - V_p C_2 + 2V_p C_1}{C_1+C_2} = V_p \left(\frac{C_1 \cdot C_2}{C_1+C_2} \right) = V_o$$

Issue: Parasitic Capacitance

$$V_b = \frac{C_1 \cdot C_2}{C_1+C_2+C_p} V_p$$

As before, C_p reduces gain → Soln: Use op amp!

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 17

Buffer-Bootstrapped Position Sensing

$+V_p$

$-V_p$

V_0

C_p

C_{gd}

Unity Gain Buffer

$C_{gd} = \text{gate-to-drain capacitance of the input MOS transistor}$

Includes capacitance from interconnects, bond pads, and C_{gs} of the op amp

- Bootstrap the ground lines around the interconnect and bond pads
 - No voltage across C_p
 - It's effectively not there!

Interconnect

Ground Plane

1x

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 18

Effect of Finite Op Amp Gain

UC Berkeley

Total ADXL-50 Sense $C \sim 100\text{fF}$

$$+V_P \quad -V_P \quad N_o = A_o(N_i^+ - N_i^-) = A_o(N_i^+ - N_o) \rightarrow N_o(1 + A_o) = A_o N_i^+ \rightarrow \frac{N_o}{N_i^+} = \frac{A_o}{1 + A_o}$$

$$\text{Get } Z_a = \frac{V_o}{i_{i^+}}: \quad i_{i^+} = (N_i^+ - N_o) s C_p = N_i^+ \left(1 - \frac{A_o}{1 + A_o}\right) s C_p = N_i^+ \frac{1}{1 + A_o} s C_p$$

$$\therefore \frac{N_i^+}{i_{i^+}} = Z_a = \frac{1}{s \left[\frac{C_p}{1 + A_o} \right]} \quad \boxed{C_{eff} = \frac{C_p}{1 + A_o}}$$

No longer zero!

Ex: $A_o = 100, C_p = 2\text{pF}$
 $\Rightarrow C_{eff} = \frac{2\text{pF}}{101} = 20\text{fF}$

Not negligible compared w/ ADXL-50 $C_{tot} \sim 100\text{fF}$!

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 19

Integrator-Based Diff. Position Sensing

UC Berkeley

$$+V_P \quad -V_P \quad i_1 \quad i_2 \quad i_o = i_1 + i_2 = N_p(sC_1) - N_p(sC_2) = V_p s(C_1 - C_2)$$

$$\therefore N_o = -i_o \left(\frac{1}{sC_F} \right) = -N_p \left(\frac{C_1 - C_2}{C_F} \right)$$

$$\boxed{\frac{N_o}{V_p} = -\frac{C_1 - C_2}{C_F}}$$

Can drive next stage's R_i w/o interference to transfer function!

A seemingly perfect differential Sensor/amplifier output! ...but only when the op amp is ideal...

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 20