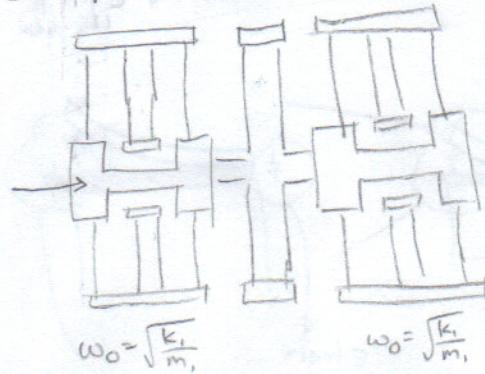


Discussion 11/15

① Coupled structures, estimated modes.



$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\omega_0 = \sqrt{\frac{k_1}{m_1}}$$

$$k_1 \approx k_c \\ m_1 \approx m_{sh}$$

Consider the coupled folded flexure resonator.

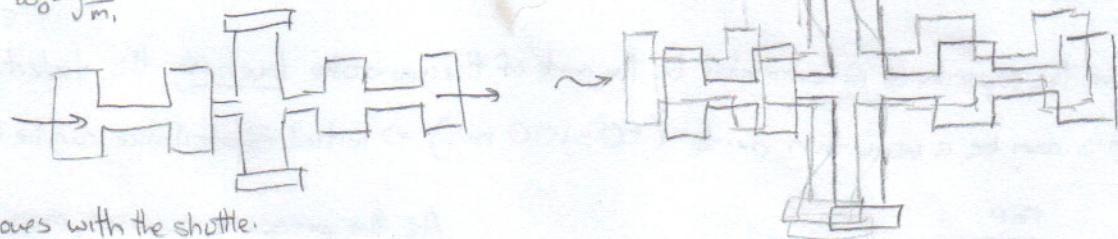
Assume one resonator has a characteristic resonance ω_0 .

Are the resonances of the new system higher or lower?

How many are they? What is their mode shape?

I probably should have used 2 colors... just imagine a translation from left to right

Mode 1: IN-PHASE:



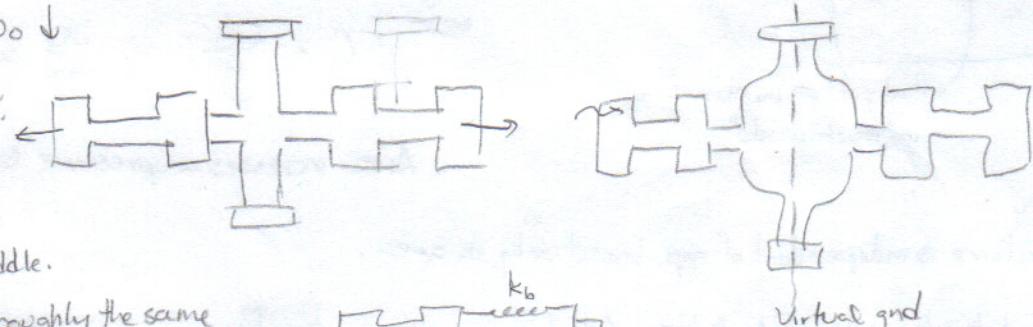
Hence the coupling beam moves with the shuttle.

\Rightarrow the effective dynamic mass is increased.

The coupling beams add no stiffness since both shuttles displace the same. (i.e. they do not flex, Like HWSP1 & midterm)

$$\Rightarrow \omega'_0 = \sqrt{\frac{k}{m+m_c}} \Rightarrow \omega_0 \downarrow$$

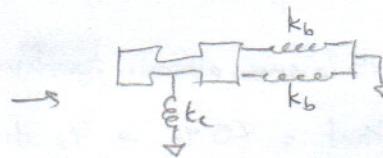
Mode 2: OUT-OF-PHASE:



We now have a half-circuit with virtual ground in the middle.

The effective mass remains roughly the same while the stiffness increases.

$$\Rightarrow \omega'_0 = \sqrt{\frac{k+k_c}{m}} \Rightarrow \omega_0 \uparrow$$

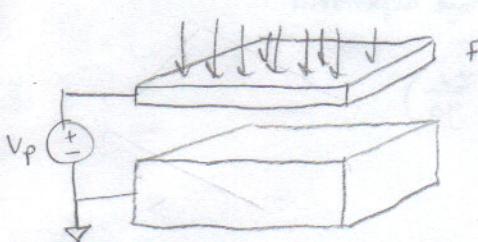


② Comb drive vs parallel-plate. Linearization of parallel-plate sensors.

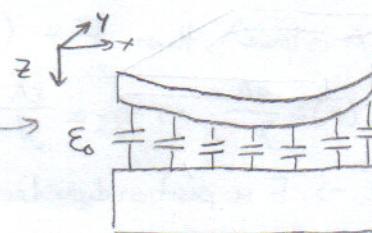
$$\frac{dC}{dx} = \text{const} \quad \frac{dC}{dx} \propto \frac{1}{x^2} \Rightarrow \text{more dynamically sensitive} \Rightarrow \text{good for sensing}$$

\hookrightarrow easier to control \Rightarrow good for driving

Consider a capacitive pressure sensor whose membrane deflects due to a static pressure.



Fixed boundaries

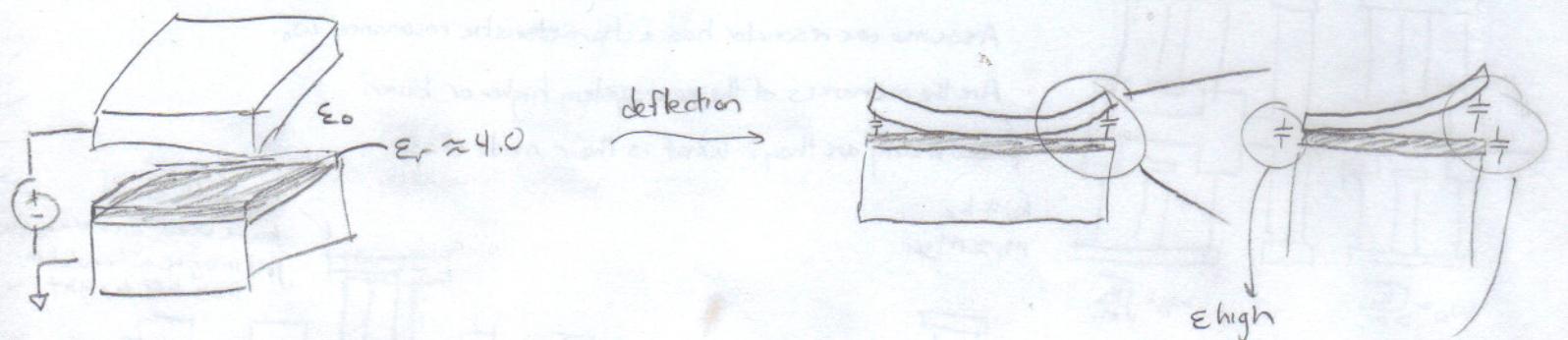


$\frac{dC}{dz} \propto \frac{1}{z^2}$, and changes over area (which can be integrated from HW1 p2)

Sensitivity of $\frac{1}{z^2}$ is good, but not always desirable.

How can we linearize $\frac{dC}{dz}$?

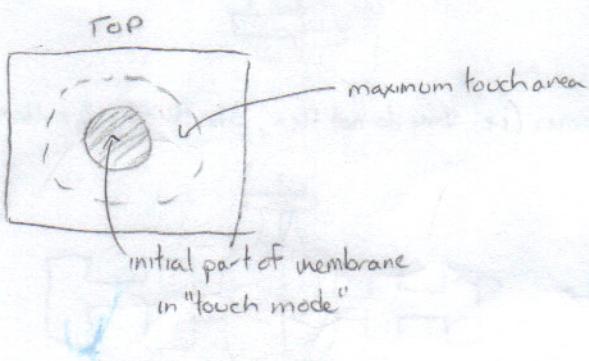
(2) Now consider a "touch-mode" capacitive sensor with a high dielectric in between the membrane and ground.



Here the capacitance is dominated by the part of the membrane taking the dielectric.

This can be a very thin oxide (50-100 nm) \Rightarrow initial capacitance can be high!

Series caps
dominated by
 $\epsilon_0 \rightarrow$ neglect



As the pressure increases, more of the membrane area touches the dielectric. Assuming parts of the membrane not in contact contribute negligible capacitance,

$$C = \frac{\epsilon A}{g} \Rightarrow \frac{dC}{dx} = \frac{\epsilon A'}{g}$$

Area increases as pressure is applied $\Rightarrow \frac{dC}{dx}$ is predictable
controllable
no pull-in *

Capacitance is independent of gap, based only on area.

Typical touch modes fabricated w/ 15% area already touching.

Most stable area that can make contact is 60% \rightarrow 4x shift in capacitance.

Issues: Pull-in still exists. After repeated use membrane can stick to dielectric.

High stress at fixed boundaries - can cause interconnect to break!

(3) Basic review / what to expect this week. (Why does this matter?)

$$\text{Voltage controlled force: } F = \frac{1}{2} \frac{dC}{dx} V_p^2$$

If $\frac{dC}{dx}$ is constant (CCA is linear) then $F = (\text{constant}) V_p^2 \rightarrow$ only voltage dependent

$$\text{But take parallel plate: } C(x) = \frac{\epsilon A}{g_0 - x} \Rightarrow \frac{dC}{dx} = \frac{\epsilon A}{g_0^2} \left(1 - \frac{x}{g_0}\right)^{-2} \approx \frac{C_0}{g_0} \left(1 + \frac{2x}{g_0}\right)$$

$\frac{dC}{dx}$ is position dependent \rightarrow F is position dependent!

Instability and positive feedback can result easily.

Clark will discuss compensation schemes this week.