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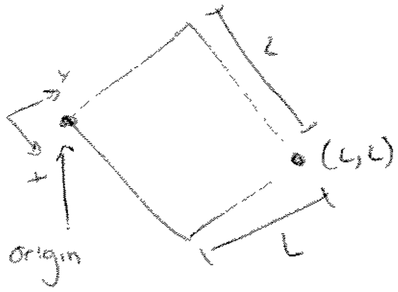
① Problem 2. (Senturia 9.7)



Fixed boundary conditions: $z(0,0)=z(0,L)=z(L,0)=z(L,L)=0$

Removal of infinite series: Hint. $\int_0^\pi \sin ax \cdot \sin bx \, dx = 0 \ \forall a \neq b$

$$\iint_0^L \sum_{x,y} Z_{x,y} \sin A_x x \sin B_y y = \text{?} \quad \text{hard to solve}$$

$$(\sin A_x x \sin B_y y)\iint_0^L \sum Z_{x,y} \sin A_x x \sin B_y y$$

$$= (\qquad\qquad) \iint (\sin A_1 x + \ldots + \sin A_\infty x)(\sin B_1 y + \ldots) \sin B_\infty y)$$

$$= \iint \sin^2 A_x x \sin^2 B_y y \cdot Z_{x,y}$$

② Domain Analogues (Senturia 5)

	Relation	Relation
Electrical:	$V = IR$	$Q = CV$
Mechanical:	$F = \dot{x} b$	$x = \frac{1}{k} F$
Thermal:	$\Delta T = P \cdot R_{th}$	$Q = mc_p \Delta T$
	resistive relation	capacitive relation

Quantities

$(V, F, \Delta T)$ represent effort variables

$(I, \dot{x}, P)$ represent flow variables

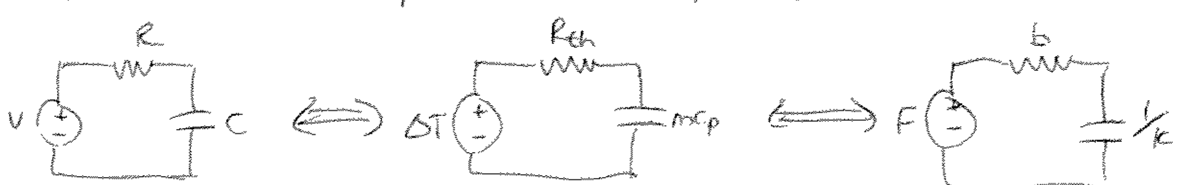
(Q, x, Q) is the accumulation (integral) of flow

(R, b, R_{th}) represent resistance to flow

$(C, \frac{1}{k}, mc_p)$ represent capacity to store effort

So an electrical RC circuit behaves the same in response to voltage/current as a thermal circuit does to temperature/heat flow (power).

i.e.



all circuits have the same time constant $\tau = RC$!

they also have the same steady state response in terms of their respective domains

For thermal circuits remember:

$\boxed{V \Leftrightarrow \Delta T}$ $\boxed{I \Leftrightarrow P}$ $\boxed{R \Leftrightarrow R_{th}}$ $\boxed{C \Leftrightarrow C_{th} = \rho V c_p}$

can use exact same circuit techniques to solve

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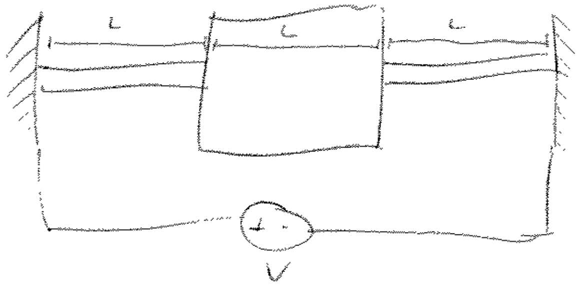
③ Thermal circuit example

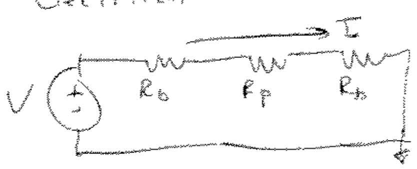


This is a multiphysical problem - need to solve in 2 domains

Important to understand.

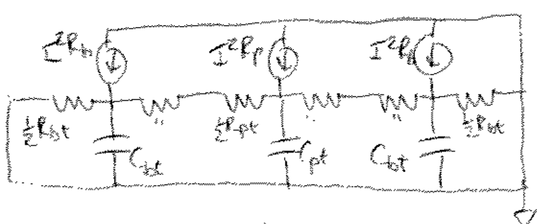
* This is first an electrical circuit! Directly apply V
* Joule heating phenomenon brings into thermal domain
* assume thermal domain does not affect electrical properties

Electrical ckt:

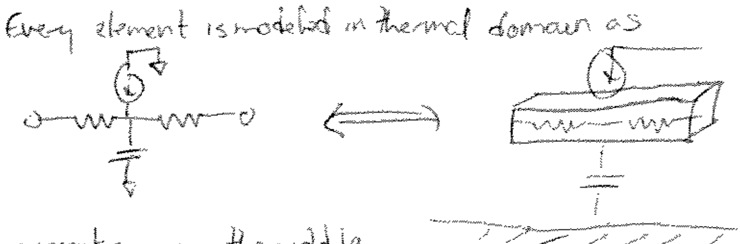


Simple enough. Given dimensions we can solve for each resistance using $R = \frac{\rho L}{A}$. Remember A is cross-sectional area NORMAL to direction of current flow. Then solve for I.

Thermal ckt:



Basics. Every element is modeled in thermal domain as

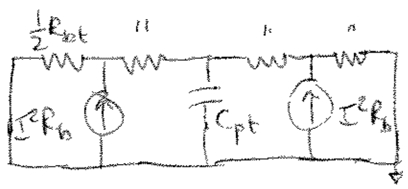


We place current source in the middle otherwise results look nonsensical. (assume heating occurs throughout beam)

Simplify!

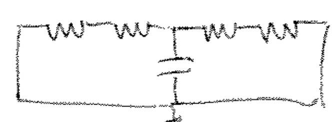
* can remove → R of items with large area

can remove —||— of items with small area



Note power sources are based on electrical resistance!

Time constant: * remove current/power sources, replace with open:



* look at node (only one here) and determine R_{eq} & C_{eq}

here $R_{eq} = \left(\frac{1}{2}R_{bt} + \frac{1}{2}R_{bt}\right) \| \left(\frac{1}{2}R_{bt} + \frac{1}{2}R_{bt}\right) = \frac{1}{2}R_{bt}$ & $C_{eq} = C_{pt}$

$\Rightarrow \tau = \frac{1}{2}R_{bt}C_{pt}$ (check units - this should be in seconds!)

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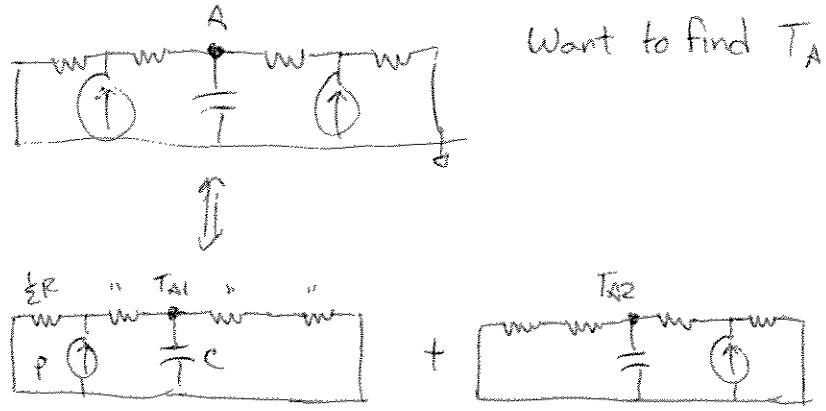
(4) Steady state analysis. Proceed with Kirchoff Laws and superposition.

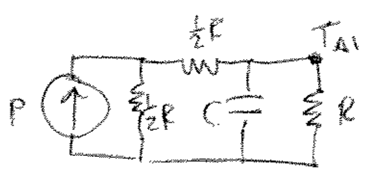


Want to find T_A.

Superposition allows us to deal with one source at a time.

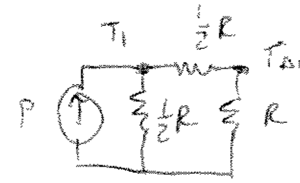
Can easily solve with conventional techniques.

Source 1:



combined $\frac{1}{2}R + \frac{1}{2}R$ in series

For steady state you can replace capacitor with open:



Now use voltage division to get $\frac{T_{A1}}{T_1}$, and ohm's law to get $\frac{T_1}{P}$, to get transfer function between T_{A1} & P. (Do same for P & T_{A2}, then add $T_{A1} + T_{A2} = T_A$)

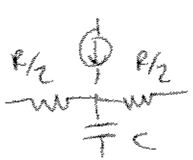
$$\frac{T_{A1}}{T_1} = \frac{R}{R + \frac{1}{2}R}, \quad \frac{T_1}{P} = \frac{1}{2}R \,\|\, (\tfrac{1}{2}R + R) = \frac{3}{8}R \Rightarrow \frac{T_{A1}}{P} = \frac{1}{4}R \quad (T_{A1} = \tfrac{1}{4}PR)$$

So $T_A = \frac{1}{2}PR$. Does this make sense?

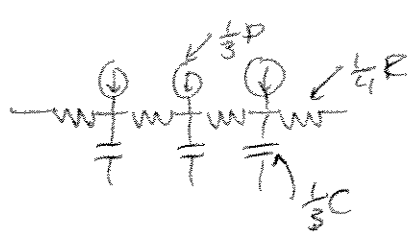
To first order yes - roughly half the input power will heat the plate, the other half shunts through the beams to ground.

The way we have modeled the circuit places the beams/plate at same temp!

Can fix this by DISTRIBUTING the model:



becomes



much more complex!

this is basis for FEA.