EE245 Discussion 9/20/10

Thursday, September 16, 2010
5:10 PM

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10:30-12 on Tuesdays in Cory 481

Today:
Oxidation and diffusion
HW#2

Modeling oxidation

(e.g. the thermal growth of SiO_2 over Si)

$\Rightarrow$ The growth rate decreases as time goes on.

Q: What is diffusion?

A:
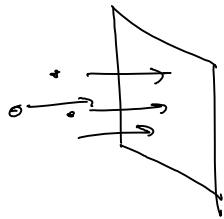
How many particles pass through a unit area per unit time.
e.g. 5×10^{12} molecules/cm^2/s

The diffusion rate (i.e. the time rate of particle flux) ~ [particles/(area·time)] and direction are described by Fick's first law of diffusion:

$$\vec{J}(x,y,z,t) = -D\,\nabla N(x,y,z,t)$$

✶ Recall: $\nabla N = \frac{\partial N}{\partial x}\hat{x} + \frac{\partial N}{\partial y}\hat{y} + \frac{\partial N}{\partial z}\hat{z}$; $\hat{x},\hat{y},\hat{z}$ are unit vectors

Where D is the diffusion coefficient [cm^2/s] and N is the particle concentration [cm^{-3}].

If $\frac{\partial N}{\partial y} = \frac{\partial N}{\partial z} = 0$, this equation simplifies to:

$$J(x,t) = -D\frac{\partial N(x,t)}{\partial x}$$

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<u>Intuition:</u> Particles move from regions of high concentration to regions of low concentration due to random thermal motion.

<u>Objective:</u>
Derive the oxide thickness as a function of t.

For silicon oxidation, assume the oxygen flux is constant everywhere in the oxide. (Oxygen does not accumulate.)
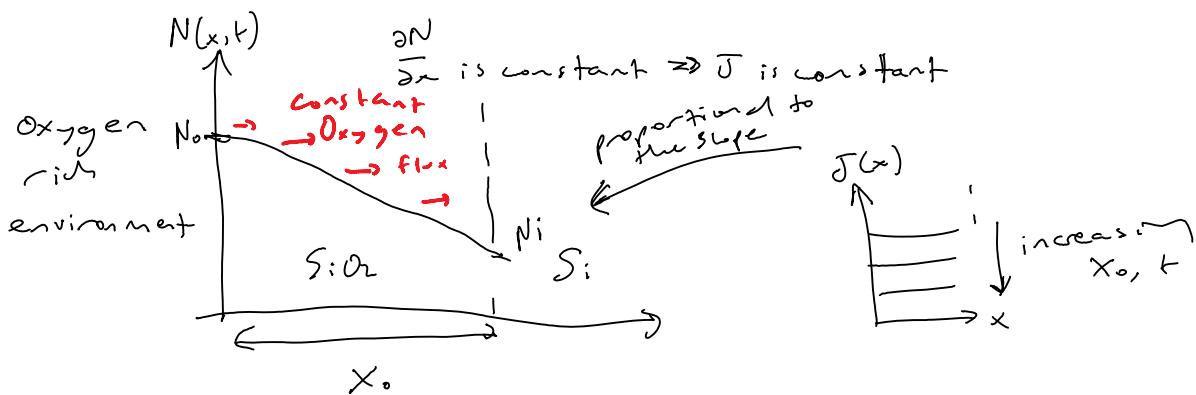
where X_o is the thickness and N_o and N_i are the oxygen concentrations at the oxide's surface and $Si-SiO_2$ interface respectively.

At the interface,

$$J = k_s N_i \Rightarrow N_i = \frac{J}{k_s}, \quad k_s \text{ is the reaction rate.}$$

Substitute,

$$\Rightarrow J = -D\left(\frac{J}{k_s X_o} - \frac{N_o}{X_o}\right)$$

$$J\left(1 + \frac{D}{k_s X_o}\right) = \frac{D N_o}{X_o}$$

$$\Rightarrow \boxed{J = \frac{D N_o}{X_o + \frac{D}{k_s}}}$$

The rate of change of oxide thickness equals the oxidizing flux divided by the number of

molecules M incorporated into a unit volume in the oxide.

$$\frac{dx_o}{dt} = \frac{J}{M} = \frac{D N_o}{M(x_o + \frac{D}{k_s})} \quad \sim \textcolor{red}{\left[\frac{\#}{cm^2 \cdot s} \cdot \frac{cm^3}{\#}\right] = [cm/s] \checkmark}$$

$$\Rightarrow \frac{M}{D N_o}\left(x_o + \frac{D}{k_s}\right) dx_o = dt$$

$$\int_{x_i}^{x_o} \frac{M x_o}{D N_o} + \frac{D}{k_s} \, dx_o = \int_{\tau}^{t-\tau} dt = t$$

$$\Rightarrow \left(\frac{M x_o^2}{2 D N_o} + \frac{M x_o}{N_o k_s}\right) - \left(\frac{M x_i^2}{2 D N_o} + \frac{M x_i}{N_o k_s}\right) = t$$

$$\Rightarrow \quad \tau = \frac{M x_i^2}{2} + \frac{D x_i}{k_s}, \quad \text{Time required to achieve initial oxide thickness } x_i$$

$$t = \frac{M x_o^2}{2 D N_o} + \frac{M x_o}{N_o k_s} - \tau \quad \text{additional time required to achieve a thickness } X_o.$$

Note: You can choose $x_i = 0$, $\tau = 0$ if you'd like.

$$t + \tau = \frac{M x_o^2}{2 D N_o} + \frac{M x_o}{N_o k_s}, \quad \text{time required to grow to a thickness of } X_o.$$

$$\frac{M x_o^2}{2 D N_o} + \frac{M x_o}{N_o k_s} - (t+\tau) = 0 \qquad \text{Use the quadratic formula}$$

$$X_o(t+\tau) = \frac{-\frac{M}{N_o k_s} \textcolor{blue}{\oplus} \sqrt{\frac{M^2}{N_o^2 k_s^2} + 2\frac{M}{D N_o}(t+\tau)}}{\frac{M}{D N_o}} \quad \textcolor{blue}{\left(\frac{\frac{D N_o}{M}}{\frac{D N_o}{M}}\right)}$$

$$= -\frac{D}{k_s} + \sqrt{\frac{D^2}{k_s^2} + 2\frac{D N_o}{M}(t+\tau)}$$

$$\boxed{A = \frac{2D}{k_s}} \quad (\text{Jaeger})$$
$$B = \frac{2 D N_o}{M}$$

$$\Rightarrow \colorbox{yellow}{$X_o(t+\tau) = \frac{D}{k_s}\left[\sqrt{1 + 2\frac{N_o k_s^2}{D M}(t+\tau)} - 1\right]$}$$

$*$ Recall: $\sqrt{1+x} = 1 + \frac{x}{2}$ if $x \ll 1$.

Two useful simplifications:

Reaction rate limited regime:

If $t + \tau \ll \dfrac{DM}{2 k_s^2 N_0}$, then $X_0 (t + \tau) \cong \dfrac{k_s N_0}{M} (t + \tau)$

Diffusion rate limited regime:

If $t + \tau \gg \dfrac{DM}{2 k_s^2 N_0}$, then $X_0 (t + \tau) \cong \sqrt{\dfrac{2 D N_0}{M} (t + \tau)}$

$D (E_A, T, D_0) = \boxed{D_0 \, e^{-E_A / kT}}$ D_0 depends on (100) or (111) orientation

$\Rightarrow$ Huge temperature dependence.

See Jaeger p. 47.

Homework #2

Topics:
  ① etching
 ② diffusion & sheet resistance.
  ③ lithography

Hints:

① etching problem.

Hint 1:   vertical etch rate, e_{V_1}

$e_{V_{1T}} \downarrow$ $e_{V_1} + e_{H_1} \cot \theta_2$

$\longleftarrow$ lateral etch rate, e_{H_1}

$e_{H_1} + e_{V_1} \cot \theta_1$

θ_1

$e_1 \diagdown \theta \diagup e_2$

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Hint 2:

vertical etching only

e_1

$e_1 t$

$e_2 t$

$e_1 t - e_2 t$

θ_1

e_2

x

θ_0

initial conditions
after etching for t seconds
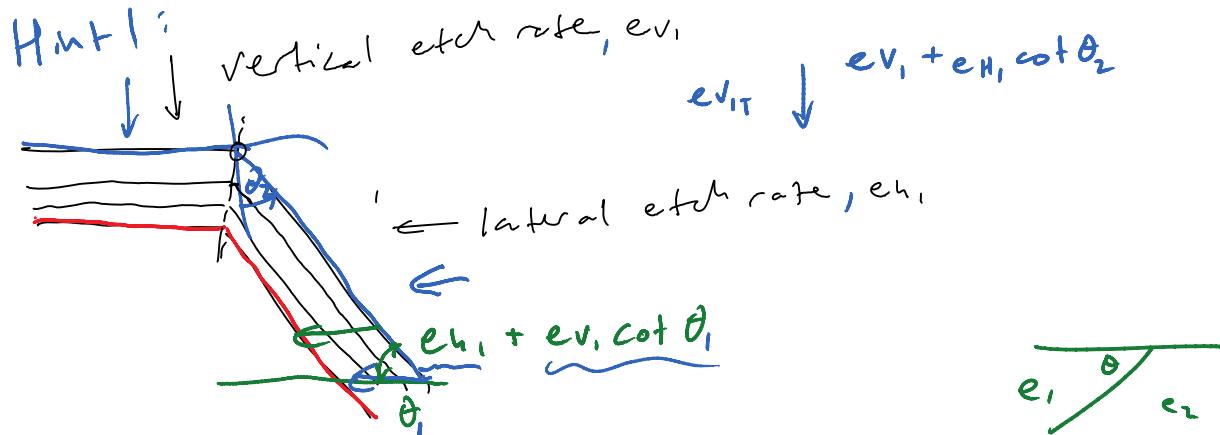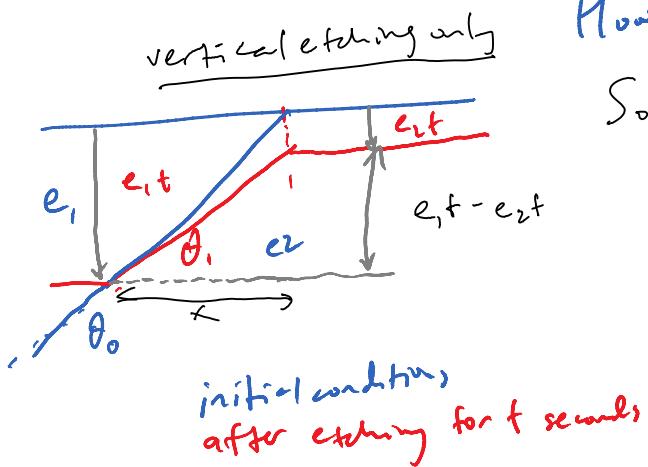
How can we define θ_1 in terms of θ_0?

Solution: $\tan \theta_0 = e_1 t / x$

$\tan \theta_1 = (e_1 t - e_2 t) / x$

$\Rightarrow \dfrac{\tan \theta_1}{e_1 t - e_2 t} = \dfrac{\tan \theta}{e_1 t}$

$$\Rightarrow \boxed{\theta_1 = \tan^{-1}\left(\tan \theta_0 \left(\frac{e_1 - e_2}{e_1}\right)\right)}$$

② Diffusion

$N(x,t)$

increasing time

x_i

x_i

x_i

x
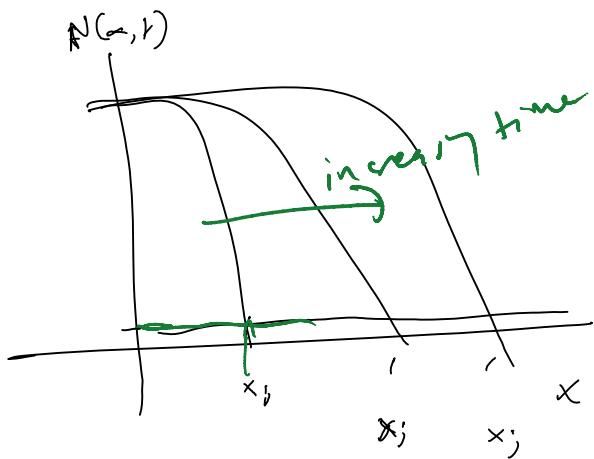
Hint 1: Assume a constant source diffusion.

Hint 2: Can you express Dt as an integral?

Hint 3: a) Use a numerical solver.

Hint 4: b) Integrate over the depth of the diffusion.

c) $R_\square = \left(\displaystyle\int_0^{x_i} q\mu N_0(x)\, dx\right)^{-1} \sim \Omega$

$1.6 \times 10^{-19}\, C$ ↑

$R = R_\square \cdot \text{\# of squares}$

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$R = \dfrac{\rho L}{A} = \dfrac{\rho L}{t w} = \dfrac{\rho}{t}\displaystyle\int_0^t w(t)\, dt$

1 square corner
$= 0.56\ \square$

$$R = \frac{L}{A} = r_t w \quad t \quad = \boxed{0.5b \; L}$$

$A = tw$

$$R = \rho \frac{L}{A} = \frac{\rho}{t} \cdot \frac{L}{w}$$
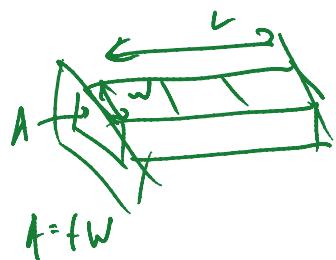
$R_{\square}$ ↗ 3

↘ ↓

③ Assume "underexposed" and "overexposed"