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EE C245 – ME C218 Introduction to MEMS Design Fall 2011

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Lecture Module 7: Mechanics of Materials

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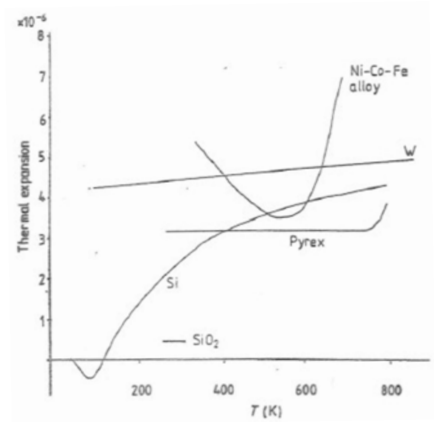
α_T As a Function of Temperature



[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

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MEMS Material Properties

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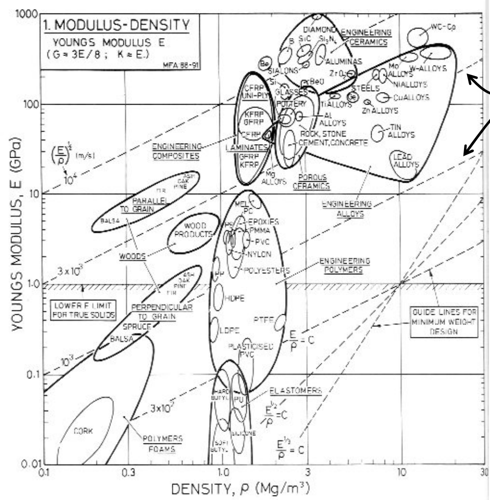
Material Properties for MEMS

Material	Density, ρ, Kg/m³	Modulus, E, GPa	E/ ρ GN/kg-m
Silicon	2330	165	72
Silicon Oxide	2200	73	36
Silicon Nitride	3300	304	92
Nickel	8900	207	23
Aluminum	2710	69	25
Aluminum Oxide	3970	393	99
Silicon Carbide	3300	430	130
Diamond	3510	1035	295

Units: $(m/s)^2$ → $\sqrt{(E/\rho)}$ is acoustic velocity

[Mark Spearing, MIT]

Young's Modulus Versus Density



Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]

Yield Strength

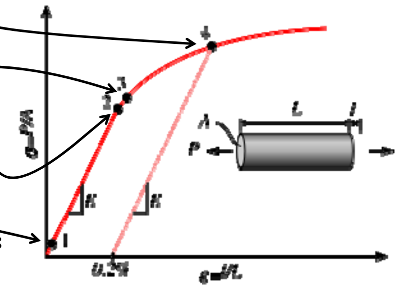
- Definition: the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
- Below the yield point: material deforms elastically → returns to its original shape when the applied stress is removed
- Beyond the yield point: some fraction of the deformation is permanent and non-reversible

Yield Strength: defined at 0.2% offset pt.

Elastic Limit: stress at which permanent deformation begins

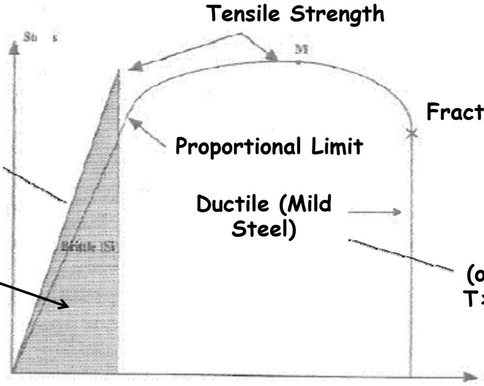
Proportionality Limit: point at which curve goes nonlinear

True Elastic Limit: lowest stress at which dislocations move



Yield Strength (cont.)

- Below: typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g. steel) materials



[Maluf]

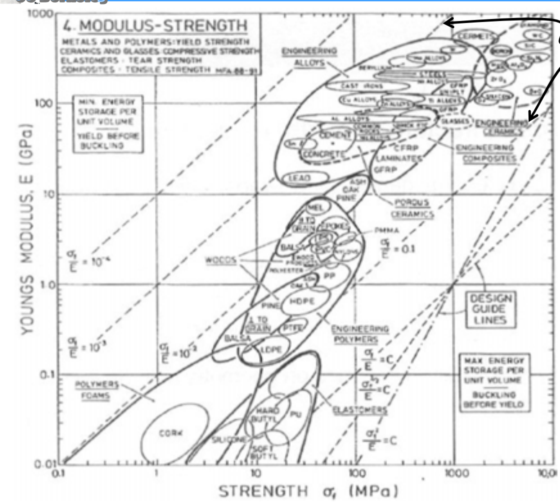
Young's Modulus and Useful Strength

Stored mechanical energy

Material	Modulus, E, GPa	Useful Strength*, σ_f, MPa	$\frac{\sigma_f}{E}$ (-) x 10^{-3}	$\frac{\sigma_f^2}{E}$ MJ/m³
Silicon	165	4000	24	97
Silicon Oxide	73	1000	13	14
Silicon Nitride	304	1000	3	4
Nickel	207	500	2	1.2
Aluminum	69	300	4	1.3
Aluminum Oxide	393	2000	5	10
Silicon Carbide	430	2000	4	9.3
Diamond	1035	1000	1	0.9

From Mark Spearing, MIT, *Future of MEMS Workshop*, Cambridge, England, May 2003

Young's Modulus Versus Strength

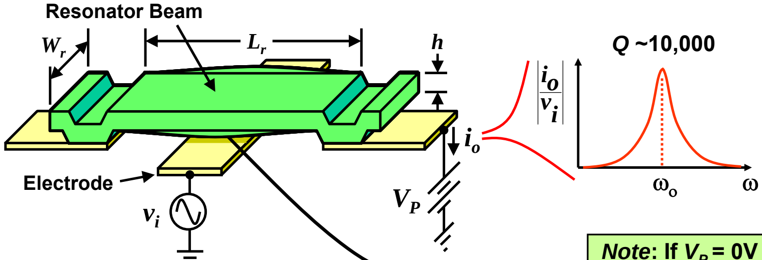


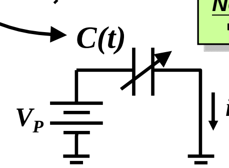
Lines of constant maximum strain

[Ashby, Mechanics of Materials, Pergamon, 1992]

Quality Factor (or Q)

Clamped-Clamped Beam μResonator





$$i_o = V_P \frac{dC}{dt}$$

Note: If V_P = 0V ⇨ device off

Frequency:

Stiffness, Young's Modulus, Density, Mass

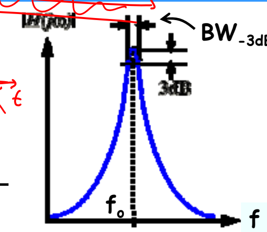
$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L_r^2}$$

(e.g., $m_r = 10^{-13}$ kg) → Smaller mass ⇨ higher freq. range and lower series R_x

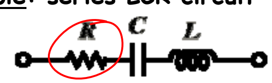
Quality Factor (or Q)

- Measure of the frequency selectivity of a tuned circuit
- Definition:

$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

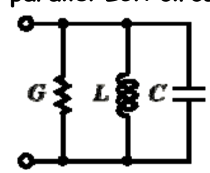


- Example: series LCR circuit



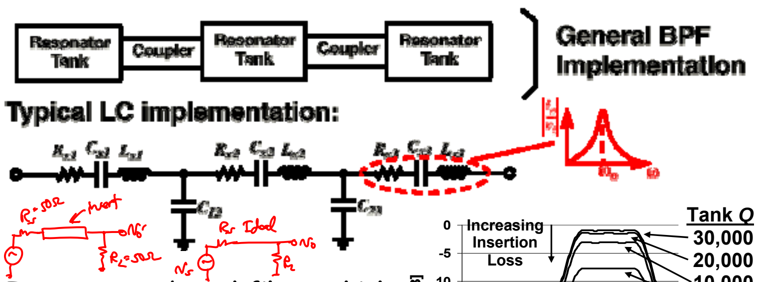
$$\Rightarrow\quad Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

- Example: parallel LCR circuit

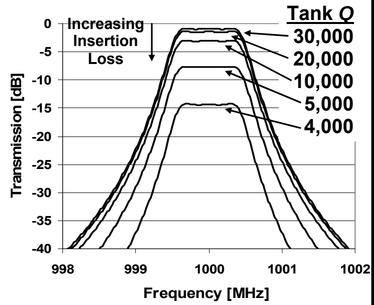


$$\Rightarrow\quad Q = \frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$

Selective Low-Loss Filters: Need *Q*

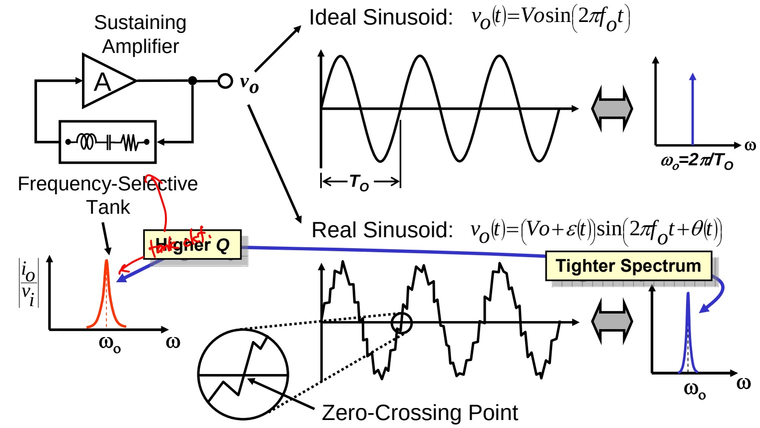


- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - ↳ heavy insertion loss for resonator Q < 10,000



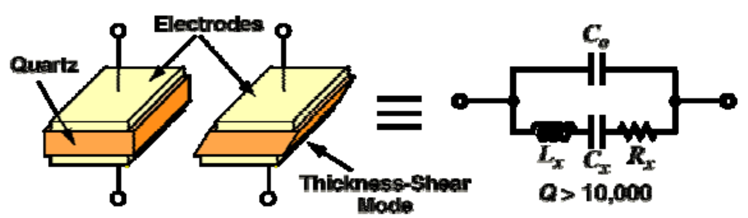
Oscillator: Need for High *Q*

- Main Function: provide a stable output frequency
- Difficulty: superposed noise degrades frequency stability



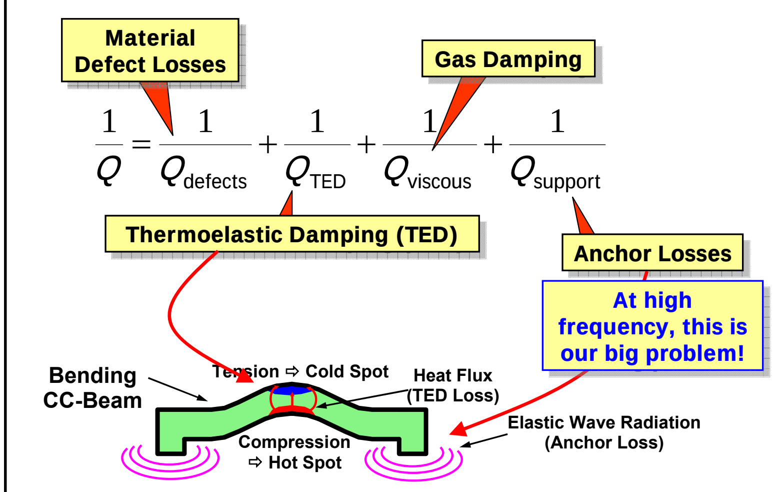
Attaining High *Q*

- Problem: IC's cannot achieve Q's in the thousands
  - ↳ transistors ⇨ consume too much power to get Q
  - ↳ on-chip spiral inductors ⇨ Q's no higher than ~10
  - ↳ off-chip inductors ⇨ Q's in the range of 100's
- Observation: vibrating mechanical resonances ⇨ Q > 1,000
- Example: quartz crystal resonators (e.g., in wristwatches)
  - ↳ extremely high Q's ~ 10,000 or higher (Q ~ 10^6 possible)
  - ↳ mechanically vibrates at a distinct frequency in a thickness-shear mode



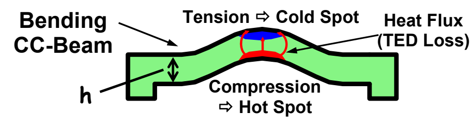
Energy Dissipation and Resonator *Q*

$$\frac{1}{Q}=\frac{1}{Q_{defects}}+\frac{1}{Q_{TED}}+\frac{1}{Q_{viscous}}+\frac{1}{Q_{support}}$$



Thermoelastic Damping (TED)

- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss



$$\varsigma = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2\left[\frac{f_{TED} f}{f_{TED}^2 + f^2}\right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

ζ = thermoelastic damping factor
α = thermal expansion coefficient
T = beam temperature
E = elastic modulus
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
f = beam frequency
h = beam thickness
f_{TED} = characteristic TED frequency

TED Characteristic Frequency

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

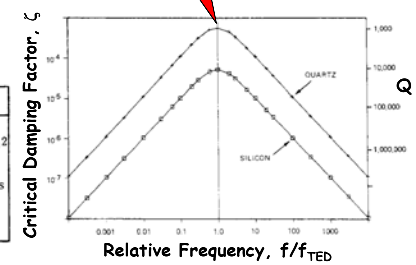
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
h = beam thickness
f_{TED} = characteristic TED frequency

- Governed by
  - Resonator dimensions
  - Material properties

TABLE 1. MATERIAL PROPERTIES

Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10^{12} dyne/cm^2
Material density	2.33	2.60	g/cm^3
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10^7 dyne/°K/s
Peak damping @ 300°k	1.06	11.34	10^{-4}

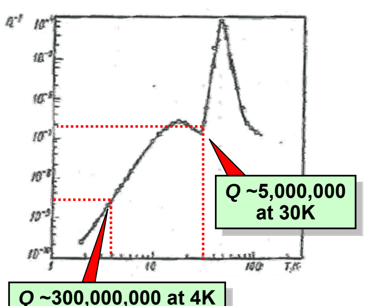
Peak where Q is minimized



[from Roszhart, Hilton Head 1990]

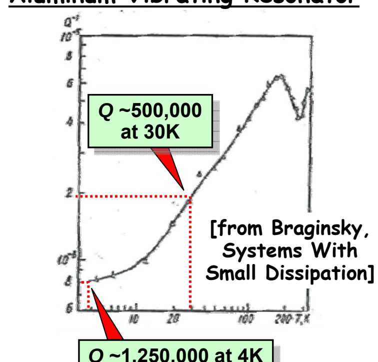
Q vs. Temperature

Quartz Crystal



Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇨ less energy loss per cycle

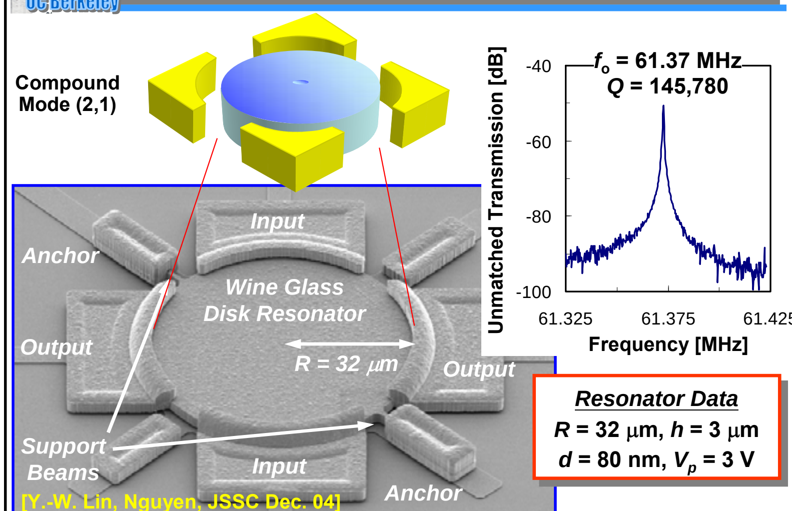
Aluminum Vibrating Resonator



[from Braginsky, Systems With Small Dissipation]

Even aluminum achieves exceptional Q's at cryogenic temperatures

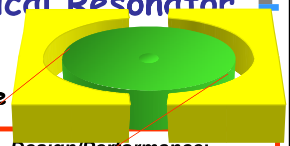
Polysilicon Wine-Glass Disk Resonator



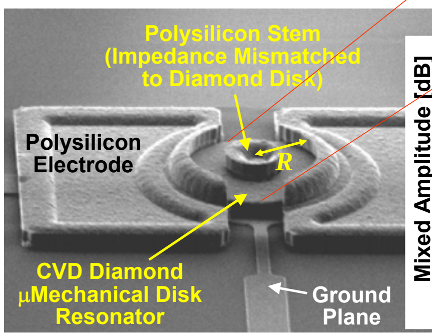
Resonator Data
R = 32 μm, h = 3 μm
d = 80 nm, V_p = 3 V

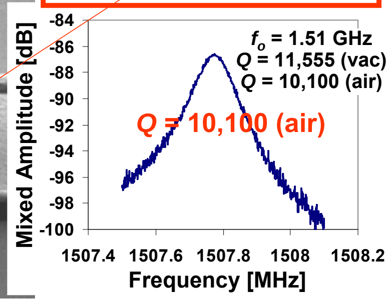
1.51-GHz, *Q*=11,555 Nanocrystalline Diamond Disk μMechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- *Q* ~11,555 (vacuum); *Q* ~10,100 (air)
- *Below*: 20 μm diameter disk



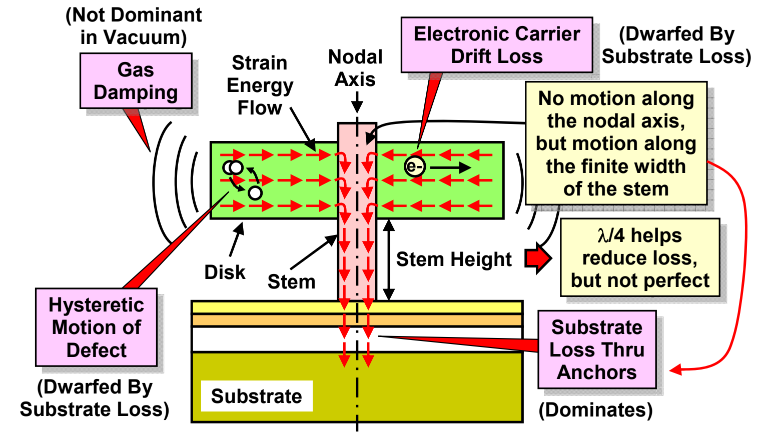
Design/Performance:
R=10μm, t=2.2μm, d=800Å, V_P=7V
f_o=1.51 GHz (2nd mode), Q=11,555





[Wang, Butler, Nguyen MEMS'04]

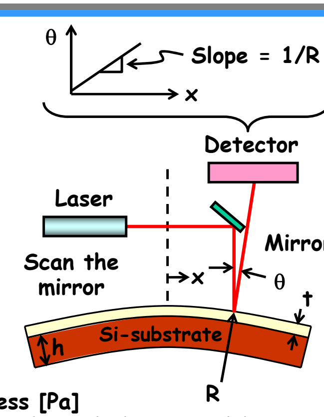
Disk Resonator Loss Mechanisms



MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:

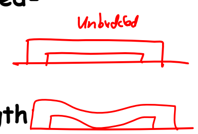


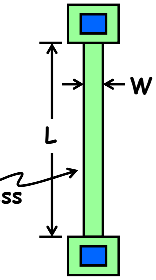
$$\sigma = \frac{E'h^2}{6Rt}$$

σ = film stress [Pa]
E' = E/(1-ν) = biaxial elastic modulus [Pa]
h = substrate thickness [m]
t = film thickness
R = substrate radius of curvature [m]

MEMS Stress Test Structure

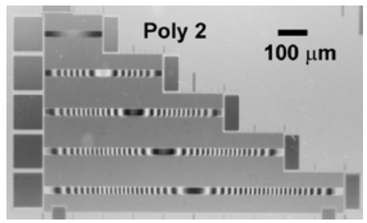
- **Simple Approach**: use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where





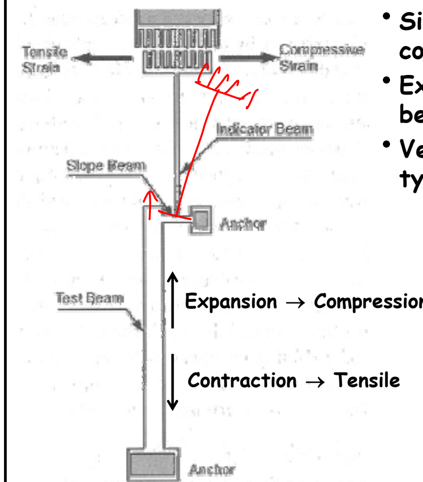
$$\sigma_{critical} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}$$

E = Young's modulus [Pa]
I = (1/12)Wh³ = moment of inertia
L, W, h indicated in the figure

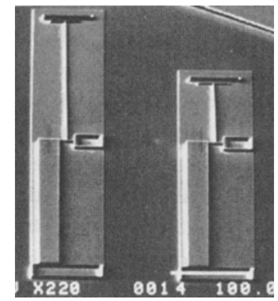


- **Limitation**: Only compressive stress is measurable

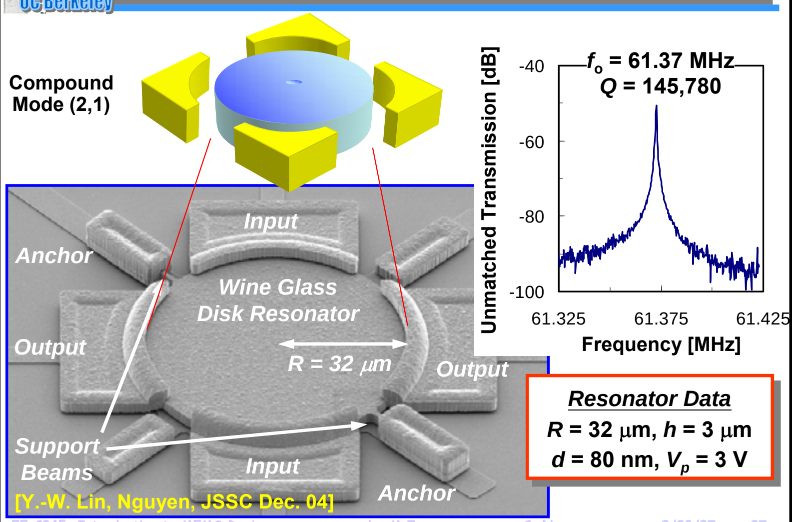
More Effective Stress Diagnostic



- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress

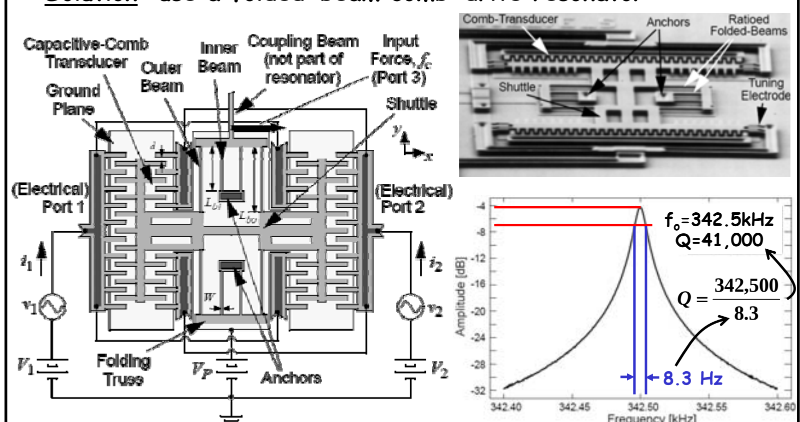


Q Measurement Using Resonators



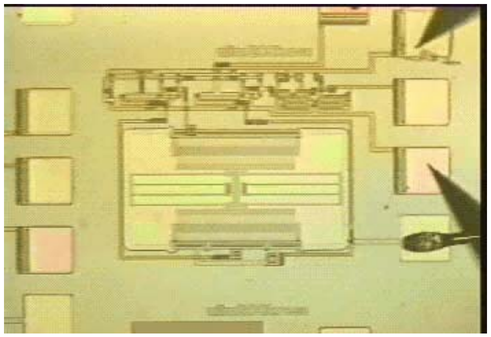
Folded-Beam Comb-Drive Resonator

- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator



Comb-Drive Resonator in Action

- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach



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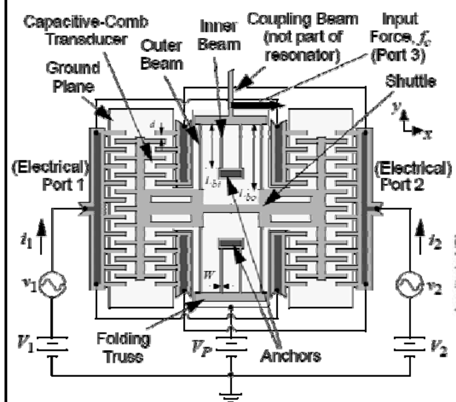
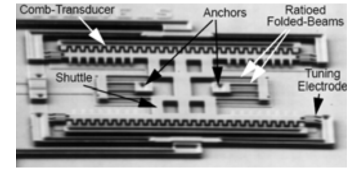
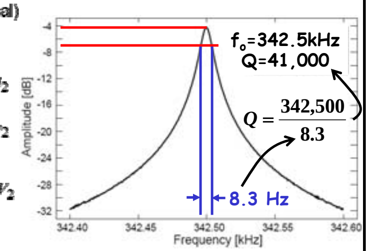






Measurement of Young's Modulus

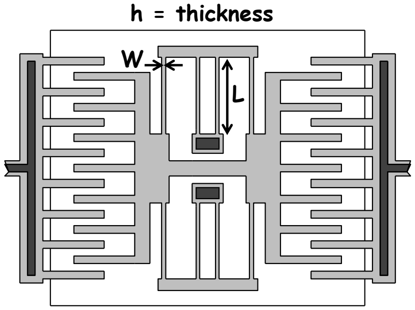
- Use micromechanical resonators
  - Resonance frequency depends on E
  - For a folded-beam resonator:

Young's modulus

$$\text{Resonance Frequency} = f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}}\right]^{1/2}$$

Equivalent mass

h = thickness



- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Anisotropic Materials

Elastic Constants in Crystalline Materials

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- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&C_{14}&C_{15}&C_{16}\\ C_{12}&C_{22}&C_{23}&C_{24}&C_{25}&C_{26}\\ C_{13}&C_{23}&C_{33}&C_{34}&C_{35}&C_{36}\\ C_{14}&C_{24}&C_{34}&C_{44}&C_{45}&C_{46}\\ C_{15}&C_{25}&C_{35}&C_{45}&C_{55}&C_{56}\\ C_{16}&C_{26}&C_{36}&C_{46}&C_{56}&C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}$$

Stresses — Stiffness Coefficients — Strains

Stiffness Coefficients of Silicon

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- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ C_{12}&C_{22}&C_{23}&0&0&0\\ C_{13}&C_{23}&C_{33}&0&0&0\\ 0&0&0&C_{44}&0&0\\ 0&0&0&0&C_{55}&0\\ 0&0&0&0&0&C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}$$

where $\begin{cases}C_{11} = 165.7\text{ GPa}\\ C_{12} = 63.9\text{ GPa}\\ C_{44} = 79.6\text{ GPa}\end{cases}$

Young's Modulus in the (001) Plane

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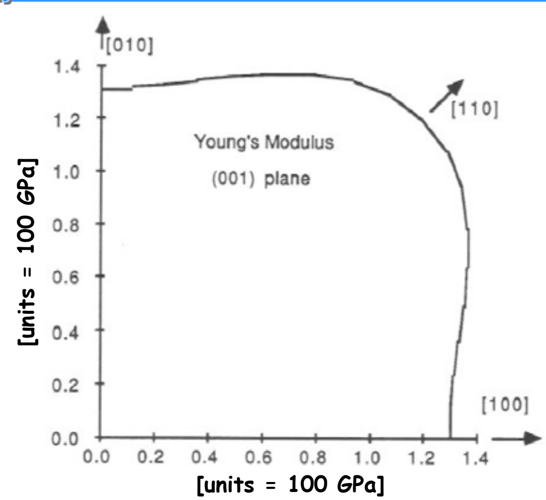

Poisson Ratio in (001) Plane

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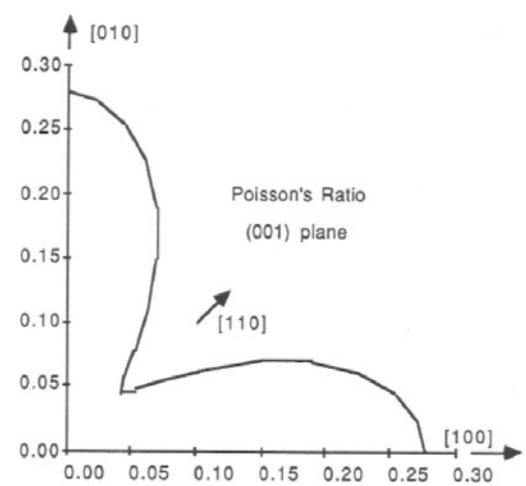
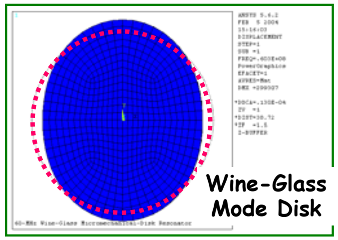
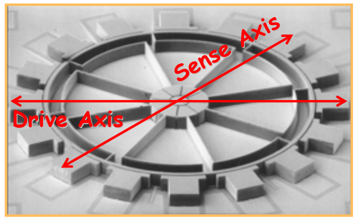


Anisotropic Design Implications

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
  - Mode matching is required, where frequencies along different axes of a ring must be the same
  - Not okay to ignore anisotropic variations, here





Ring Gyroscope