Lecture 16: Beam Combos I

- **Announcements**:
- This is our makeup lecture for Thursday next week
- Midterm is nearing: Thursday, Oct. 27
  - I will soon pass out materials associated with the midterm, including and information sheet and old exams
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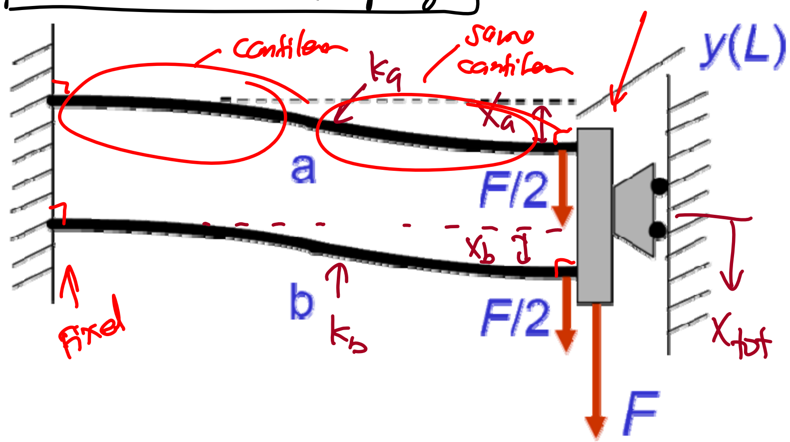
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- ------------------------------------
- Last Time:

Parallel Combination of Springs



Parallel: $x_{tot} = x_a = x_b$

$y(L) = \frac{F}{k} = \frac{F_a}{k_a} = \frac{F_b}{k_b} = \left(\frac{F}{2}\right)\left(\frac{1}{k_a}\right)$ ($F_a \to \frac{F}{2}$, $F_b \to \frac{F}{2}$)

↑ of the whole thing

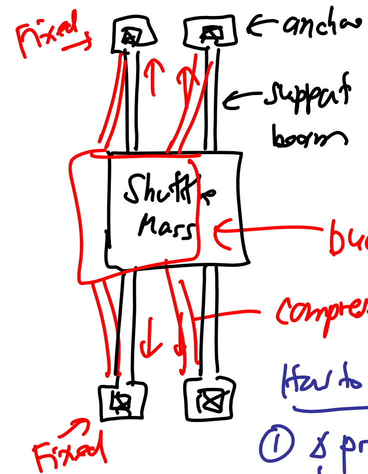
$\boxed{k = 2k_a}$

In general: $\boxed{k_{tot} = k_a + k_b}$

For EE's: springs combine like capacitors

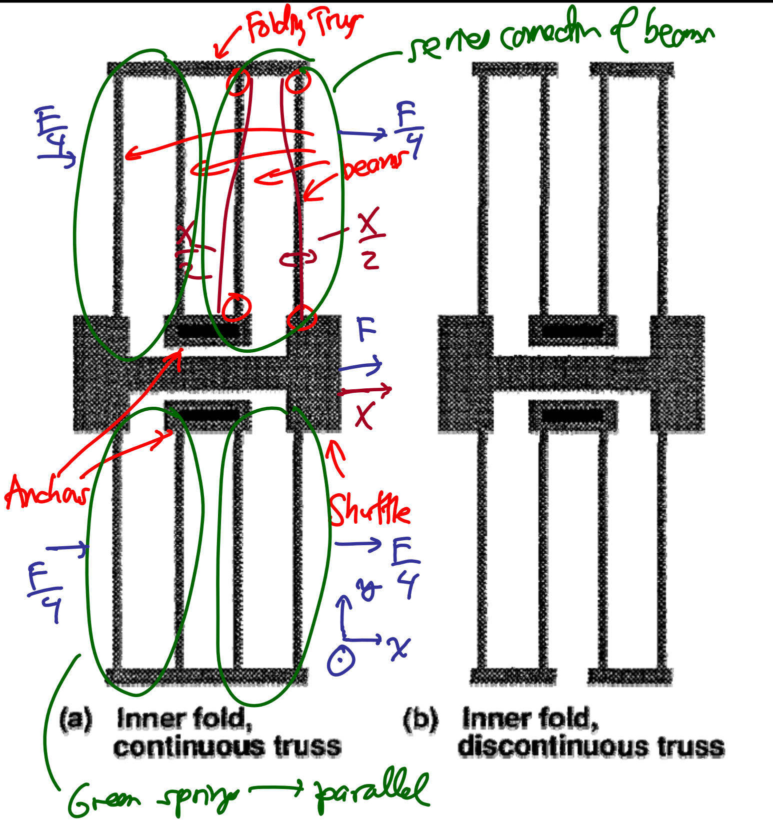
Folded-Beam Suspension

→ Module 8, Slide 22

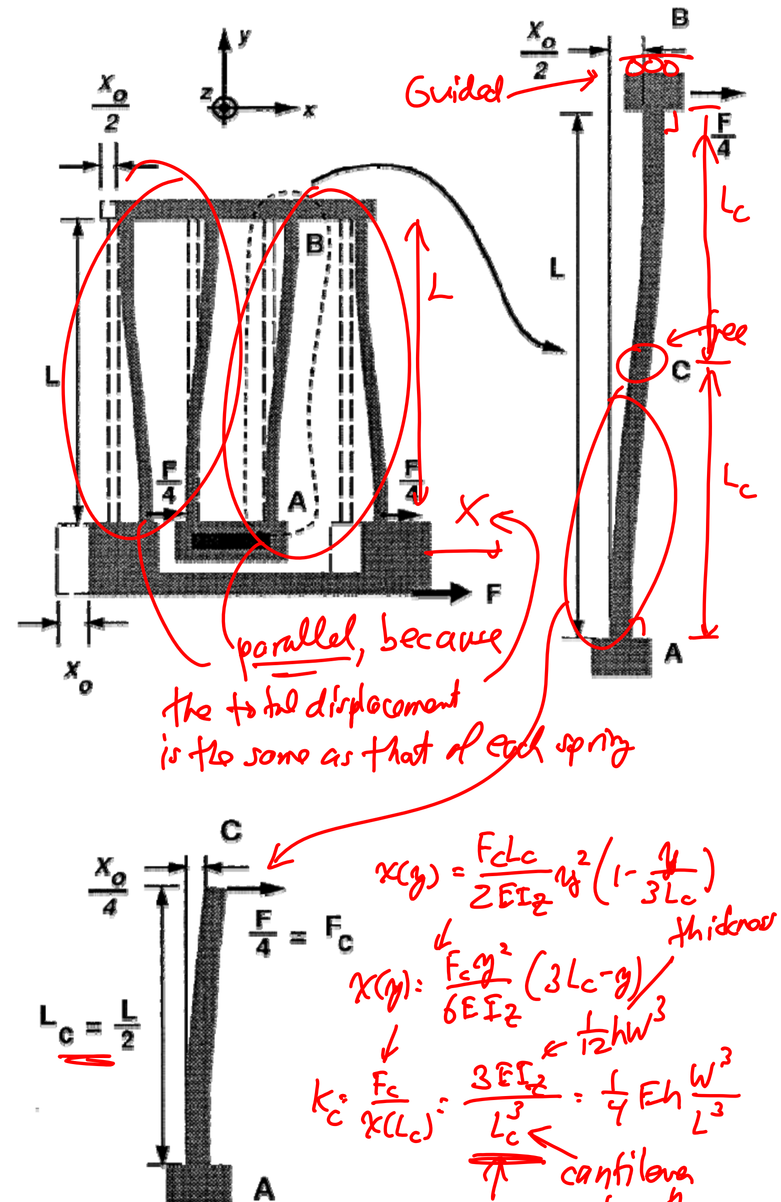


① Dunng dep. @ high T
↳ stress free
② Cool to RT → stress

How to Defend Against this:
① δ process parameters → deposition
↳ can't always do this
↓
② Design → fold-beam!



(a) Inner fold, continuous truss
(b) Inner fold, discontinuous truss



$$x(y) = \frac{F_c L_c}{2EI_z} y^2 \left(1 - \frac{y}{3L_c}\right)$$

$$x(y) = \frac{F_c y^2}{6EI_z}(3L_c - y)$$

thickness

$\frac{1}{12}hW^3$

$$k_c = \frac{F_c}{x(L_c)} = \frac{3EI_z}{L_c^3} = \frac{1}{4}Eh\frac{W^3}{L^3}$$

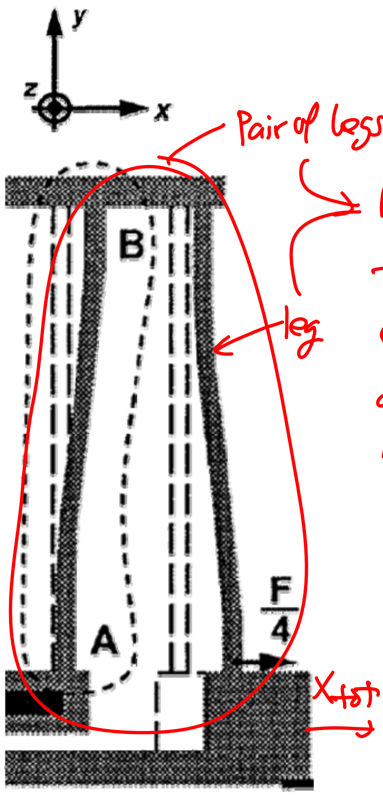
cantilever length

⇒ insert L_c

$$\boxed{k_c = \frac{3EI_z}{(L/2)^3} = \frac{24EI_z}{L^3}}$$

stiffness of cantilever of length L_c

full beam length



legs in series, because the total displacement X_{tot} equals the sum of the displacements of the individual legs

$F_{pair} = \frac{F}{4}$ ← applied to shuttle

$$x = \frac{F_{pair}}{k_{pair}} = \frac{F_{pair}}{(k_{leg} \| k_{leg})} = \frac{F}{4}\left(\frac{1}{k_{leg}} + \frac{1}{k_{leg}}\right)$$

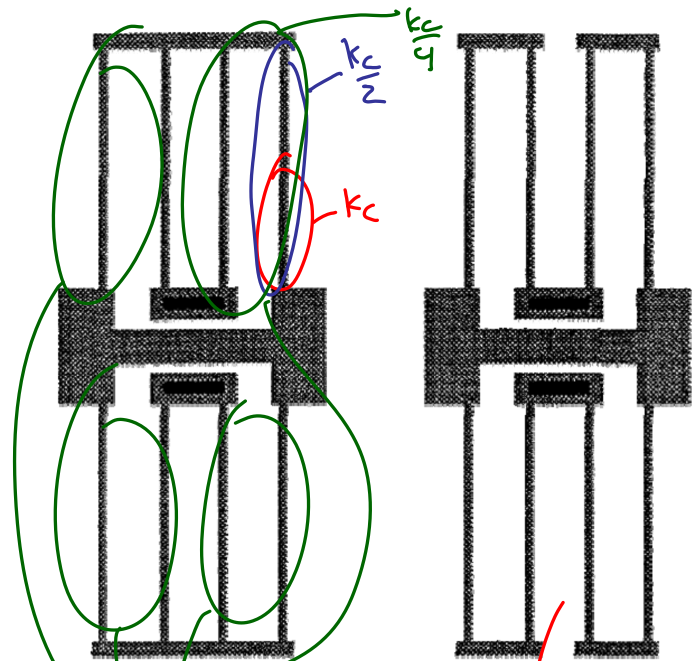
From before: $k_{leg} = k_c \| k_c = \frac{k_c}{2}$

Thus:

$$x = \left(\frac{F}{4}\right)\left(\frac{2}{k_c} + \frac{2}{k_c}\right) = \frac{F}{k_c} = \frac{F}{k_{tot}}$$

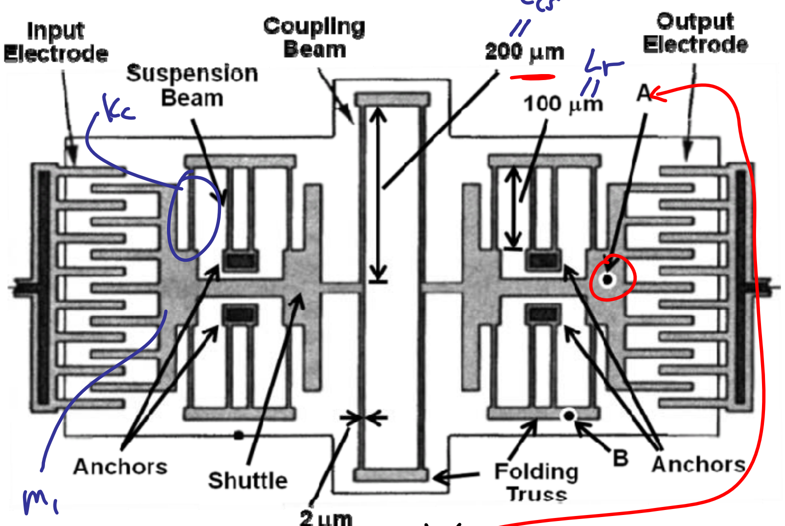
$$\boxed{k_{tot} = k_c = \frac{24EI_z}{L^3}}$$

Better Way to Do It → Just consider stiffnesses!



(a) Inner fold, continuous truss (b) Inner fold, discontinuous truss

Micromechanical Filter



→ Find the stiffness at point A.

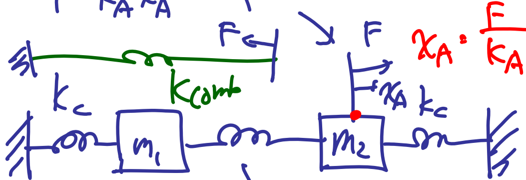
(shuttles are rigid)

↳ apply F @ A → what is x_A?

k_A ≙ stiffness at point A

$F = k_A x_A$

$x_A = \frac{F}{k_A}$ ← want this



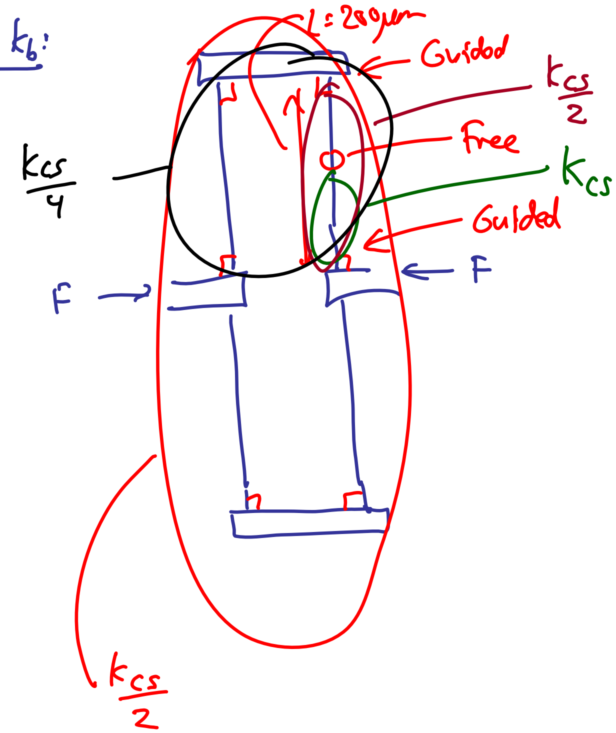
anchor

mass of shuttle 1

mass of shuttle 2 identical to m_1 (two resonators are identical)

$k_b = ? \rightarrow \frac{k_{cs}}{2}$

Get k_b:

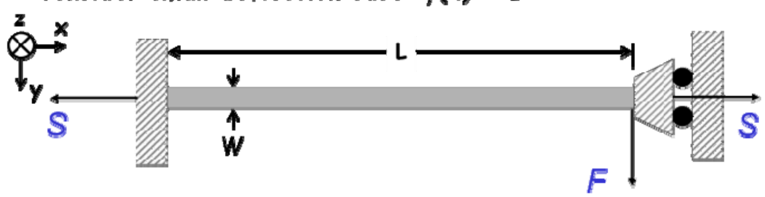


∴ $k_A = k_c + k_{comb}$

$$k_A = k_c + k_c \,\|\, \frac{k_{cs}}{2} \quad \text{where } k_c = \frac{24EI_z}{L_r^3}, \quad k_{cs} = \frac{24EI_z}{L_{cs}^3}$$

Tensioned Spring (Non-Ideality)

- Important case for MEMS suspensions, since the thin films comprising them are often under residual stress
- Consider small deflection case: $y(x) \ll L$



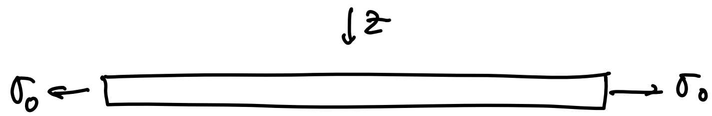
Governing differential equation: (Euler Beam Equation)

$$EI_z \frac{d^4 y}{dx^4} - S\frac{d^2 y}{dx^2} = F\underbrace{\delta(x - L)}_{\text{Unit impulse @ x=L}}$$

S: Axial Load

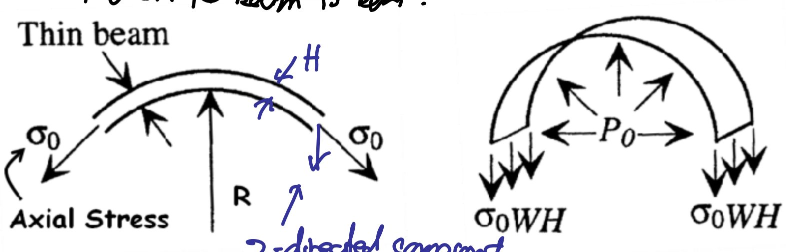
Heuristic Derivation for the Euler Beam Equation

Consider first a straight beam under an axial stress:



⇒ no effect on z-directed stiffness when the beam is straight

...but when the beam is bent:



z-directed component

now, k is affected!