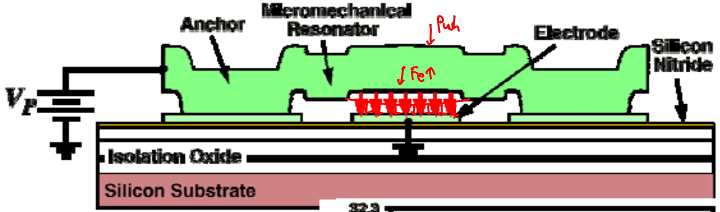
Voltage-Controllable Center Frequency



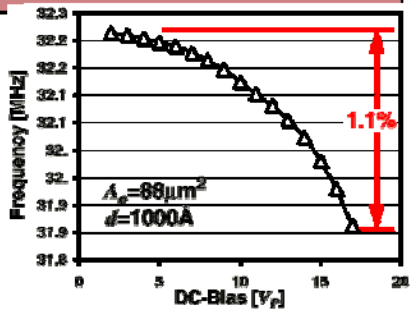
- Quadrature force ⇒ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

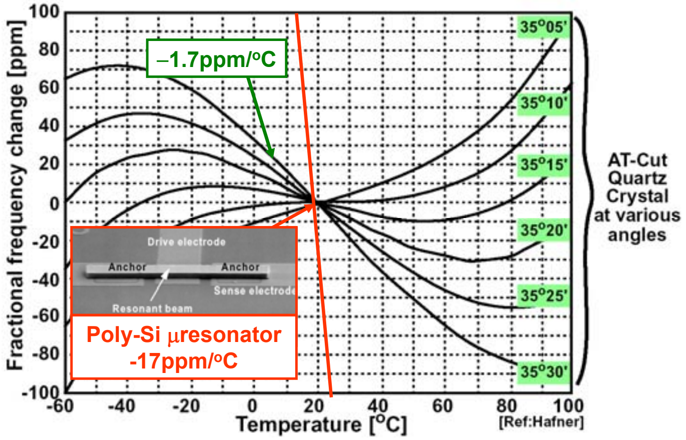
Electrode Overlap Area

Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



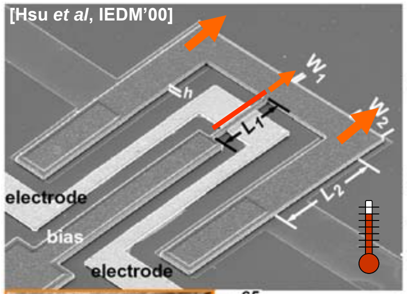
Microresonator Thermal Stability

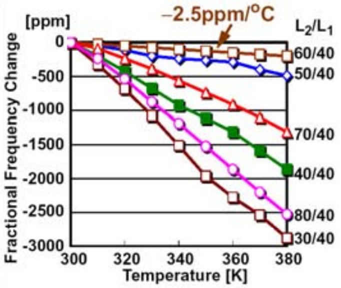


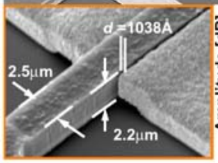
- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

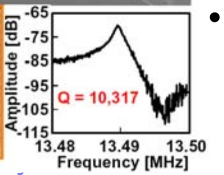
Geometric-Stress Compensation

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.





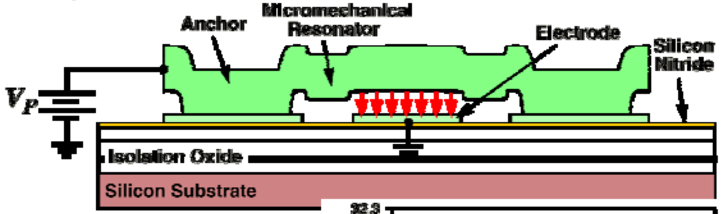




- ***Problems***:
  - ➯ stress relaxation
  - ➯ compromised design flexibility

[Hsu *et al* IEDM 2000]

Voltage-Controllable Center Frequency



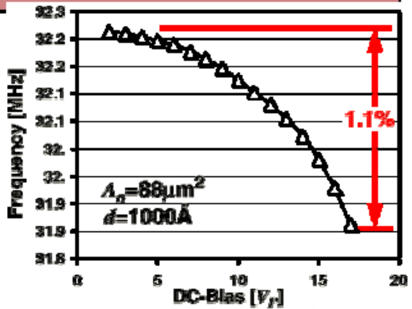
- Quadrature force ⇒ voltage-controllable electrical stiffness:

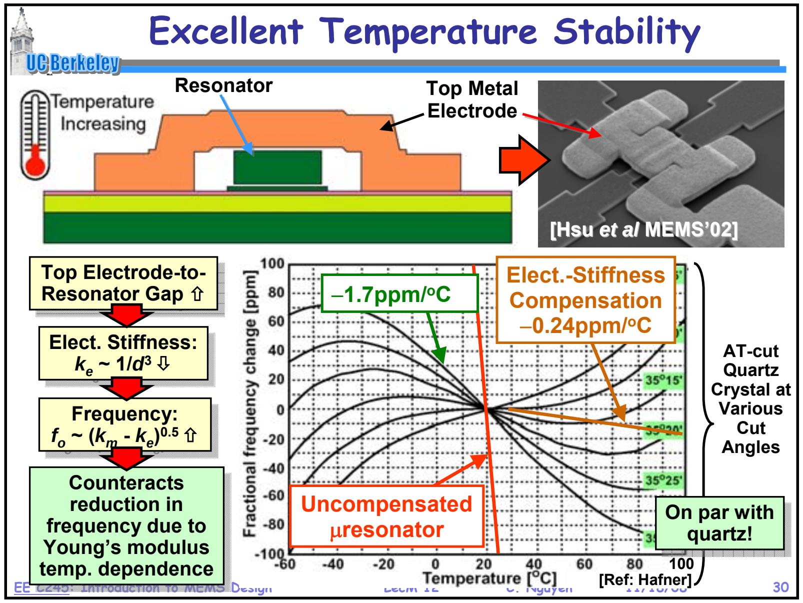
$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

Electrode Overlap Area

Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$





Excellent Temperature Stability

Top Electrode-to-Resonator Gap ⇧

Elect. Stiffness: $k_e \sim 1/d^3$ ⇩

Frequency: $f_o \sim (k_m - k_e)^{0.5}$ ⇧

Counteracts reduction in frequency due to Young's modulus temp. dependence

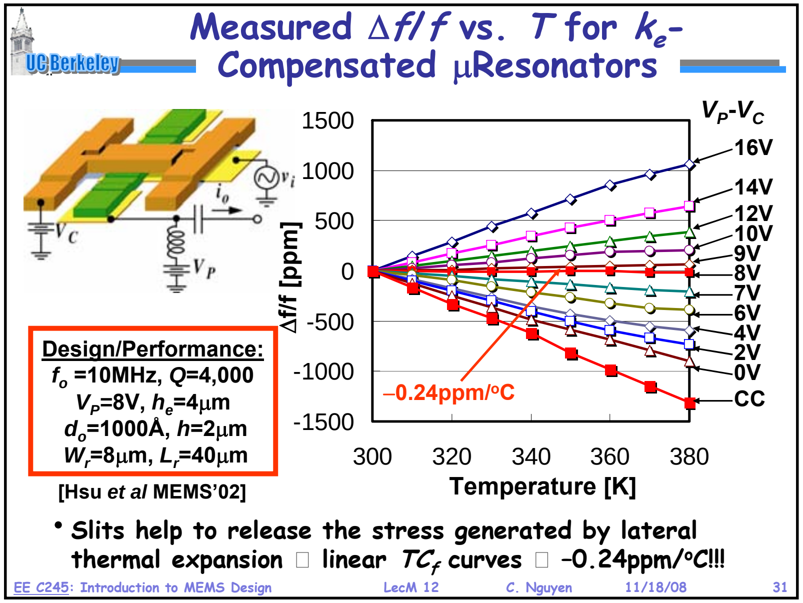
–1.7ppm/°C

Elect.-Stiffness Compensation –0.24ppm/°C

Uncompensated μresonator

AT-cut Quartz Crystal at Various Cut Angles

On par with quartz!



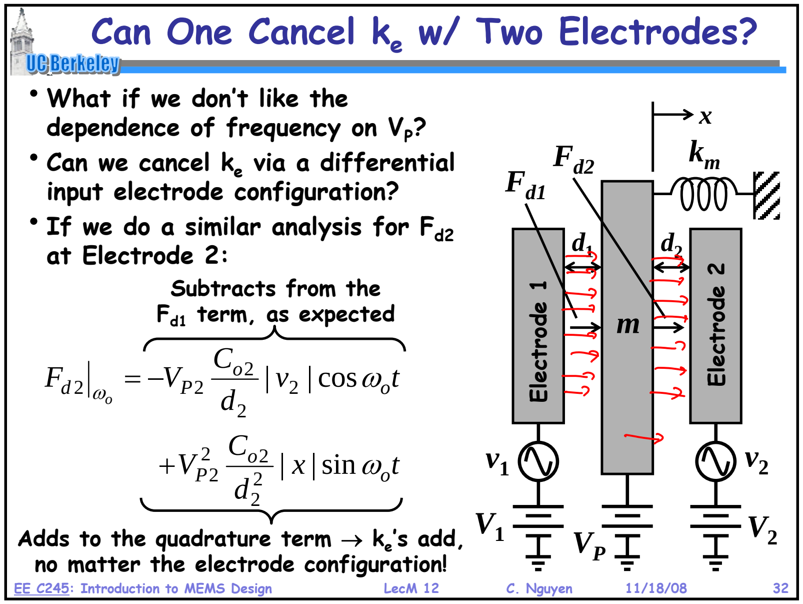
Measured $\Delta f/f$ vs. T for k_e-Compensated μResonators

–0.24ppm/°C

Design/Performance:
f_o =10MHz, Q=4,000
V_P=8V, h_e=4μm
d_o=1000Å, h=2μm
W_r=8μm, L_r=40μm

[Hsu *et al* MEMS'02]

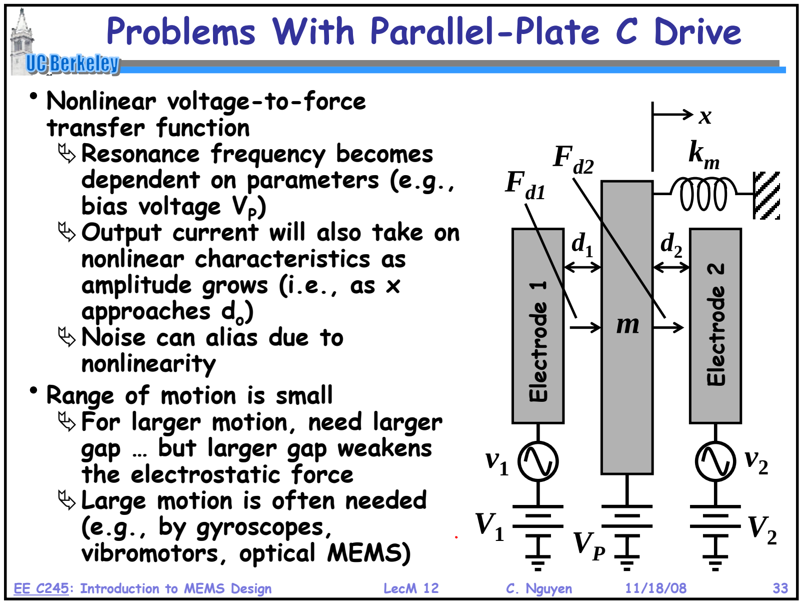
- Slits help to release the stress generated by lateral thermal expansion □ linear TC_f curves □ –0.24ppm/°C!!!



Can One Cancel k_e w/ Two Electrodes?

- What if we don't like the dependence of frequency on V_P?
- Can we cancel k_e via a differential input electrode configuration?
- If we do a similar analysis for F_{d2} at Electrode 2:

$$F_{d2}\Big|_{\omega_o} = \overbrace{-V_{P2}\frac{C_{o2}}{d_2}|v_2|\cos\omega_o t}^{\text{Subtracts from the } F_{d1} \text{ term, as expected}} \underbrace{+V_{P2}^2\frac{C_{o2}}{d_2^2}|x|\sin\omega_o t}_{\text{Adds to the quadrature term} \rightarrow k_e\text{'s add, no matter the electrode configuration!}}$$



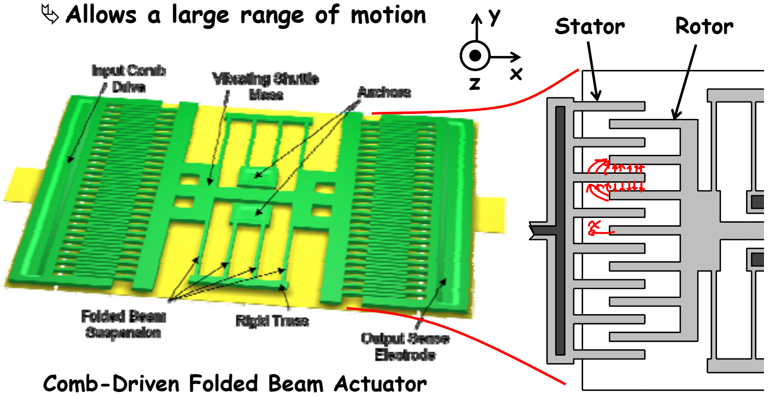
Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_o)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap … but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

Electrostatic Comb Drive

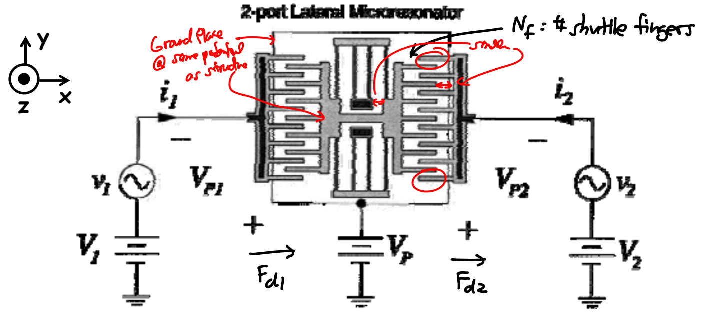
Electrostatic Comb Drive

- Use of comb-capacitive tranducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion



Comb-Driven Folded Beam Actuator

Typical Drive & Sense Configuration



Simple Analysis:

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(v_1 - V_{P1})^2 = \frac{1}{2}\left(-\frac{\epsilon_o h}{d_1}\right)(v_1^2 - 2V_{P1}v_1 + V_{P1}^2)(2N_f)$$

$$F_{d2} = \frac{1}{2}\frac{\partial C_2}{\partial x}(v_2 - V_{P2})^2 = \frac{1}{2}\left(\frac{\epsilon_o h}{d_2}\right)(v_2^2 - 2V_{P2}v_2 + V_{P2}^2)(2N_f)$$

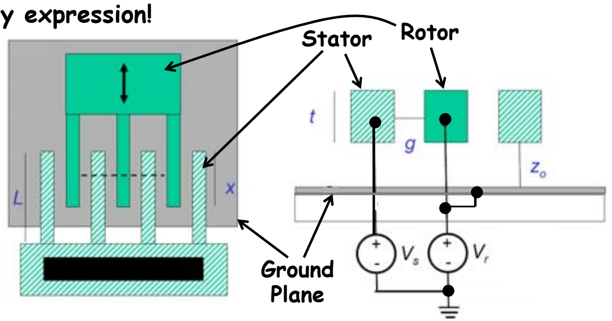
$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2}\left(\frac{\epsilon_o h}{d}\right)\left(v_2^2 - v_1^2 - 2(V_{P2}v_2 - V_{P1}v_1) + V_{P2}^2 - V_{P1}^2\right)(2N_f)$$

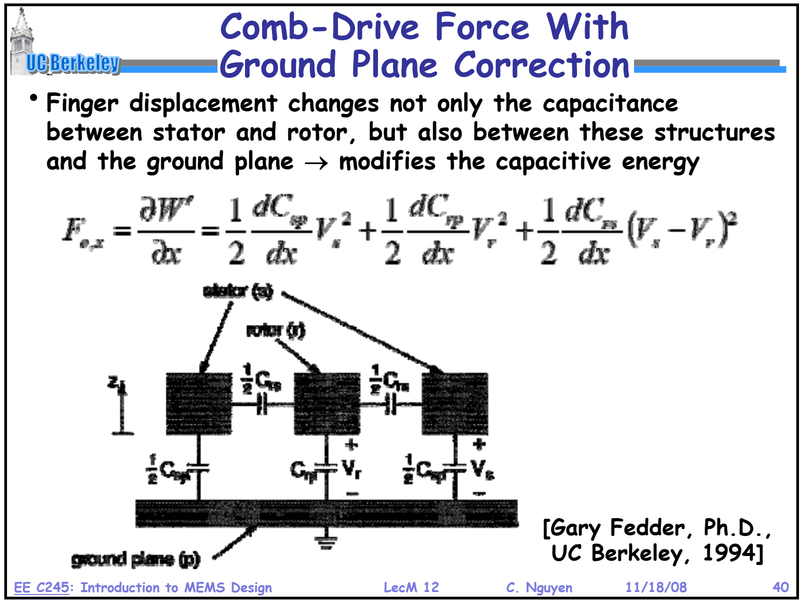
For $V_1 = V_2$, $v_1 = -v_2$

$$\boxed{F_{net} = 2(2N_f)\left(\frac{\epsilon_o h}{d}\right)V_{P1}v_1}$$

Comb-Drive Force Equation (2nd Pass)

- In our 1st pass, we accounted for
  - Parallel-plate capacitance between stator and rotor
- … but neglected:
  - Fringing fields
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!





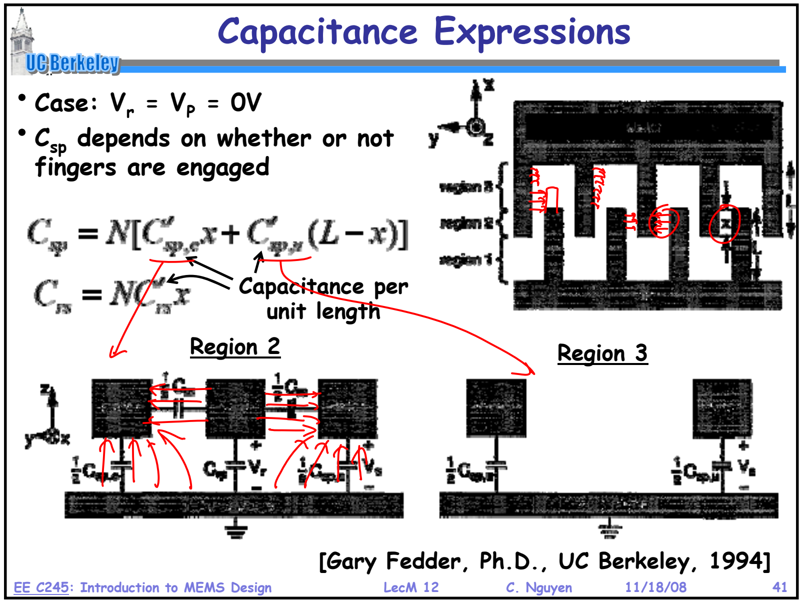
Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$

[Gary Fedder, Ph.D., UC Berkeley, 1994]

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 40



Capacitance Expressions

- Case: $V_r = V_P = 0V$
- C_{sp} depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

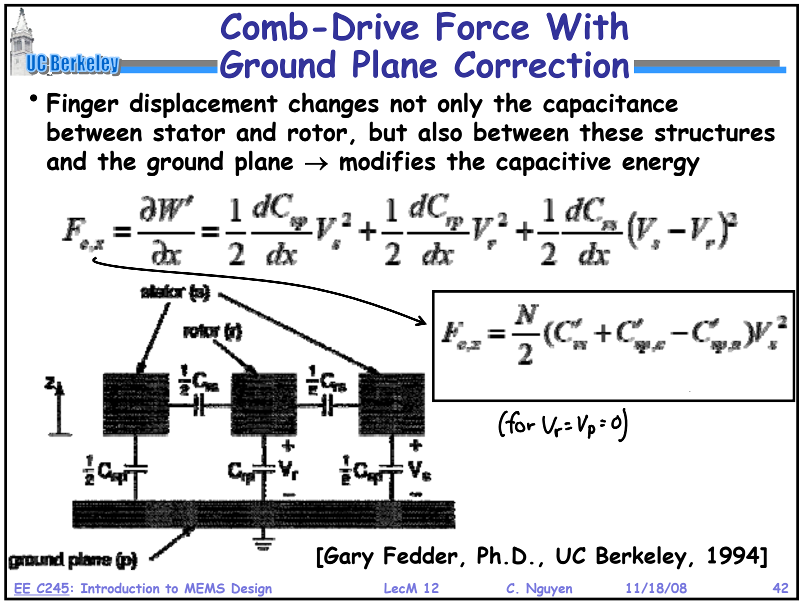
Capacitance per unit length

Region 2

Region 3

[Gary Fedder, Ph.D., UC Berkeley, 1994]

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 41



Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

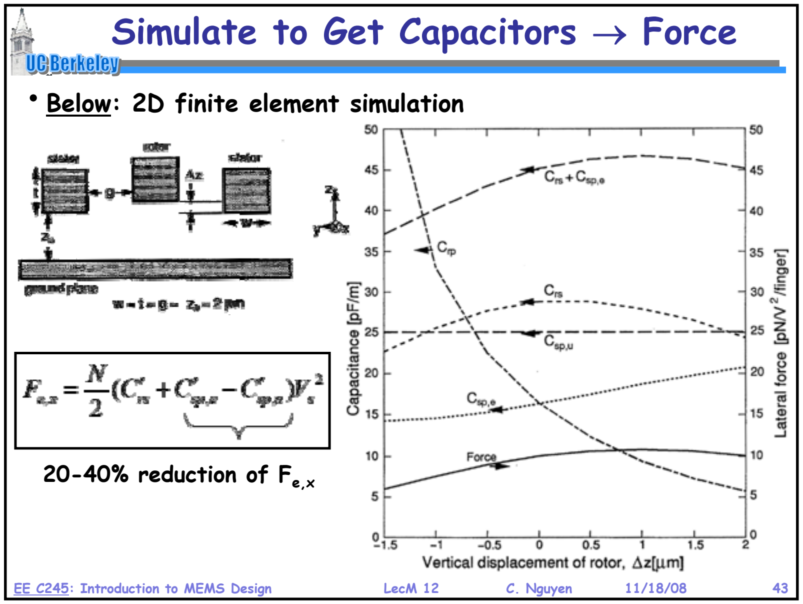
$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$

$$\boxed{F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2}$$

(for $V_r = V_p = 0$)

[Gary Fedder, Ph.D., UC Berkeley, 1994]

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 42



Simulate to Get Capacitors → Force

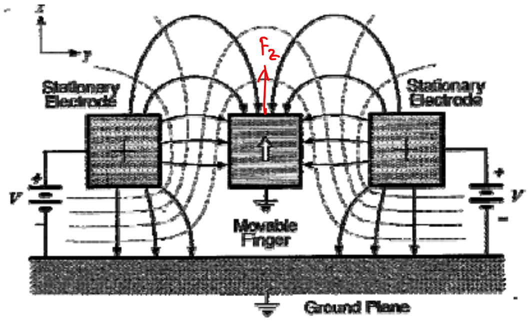
- **Below**: 2D finite element simulation

$$\boxed{F_{e,x} = \frac{N}{2}(C'_{rs} + \underbrace{C'_{sp,e} - C'_{sp,u}}_{})V_s^2}$$

20-40% reduction of $F_{e,x}$

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 43

Vertical Force (Levitation)

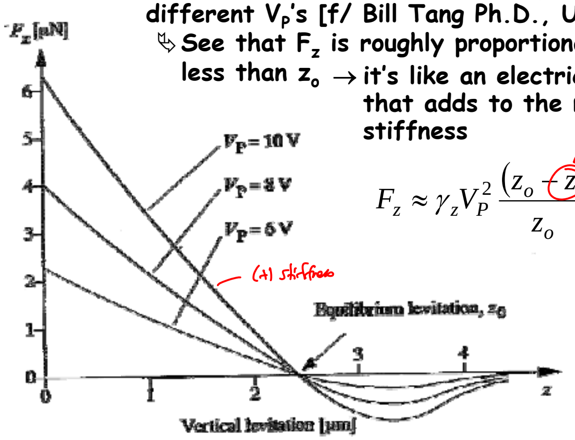


$$F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2}\frac{dC_{sp}}{dz}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dz}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dz}(V_s - V_r)^2$$

- For V_r = 0V (as shown): $F_{e,z} = \frac{1}{2}Nx\left[\frac{d(C'_{sp,e} + C'_{rs})}{dz}\right]V_s^2$

Simulated Levitation Force

- **Below**: simulated vertical force F_z vs. z at different V_P's [f/ Bill Tang Ph.D., UCB, 1990]
  - ⇨ See that F_z is roughly proportional to –z for z less than z_o → it's like an electrical stiffness that adds to the mechanical stiffness



(+) stiffness

linear fn of z

$$F_z \approx \gamma_z V_P^2 \frac{(z_o - z)}{z_o} = k_e(z_o - z)$$

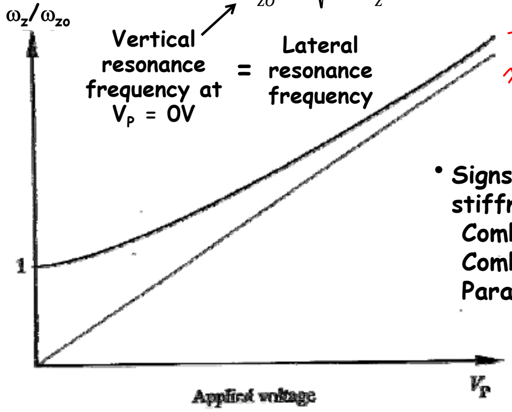
Electrical Stiffness

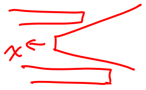
in z-direction!

Vertical Resonance Frequency

Vertical resonance frequency → $\frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}}$ where $k_e = \left(\frac{\gamma_z}{z_o}\right)V^2$

Vertical resonance frequency at V_P = 0V = Lateral resonance frequency

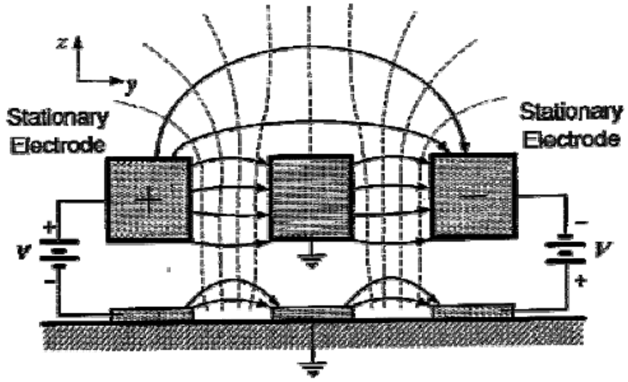




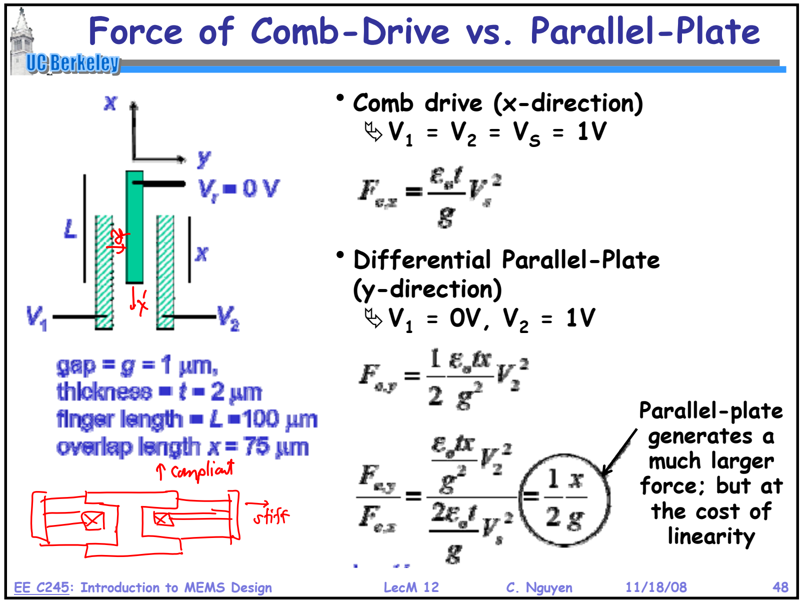
introduce an x-directed k_e

- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → k_e = 0
  - Comb (z-axis) → k_e > 0
  - Parallel Plate → k_e < 0

Suppressing Levitation



- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- **Penalty**: x-axis force is reduced



Force of Comb-Drive vs. Parallel-Plate

UC Berkeley

gap = g = 1 μm,
thickness = t = 2 μm
finger length = L = 100 μm
overlap length x = 75 μm

- Comb drive (x-direction)
  - $V_1 = V_2 = V_S = 1V$

$$F_{e,x} = \frac{\varepsilon_o t}{g} V_s^2$$

- Differential Parallel-Plate (y-direction)
  - $V_1 = 0V$, $V_2 = 1V$

$$F_{e,y} = \frac{1}{2}\frac{\varepsilon_o t x}{g^2} V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\varepsilon_o t x}{g^2} V_2^2}{\frac{2\varepsilon_o t}{g} V_s^2} = \boxed{\frac{1}{2}\frac{x}{g}}$$

Parallel-plate generates a much larger force; but at the cost of linearity

EE C245: Introduction to MEMS Design LecM 12 C. Nguyen 11/18/08 48