Lecture 23: Comb Drive

- Announcements:
- Reminder: 2nd project slide due this Friday
- Module 13 now online
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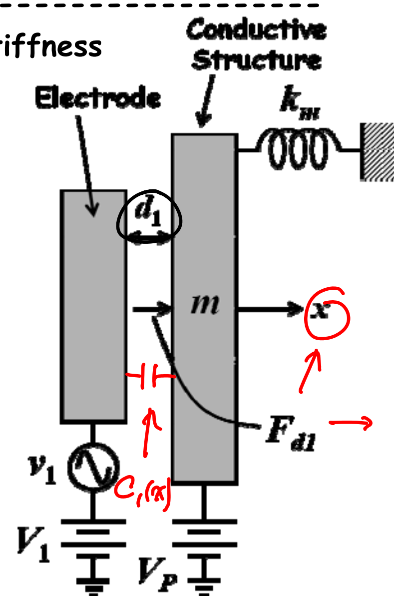
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- ------------------------------------
- Last Time: Electrical Stiffness



More Complete Expression)

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_0\left(1+\frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1+\frac{x}{d_1}\right)^{-2}$$

[Expand the Taylor series further]

$$\frac{\partial C_1}{\partial x} = -\frac{C_{01}}{d_1}\left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots\right)$$

where $A_1 = -\frac{2}{d_1}$, $A_2 = \frac{3}{d_1^2}$, $A_3 = -\frac{4}{d_1^3}$, ...

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}\left(V_P - V_1 - v_1\right)^2 = \frac{1}{2}\frac{\partial C_1}{\partial x}\left(V_{P1} - v_1\right)^2$$

$V_{P1} = V_P - V_1$

[small displacement: $x \ll d_1$]

for ∂x!

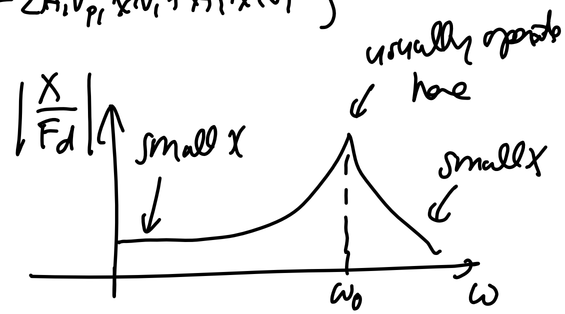
$$F_{d1} = \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)\left(1 + A_1 x\right)\left(V_{P1}^2 - 2V_{P1}v_1 + v_1^2\right)$$

@ ω_0 → $v_1 = V_1\cos\omega_0 t$

$$= \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)\left\{V_{P1}^2 - 2V_{P1}v_1 + v_1^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x v_1 + A_1 x v_1^2\right\}$$

@ ω_0

Resonance:



@ resonance:

$$x = \frac{Q F_{d1}}{jk} = \frac{Q}{jk}\frac{\partial C}{\partial x} V_{P1} v_i$$

90° phase shift → displacement x is 90° phase-shifted fr force, F_d

Thus:

$$v_i = |v_i|\cos\omega_o t \rightarrow x = |X|\sin\omega_o t$$

phase-shifted 90°

Force terms only @ ω_o:

$$F_{d1}\big|_{\omega_o} = \underbrace{V_{P1}\frac{C_{o1}}{d_1}|v_i|\cos\omega_o t}_{\text{forcing term}} + \underbrace{V_{P1}^2\frac{C_{o1}}{d_1^2}}_{k_e}|x|\sin\omega_o t$$

k_e → electrical stiffness

proportional to x

90°

in phase w/ displacement

Electrical stiffness:

① A negative spring constant!

② Derives from V_P:

$$\boxed{k_e = V_{P1}^2\frac{C_{o1}}{d_1^2} = V_{P1}^2\frac{\epsilon A}{d_1^3}}$$

overlap area of C

DC Bias

3rd power dependence on gap!

k_e → can affect resonance freq., f_o:

$\omega_o \triangleq$ rad/s resonance freq. w/ no V_p applied (i.e., $V_{P1} = 0V$)

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}} \qquad \omega_o = \sqrt{\frac{k_m}{m}}$$

mech. stiffness

electrical stiffness

$$= \sqrt{\frac{k_m}{m}}\left(1 - \frac{k_e}{k_m}\right)^{1/2}$$

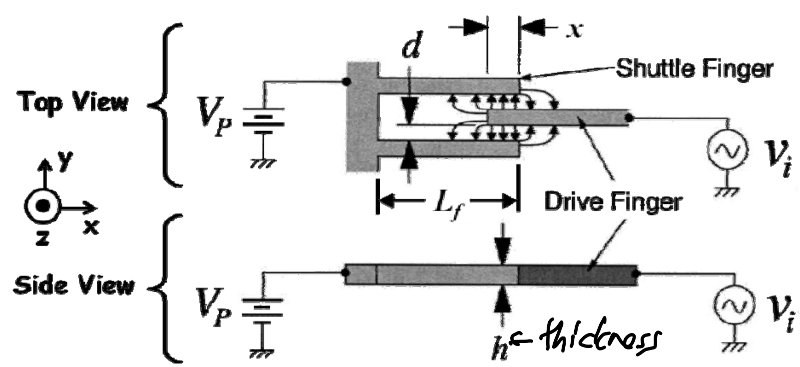
$k = k_m - k_e$

$$\boxed{\omega_o' = \omega_o\left[1 - \frac{V_{P1}^2}{k_m}\frac{\epsilon A}{d_1^3}\right]^{1/2}}$$

now a fcn of dc-bias! (voltage-controllable!)

- Go through Module 12 slides 26-35

Electrostatic Comb-Drive



$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2$$

→ Need $C(x)$.

$$C(x) = \frac{2\epsilon_0 h x}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d}$$

Not a fcn of x!

$$F_d = \cancel{\frac{1}{2}}\,\frac{\cancel{2}\epsilon_0 h}{d}\left(V_P^2 - 2V_P v_i + \cancel{v_i^2}\right)$$

small

can balance out by symmetrically placed electrodes

$$\boxed{F_d = -2V_P\frac{\epsilon_0 h}{d} v_i}$$

→ ideally, no electrical stiffness k_e

- Go through the rest of Module 12, starting from slide 38