Lecture 24: Equivalent Circuits II

- **Announcements:**
- Reminder: 2nd project slide due this Friday
- HW #6 Due Next Tuesday
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- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
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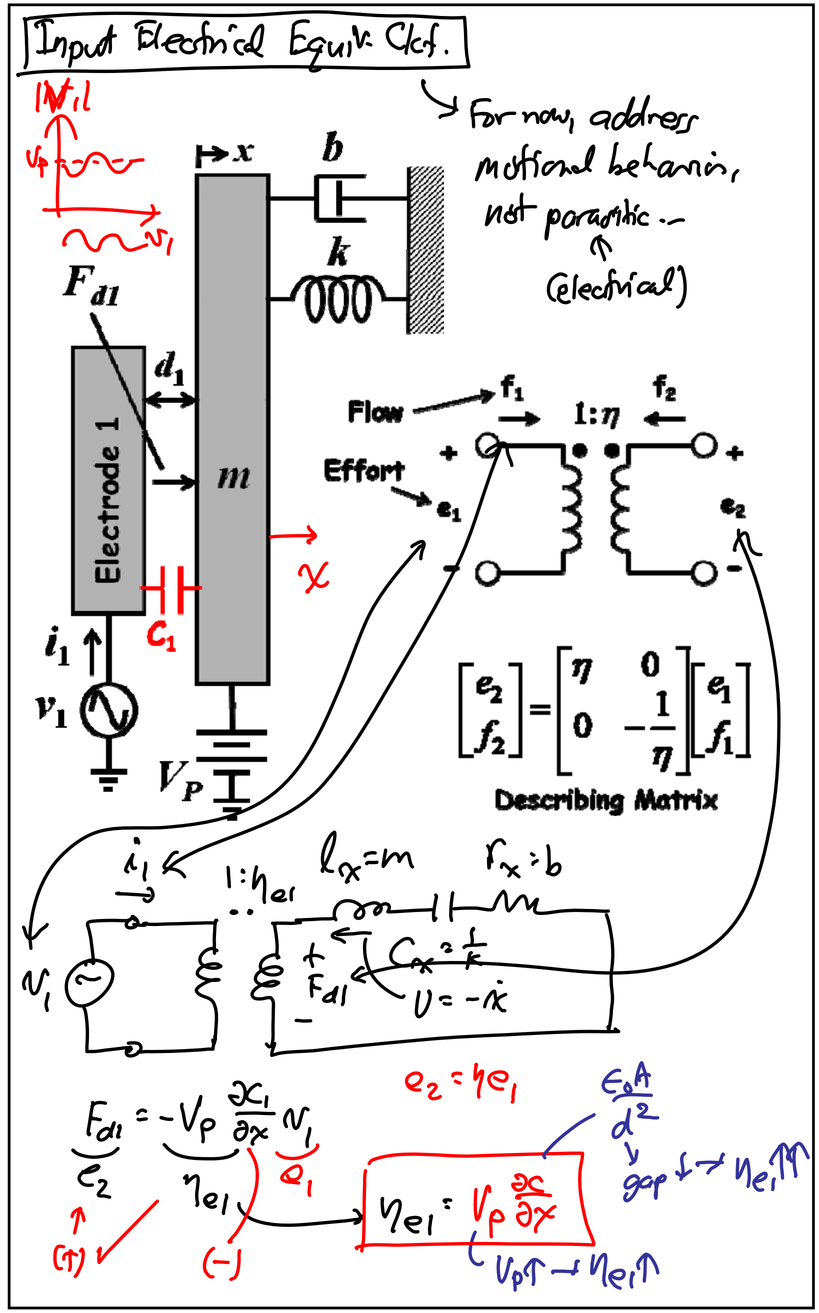
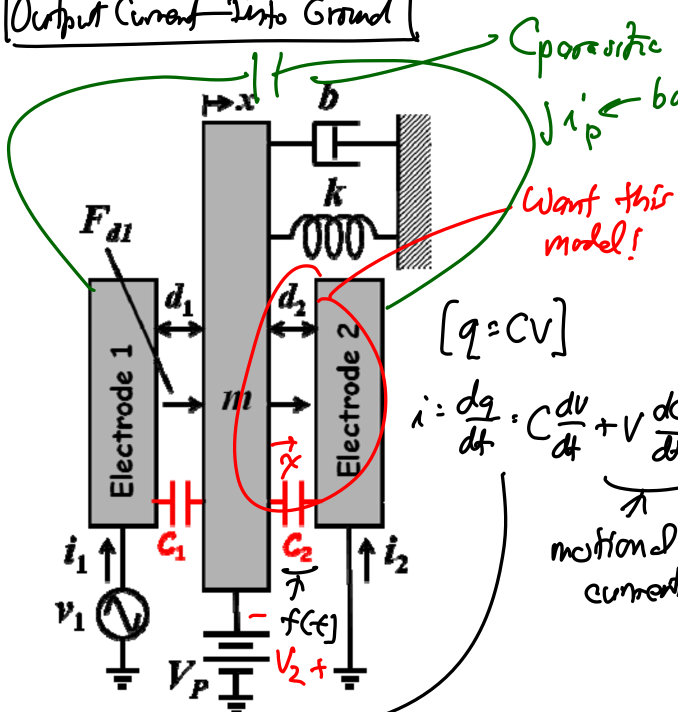
- **Last Time:**
- Finished Module 12
- Now start going through Module 13 (on Equivalent Circuits) through slide 8



Output Current Into Ground



$[q = CV]$

$$i = \frac{dq}{dt} = C\frac{dv}{dt} + \underbrace{V\frac{dC}{dt}}_{\text{motional current}}$$

$$i_2 = C_2(x,t)\cancel{\frac{dV_2(t)}{dt}} + V_2(t)\frac{dC_2(x,t)}{dt}$$

velocity

$$[V_2(t) = -V_P] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

In phasor form:

$$I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$$

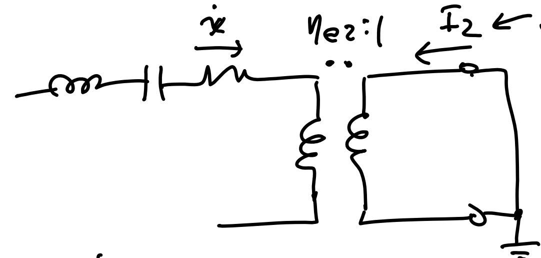
$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

motional current

If $V_P = 0V$ → $I_2 = 0$.

$$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C_2}{\partial x}\dot{x}$$

90° phase lag (+) (+) velocity

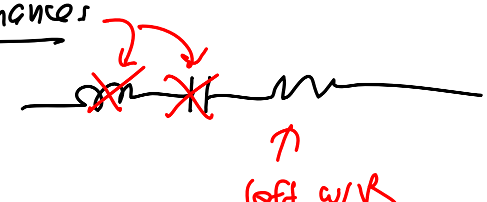


$$f_2 = -\frac{1}{\eta_{e2}}f_1 \rightarrow f_1 = -\eta_{e2}f_2$$

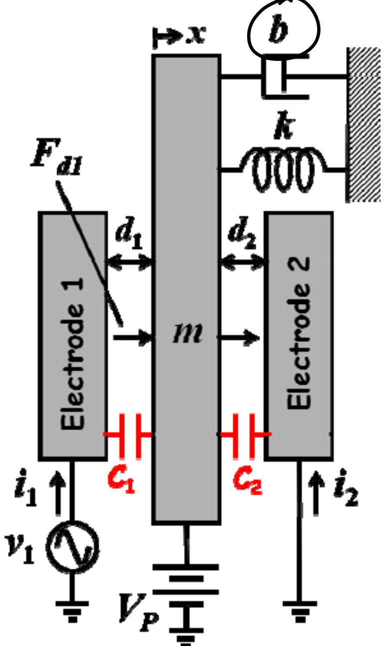
$$[f_1 = I_2,\ f_2 = \dot{x}] \Rightarrow I_2 = -\eta_{e2}\dot{x}$$

$$\boxed{\therefore\ \eta_{e2} = V_P\frac{\partial C_2}{\partial x}}$$

@ resonance:



Input Current Expression



Get $I_1(j\omega)$:

$$i_1(t) = C_1(x,t)\frac{dV_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$$

$$[V_1(t) = v_1 - V_P] \Rightarrow i_1 = C_1\frac{dv_1}{dt} + (v_1 - V_P)\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$$

$f(t)$

$$* \quad \therefore I_1(j\omega) = \underbrace{j\omega C_1 v_1}_{\text{Feedthrough Current}} + \cancel{j\omega \overset{\omega}{v_1}\frac{\partial C_1}{\partial x}\overset{\omega}{X}} - \underbrace{j\omega V_P\frac{\partial C_1}{\partial x}X}_{\text{Motional Current}}$$

@ DC: $x = \frac{F_{d1}}{k} = -\frac{1}{k}V_P\frac{\partial C_1}{\partial x}v_1$

@ resonance: $x = \frac{QF_{d1}}{jk} = -\frac{Q}{jk}V_P\frac{\partial C_1}{\partial x}v_1 = X$

90° phase lag

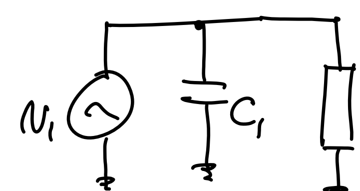
Thus: @ ω_o (resonance):

$$* \quad I_1(j\omega) = j\omega_o C_1 v_1 + \cancel{j}\omega_o \left(V_P\frac{\partial C_1}{\partial x}\right)^2\frac{Q}{\cancel{j}k}v_1$$

$$= j\omega_o C_1 v_1 + \omega_o\frac{Q}{k}\eta_{e1}^2 v_1$$

90° phase shifted from v_1 → This is a capacitor in shunt w/ the input!

In phase w/ v_1 → This is an effective resistance seen looking into electrode 1!

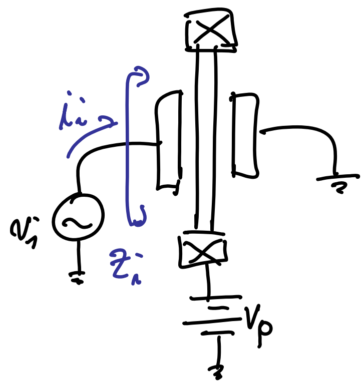


Motional Resistance:

$$R_{x1} = \frac{v_1}{I_1} = \frac{k}{\omega_o Q\eta_{e1}^2} = \frac{m\omega_o}{Q\eta_{e1}^2}$$

$$\boxed{R_{x1} = \frac{b}{\eta_{e1}^2}}$$

Input Impedance Into Port 1



What is Z_i?