Lecture 25: Gyroscopes & Sensing Circuits

- **Announcements:**
- Reminder: 3rd project slide due Dec. 2
- HW#7 online and due Thursday, Dec. 8
- Project Outbrief Signups: pick two days
  - Tuesday, Dec. 13, afternoon?yes
  - Wednesday, Dec. 14? yes
  - Thursday, Dec. 15?
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- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
- Lecture Topics:
  - Gyroscopes
- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - Detection Circuits
    - Velocity Sensing
    - Position Sensing
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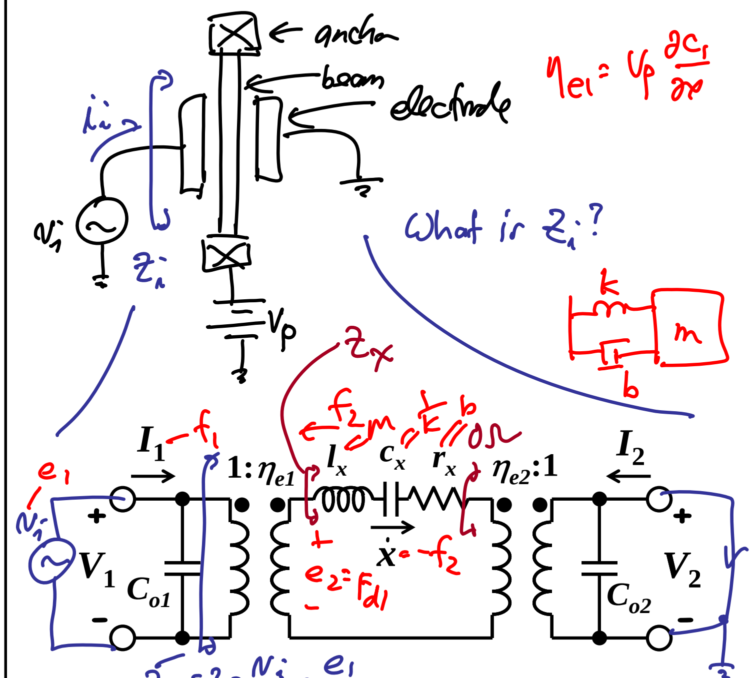
- **Last Time:**

Input Impedance Into Port 1



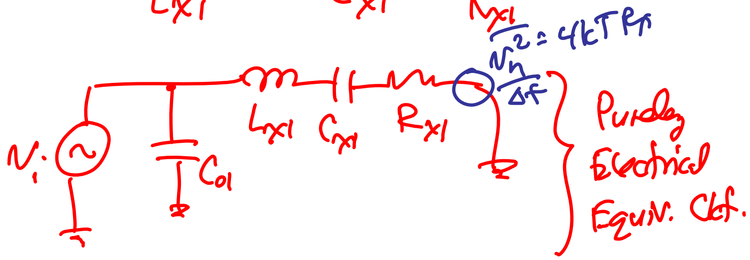
$$Z_i = ? = \frac{v_i}{i_1} = \frac{e_1}{f_1}$$

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \quad e_2 = \eta e_1 \to e_1 = \frac{e_2}{\eta}, \quad f_2 = -\frac{1}{\eta} f_1 \to f_1 = -\eta f_2$$

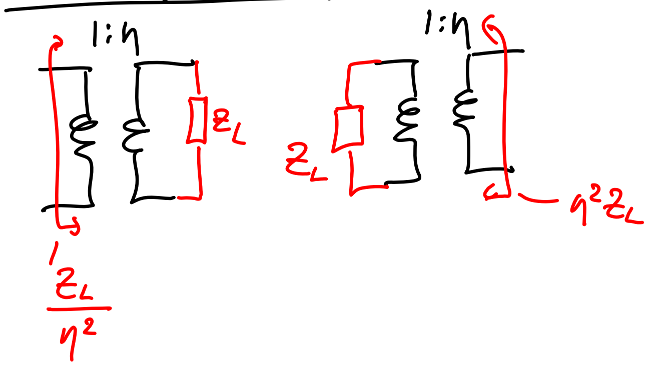
$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \to \frac{v_1}{i_1} = Z_i = -\frac{1}{\eta_{e1}^2}\underbrace{\frac{F_{d1}}{(-\dot{x})}}_{Z_x} = \frac{1}{\eta_{e1}^2} Z_x$$

$$Z_i = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right)$$

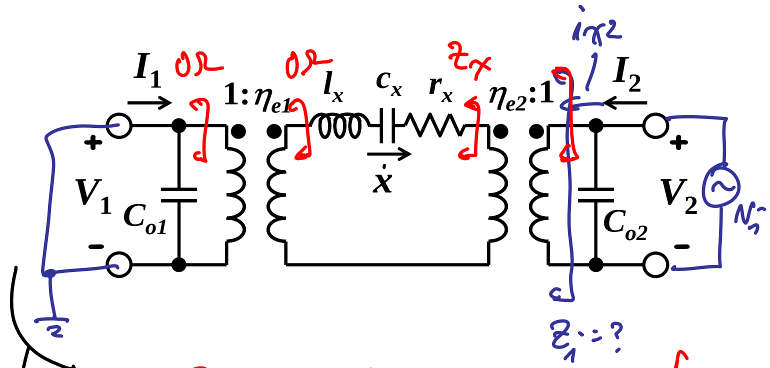
$$Z_i = \underbrace{j\omega\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \underbrace{\frac{1}{j\omega(\eta_{e1}^2 c_x)}}_{C_{x1}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$



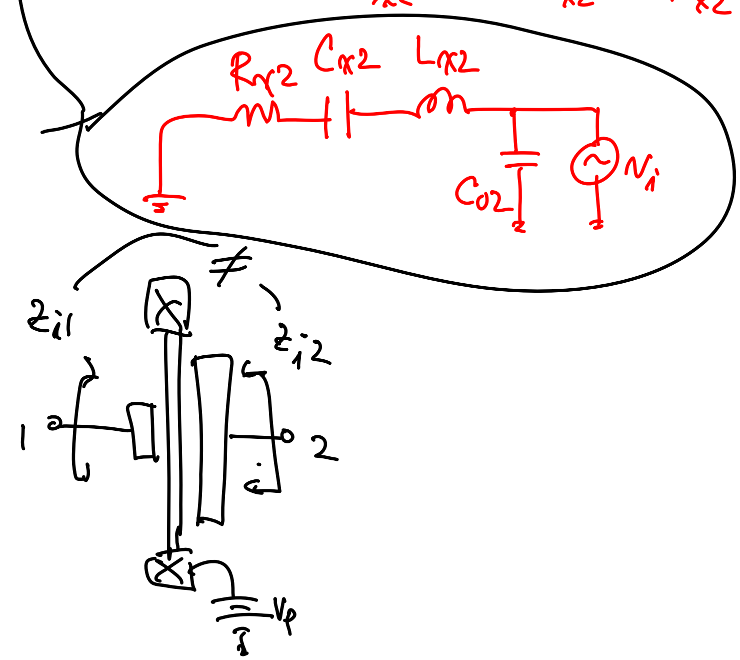
Xformer Impedance Analysis



Input Impedance Into Port 2

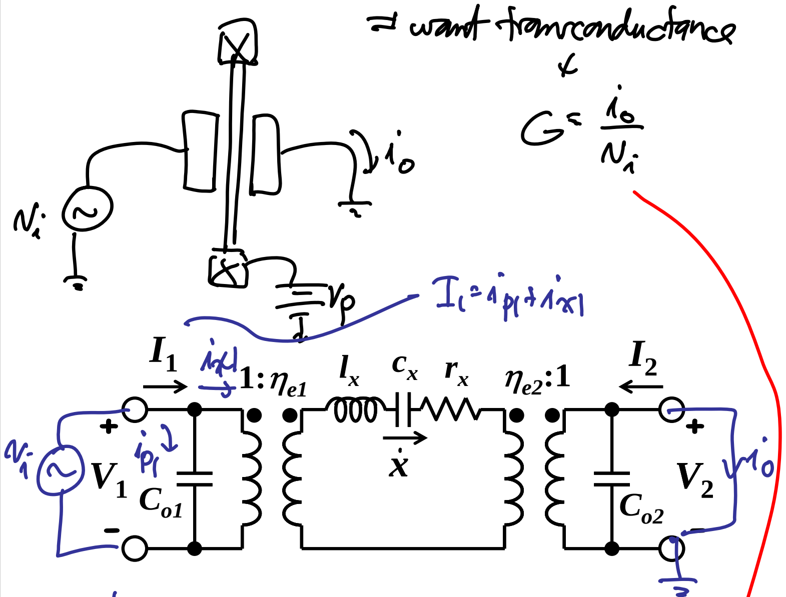


$$Z_i = \frac{v_i}{i_i} = \frac{z_x}{\eta_{e2}^2} = \underbrace{j\omega\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \underbrace{\frac{1}{j\omega(\eta_{e2}^2 c_x)}}_{C_{x2}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$



Port 1 to Port 2 Transfer Function

⇒ want transconductance & $G = \frac{i_o}{v_i}$



$\dot{x} = \frac{1}{\eta_{e1}} i_{x1}$, $i_o = \eta_{e2}\dot{x}$ → $i_o = \frac{\eta_{e2}}{\eta_{e1}} i_{x1} = \left(\frac{\eta_{e2}}{\eta_{e1}}\right)\left(\frac{v_i}{Z_x}\right)$

$$= \frac{\eta_{e2}}{\eta_{e1}} v_i \left[\frac{\eta_{e1}^2}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}\right]$$

$$\therefore \frac{i_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}$$

$$= \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}\right]^{-1} \quad *$$

$L_{x12} = \frac{l_x}{\eta_{e1}\eta_{e2}}$, $C_{x12} = \eta_{e1}\eta_{e2} c_x$, $R_{x12} = \frac{r_x}{\eta_{e1}\eta_{e2}}$

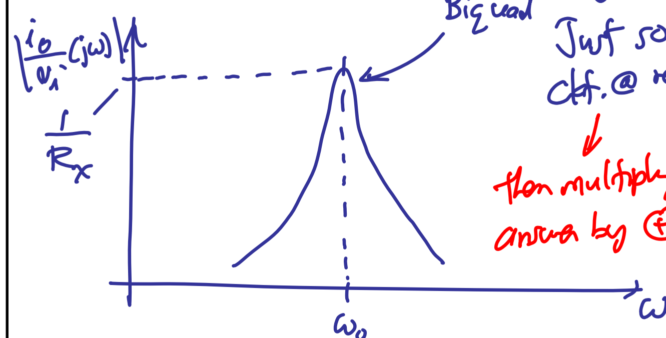
Now, separate the freq. response & magnitude:

$$* \quad \frac{i_o}{v_i}(s) = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{s\left(\frac{1}{L_x}\right)}{s^2 + \frac{1}{L_x C_x} + s\left(\frac{R_x}{L_x}\right)}$$

$$\left[\frac{1}{L_x C_x} = \omega_o^2, \quad Q = \frac{\omega_o L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o}{Q}\right]$$

$$\frac{i_o}{v_i}(s) = \frac{1}{R_x} \cdot \frac{s\left(\frac{\omega_o}{Q}\right)}{s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2} = \frac{1}{R_x} H(s)$$

- $\frac{1}{R_x}$: Gain Term
- $\frac{s(\omega_o/Q)}{s^2 + s(\omega_o/Q) + \omega_o^2}$: Freq. Shaping Term → Bandpass Biquad
- $H(s)$: resonance magnitude ↓ Just solve the ckt. @ resonance ↓ then multiply the answer by $H(s)$!



- Go through Module 15 on Gyros, slides 1-16