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**EE C245 - ME C218**  
**Introduction to MEMS Design**  
**Fall 2011**

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**Lecture Module 12: Capacitive Transducers**

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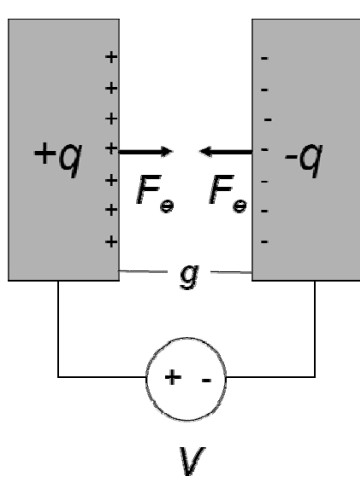
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**Lecture Outline**

- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - ↪ Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - ↪ Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - ↪ Electrostatic Comb-Drive
    - 1<sup>st</sup> Order Analysis
    - 2<sup>nd</sup> Order Analysis

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### Basic Physics of Electrostatic Actuation



- **Goal:** Determine gap spacing  $g$  as a function of input variables
- First, need to determine the energy of the system
- Two ways to change the energy:
  - ↪ Change the charge  $q$
  - ↪ Change the separation  $g$

$$\Delta W(q, g) = V\Delta q + F_e\Delta g$$

$$dW = Vdq + F_e dg$$

- **Note:** We assume that the plates are supported elastically, so they don't collapse

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### Stored Energy

• Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap

$$W = 0 + \int_0^g F_e dg'$$

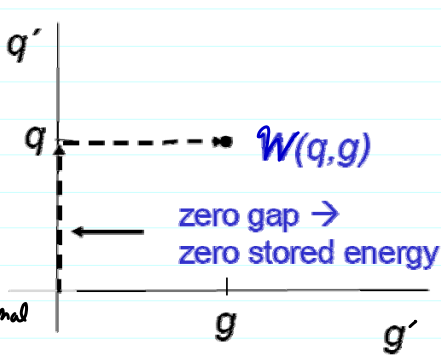
No change in charge:  $dq=0$

$$F_e = \left(\frac{q}{2}\right)\epsilon = \frac{1}{2} \frac{q^2}{\epsilon A}$$

(independent of  $g$ )

$$\therefore W = \int_0^g F_e dg' = F_e g \Big|_0^g = F_e g$$

$$\therefore W(g) = \frac{1}{2} \frac{q^2}{\epsilon A} g$$



zero gap → zero stored energy

For a capacitor  $C$ :  
 $q = CV \rightarrow V = \frac{q}{C}$

$$W(q) = \int_0^q V dq = \int_0^q \left(\frac{q}{C}\right) dq$$

$$= \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \frac{q^2}{\epsilon A} = W(g)$$

Work done to charge  $C$  to  $q$  at fixed gap

Same answer when →

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### Charge-Control Case

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- Having found stored energy, we can now find the force acting on the plates and the voltage across them:

From  $dW = Vdq + F_e dg$ :

⇒ Force is given by:

$$F_e = \left. \frac{\partial W(q,g)}{\partial g} \right|_{q=\text{const.}} = \frac{\partial}{\partial g} \left( \frac{1}{2} \frac{q^2}{\epsilon A} g \right)$$

$$\therefore \boxed{F_e = \frac{1}{2} \frac{q^2}{\epsilon A}} \Rightarrow \text{independent of gap spacing!}$$

⇒ Voltage is given by:

$$V = \left. \frac{\partial W(q,g)}{\partial q} \right|_{g=\text{const.}} = \frac{\partial}{\partial q} \left( \frac{1}{2} \frac{q^2}{\epsilon A} g \right) = \frac{qg}{\epsilon A} \therefore \boxed{V = \frac{q}{C}} \Rightarrow \text{consistent w/ what we already know} \checkmark$$

The diagram shows two vertical parallel plates. The left plate is labeled '+q' and the right plate is labeled '-q'. The gap between them is labeled 'g'. Below the plates, a voltage source 'V' is indicated with '+' on the left and '-' on the right. Two horizontal arrows labeled 'F\_e' point towards each other between the plates, representing the electrostatic force.

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### Voltage-Control Case

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- Practical situation: We control V
  - Charge control on the typical sub-pF MEMS actuation capacitor is difficult
  - Need to find  $F_e$  as a partial derivative of the stored energy  $W = W(V,g)$  with respect to  $g$  with  $V$  held constant? But can't do this with present  $W(q,g)$  formula
  - Solution:** Apply Legendre transformation and define the co-energy  $W'(V,g)$

Effort (e.g., force, voltage, ...)

Energy  $W(q) = \int_0^{q_1} e dq = \int_0^{q_1} \Phi(q) dq$

Co-Energy  $W'(e) = \int_0^{e_1} q de = \int_0^{e_1} \Phi^{-1}(e) de$

Displacement (e.g., displacement, charge)

Can define co-energy as:  $W'(e) = eq - W(q)$  (from this plot)

For a linear system, these will be equal.

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**Co-Energy Formulation**

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- For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

$$W'(V, g) = qV - W(q, g)$$

Differentially, this becomes:

$$dW'(V, g) = (q dV + V dq) - dW(q, g)$$

But  $[dW(q, g) = F_e dg + V dq]$

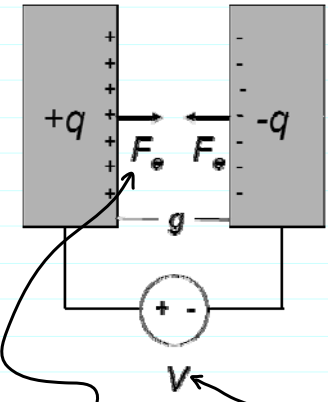
$dW'(V, g) = q dV - F_e dg$

Working Co-Energy Expression

From which:

Charge,  $Q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_{g = \text{const.}}$

Force,  $F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_{V = \text{const.}}$  ⇒ this gives force as a function of applied voltage



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**Electrostatic Force (Voltage Control)**

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- Find co-energy in terms of voltage (w/ gap held constant)

$$W' = \int_0^V q(g, V') dV' = \int_0^V \left( \epsilon \frac{A}{g} \right) V' dV' = \frac{1}{2} \left( \frac{\epsilon A}{g} \right) V^2 = \frac{1}{2} C V^2$$

(as expected)

- Variation of co-energy with respect to gap yields electrostatic force:

$$F_e = - \left. \frac{\partial W'(V, g)}{\partial g} \right|_V = - \frac{1}{2} \left( - \frac{\epsilon A}{g^2} \right) V^2 = \frac{1}{2} \frac{C}{g} V^2$$

strong function of gap!

- Variation of co-energy with respect to voltage yields charge:

$$q = \left. \frac{\partial W'(V, g)}{\partial V} \right|_g = \left( \frac{\epsilon A}{g} \right) V = C V$$

as expected

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## Spring-Suspended Capacitive Plate

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## Charge Control of a Spring-Suspended C

Force generated by charge  $q$  supplied by current  $I$ :

$$F_e = \left. \frac{\partial W(q, g)}{\partial g} \right|_q = \frac{q^2}{2\epsilon A}$$

Restoring force of spring:  $F_{\text{spring}} = kz = F_e$  (@ equilibrium)

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k}} = g \Rightarrow \text{can increase } q \text{ and drive } g \rightarrow 0$$

initial gap

$$V = \frac{q}{C} = \frac{q}{\epsilon A} g = \boxed{\frac{q}{\epsilon A} \left( g_0 - \frac{1}{2} \frac{q^2}{\epsilon A} \frac{1}{k} \right)} = V \Rightarrow V \downarrow \text{ as } g \downarrow$$

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### Voltage Control of a Spring-Suspended C

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Again,  $F_{\text{spring}} = kz = F_e$   
 But now:  

$$F_e = \frac{\partial W'(V, g)}{\partial g} \Big|_V = \frac{1}{2} \frac{\epsilon A}{g^2} V^2$$

And the gap:  

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2} \frac{\epsilon A V^2}{g^2 k} = g} \Rightarrow g \text{ shows up on both sides!}$$
 cubic nonlinearity in  $g!$   
 If  $V \uparrow \rightarrow g \downarrow \rightarrow F_e \uparrow$

Charge: (for a stable gap)  

$$q = \frac{\partial W'(V, g)}{\partial V} \Big|_g = CV = \boxed{\frac{\epsilon A}{g} V = q}$$
 Feedback!  
 If loop gain  $> 1$ , then this will go unstable!

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### Stability Analysis

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- Net attractive force on the plate:  

$$F_{\text{net}} = \underbrace{\frac{\epsilon A V^2}{2 g^2}}_{F_e} - \underbrace{k(g_0 - g)}_{F_{\text{spring}}}$$
- An increment in gap  $dg$  leads to an increment in net attractive force  $dF_{\text{net}}$   

$$dF_{\text{net}} = \frac{\partial F_{\text{net}}}{\partial g} dg = \left[ -\frac{\epsilon A V^2}{g^3} + k \right] dg$$

Thus, need this = (+)  

$$\boxed{k > \frac{\epsilon A V^2}{g^3}}$$
 (for a stable uncollapsed state)

If  $g \downarrow \rightarrow dg = (-)$ , then for stability need  $\rightarrow$  otherwise, plate collapses

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### Pull-In Voltage $V_{PI}$

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- $V_{PI}$  = voltage at which the plates collapse
- The plate goes unstable when

$$k = \frac{\epsilon A V_{PI}^2}{g_{PI}^3} \quad (1) \quad \text{and} \quad F_{net} = 0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - k(g_0 - g_{PI}) \quad (2)$$

$\leftarrow$  pull-in gap

- Substituting (1) into (2):

$$0 = \frac{\epsilon A V_{PI}^2}{2g_{PI}^2} - \frac{\epsilon A V_{PI}^2}{g_{PI}^3} (g_0 - g_{PI})$$

$$\frac{g_0 - g_{PI}}{g_{PI}} = \frac{1}{2} \rightarrow g_0 = \frac{3}{2} g_{PI}$$

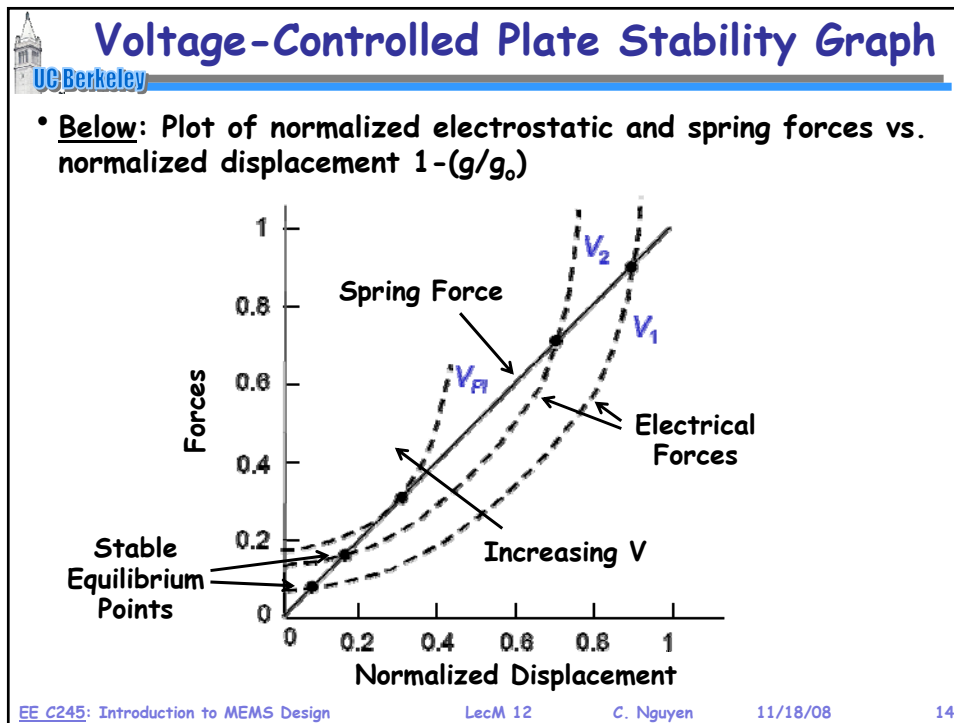
$$\therefore g_{PI} = \frac{2}{3} g_0$$


$$V_{PI} = \sqrt{\frac{k g_{PI}^3}{\epsilon A}}$$

$$\therefore V_{PI} = \sqrt{\frac{8}{27} \frac{k g_0^3}{\epsilon A}}$$

When a gap is driven by a voltage to (2/3) its original spacing, collapse will occur!

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## Advantages of Electrostatic Actuators

- Easy to manufacture in micromachining processes, since conductors and air gaps are all that's needed → low cost!
- Energy conserving → only parasitic energy loss through  $I^2R$  losses in conductors and interconnects
- Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
- Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
- Same capacitive structures can be used for both drive and sense of velocity or displacement
- Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q's for resonant structures

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## Problems With Electrostatic Actuators

- Nonlinear voltage-to-force transfer function
- Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale


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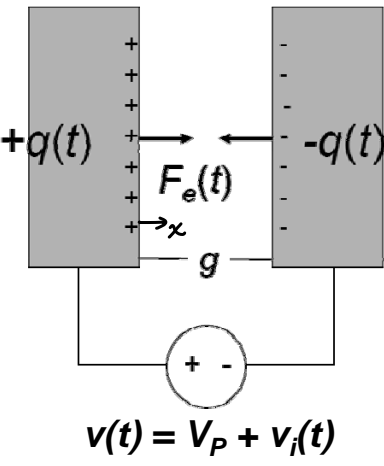
## Linearizing the Voltage-to-Force Transfer Function

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## Linearizing the Voltage-to-Force T.F.

- Apply a DC bias (or polarization) voltage  $V_p$  together with the intended input (or drive) voltage  $v_i(t)$ , where  $V_p \gg v_i(t)$



$v(t) = V_p + v_i(t)$

$$F_e(t) = \frac{\partial W}{\partial x} = \frac{\partial}{\partial x} \left( \frac{1}{2} C [v(t)]^2 \right)$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} [v(t)]^2 = \frac{1}{2} (V_p + v_i(t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2} [V_p^2 + 2V_p v_i(t) + [v_i(t)]^2] \frac{\partial C}{\partial x}$$

$[V_p \gg v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2} V_p^2 \frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_p \frac{\partial C}{\partial x} v_i(t)}_{\text{AC drive signal}}$

$$C(x) = \frac{\epsilon A}{g_0 \cdot x} = C_0 \left(1 - \frac{x}{g_0}\right)^{-1} \approx C_0 \left(1 + \frac{x}{g_0}\right)$$

$[x \ll g_0]$        $\sim \text{const.} \therefore \text{linear}$

$$\therefore \frac{\partial C}{\partial x} \approx \frac{C_0}{g_0} \Rightarrow F_e(t) = \frac{1}{2} \frac{C_0}{g_0} V_p^2 + V_p \frac{C_0}{g_0} v_i(t)$$

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**Differential Capacitive Transducer**

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- The net force on the suspended center electrode is

$$F_{net} = F_{er}(t) - F_{el}(t)$$

Do the math.

Assume matched gaps.

$$F_{net}(t) = \frac{1}{2} \frac{\partial C}{\partial x} \{ [V_R(t)]^2 - [V_L(t)]^2 \}$$

$$= \frac{1}{2} \frac{\partial C}{\partial x} \{ (V_p^2 + 2V_p v(t) + [v(t)]^2) - (V_p^2 - 2V_p v(t) + [v(t)]^2) \}$$

$$\therefore F_{net}(t) = 2V_p \frac{\partial C}{\partial x} v(t) = 2V_p \frac{C_0}{g_0} v(t) \Rightarrow \text{Linear w/ } v(t)! \text{ (gap match limited)}$$

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**Remaining Nonlinearity  
(Electrical Stiffness)**

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### Parallel-Plate Capacitive Nonlinearity

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- Example:** clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:**

$V_a$  or  $v_A$   
 $v_a = |v_a| \cos \omega t$   
 $V_a$

$V_a$  or  $v_A = V_A + v_a$   
 Total Value  
 DC Component (upper case variable; upper case subscript)  
 AC or Signal Component (lower case variable; lower case subscript)

Electrode  
 Conductive Structure  
 $k_m$   
 $d_1$   
 $m$   
 $x$   
 $F_{d1}$   
 $v_1$   
 $V_1$   
 $V_P$

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### Parallel-Plate Capacitive Nonlinearity

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- Example:** clamped-clamped laterally driven beam with balanced electrodes
- Expression for  $\partial C / \partial x$ :**

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_0 \left(1 + \frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + \frac{x}{d_1}\right)^{-2}$$

[Expand the Taylor series further]

$$\frac{\partial C_1}{\partial x} = -\frac{C_0}{d_1} \left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \dots\right)$$

where  $A_1 = -\frac{2}{d_1}$   
 $A_2 = \frac{3}{d_1^2}$   
 $A_3 = -\frac{4}{d_1^3}$   
 $\vdots$

Electrode  
 Conductive Structure  
 $k_m$   
 $d_1$   
 $m$   
 $x$   
 $F_{d1}$   
 $v_1$   
 $V_1$   
 $V_P$

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### Parallel-Plate Capacitive Nonlinearity

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- Thus, the expression for force from the left side becomes:

$$F_{d1} = \frac{1}{2} \frac{\partial C}{\partial x} (V_P - V_1 - N_i)^2 = \frac{1}{2} \frac{\partial C}{\partial x} (V_{P1} - N_i)^2$$

{small displacements:  $x \ll d_1$ }

$$F_{d1} = \frac{1}{2} \left( -\frac{C_{o1}}{d_1} \right) (1 + A_1 x) (V_{P1}^2 - 2V_{P1}N_i + N_i^2)$$

$$= \frac{1}{2} \left( -\frac{C_{o1}}{d_1} \right) \left\{ V_{P1}^2 - 2V_{P1}N_i + N_i^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x N_i + A_1 x N_i^2 \right\}$$

@ resonance:  $x = \frac{Q F_{d1}}{j k} \approx \frac{Q}{j k} \frac{\partial C}{\partial x} V_{P1} V_1$

Thus:

$$N_i = |N_i| \cos \omega_o t \rightarrow x = |x| \sin \omega_o t$$

↑  
 $x$  90° phase-shifted from  $N_i$

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### Parallel-Plate Capacitive Nonlinearity

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- Retaining only terms at the drive frequency:

$$F_{d1}|_{\omega_o} = \underbrace{V_{P1} \frac{C_{o1}}{d_1}}_{\text{Drive force arising from the input excitation voltage at the frequency of this voltage}} |v_1| \cos \omega_o t + V_{P1}^2 \frac{C_{o1}}{d_1^2} |x| \sin \omega_o t$$

↑      ↑  
 Proportional to displacement  
 90° phase-shifted from drive, so in phase with displacement

- These two together mean that this force acts against the spring restoring force!
- ↪ A negative spring constant
- ↪ Since it derives from  $V_p$ , we call it the electrical stiffness, given by:

$$k_e = V_{P1}^2 \frac{C_{o1}}{d_1^2} = V_{P1}^2 \frac{\epsilon A}{d_1^3}$$

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### Electrical Stiffness, $k_e$

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- The electrical stiffness  $k_e$  behaves like any other stiffness
- It affects resonance frequency:

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$

$$= \sqrt{\frac{k_m}{m} \left(1 - \frac{k_e}{k_m}\right)^{1/2}}$$

$$\omega_o' = \omega_o \left(1 - \frac{V_{P1}^2 \epsilon A}{k_m d_1^3}\right)^{1/2}$$

Frequency is now a function of dc-bias  $V_{P1}$

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### Voltage-Controllable Center Frequency

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- Quadrature force  $\Rightarrow$  voltage-controllable electrical stiffness:

$$k_e = \frac{\epsilon_o A_o}{d^3} V_P^2$$

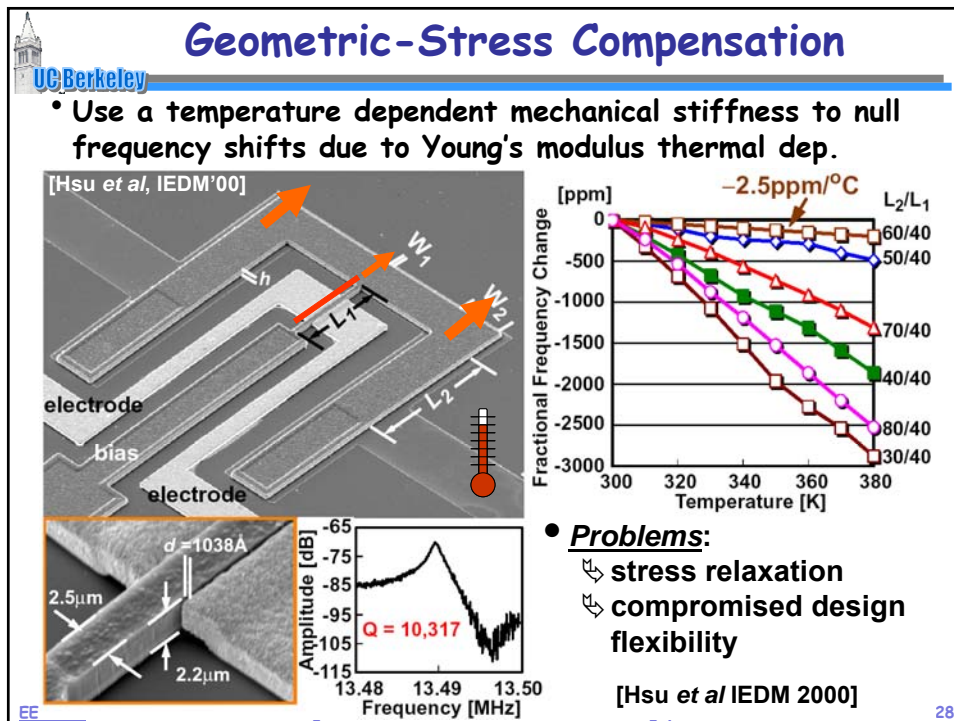
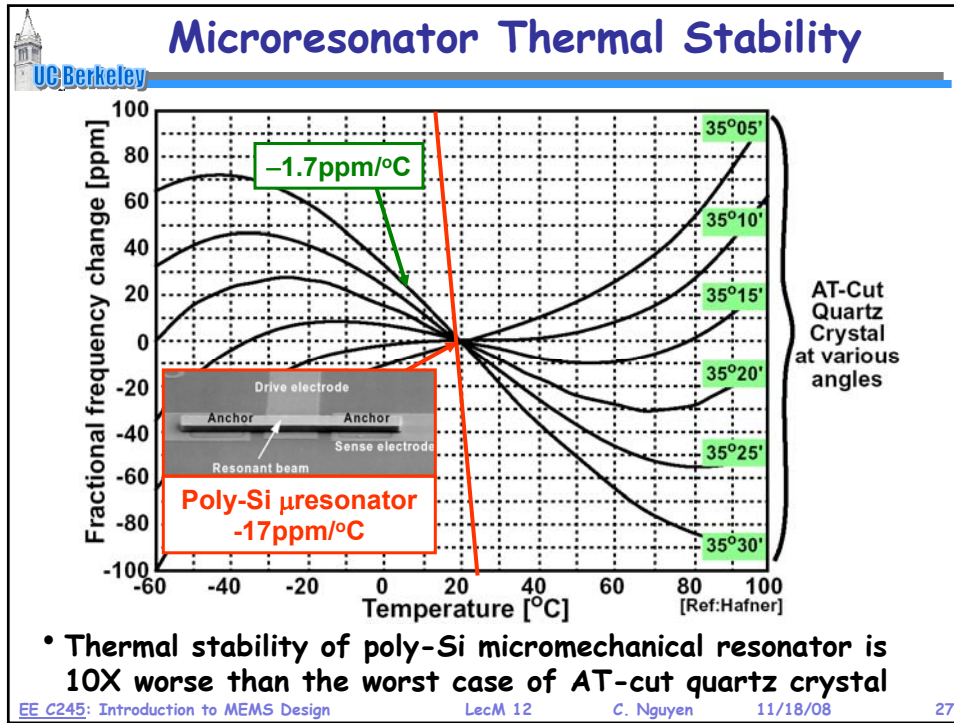
Electrode Overlap Area

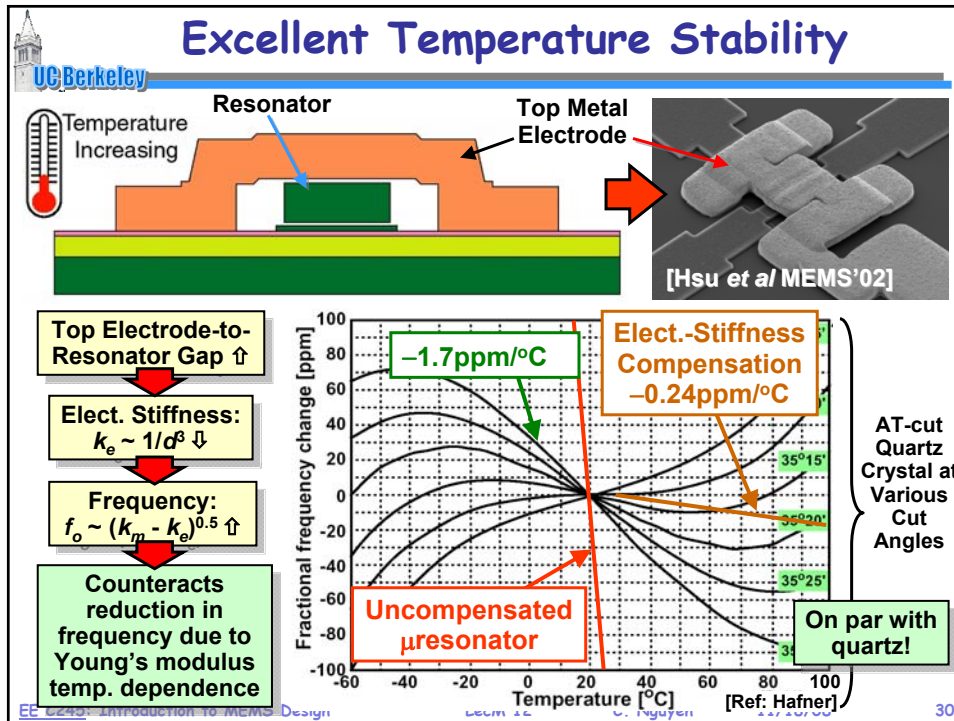
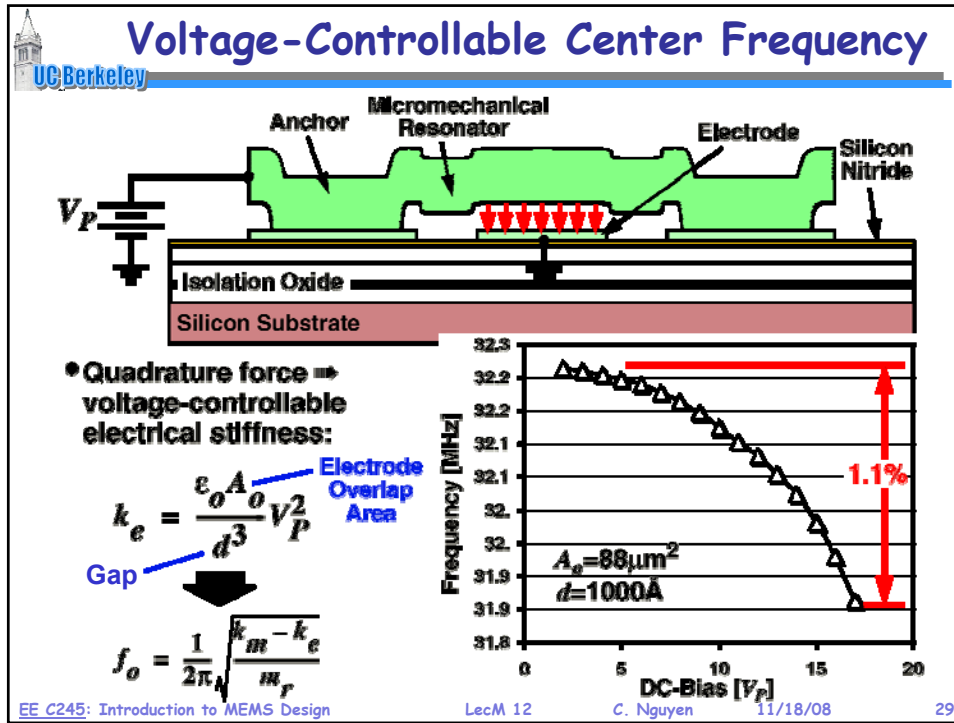
Gap

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k_m - k_e}{m_r}}$$

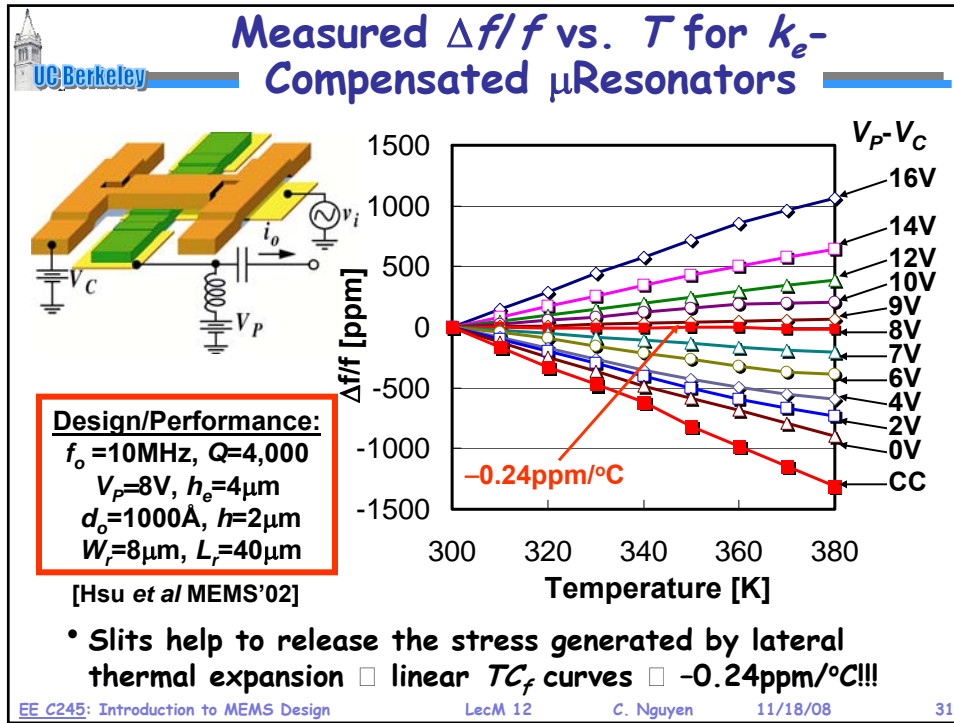
$A_o = 88 \mu\text{m}^2$   
 $d = 1000 \text{ \AA}$

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**Can One Cancel  $k_e$  w/ Two Electrodes?**

- What if we don't like the dependence of frequency on  $V_p$ ?
- Can we cancel  $k_e$  via a differential input electrode configuration?
- If we do a similar analysis for  $F_{d2}$  at Electrode 2:


Subtracts from the  $F_{d1}$  term, as expected

$$F_{d2}|_{\omega_o} = -V_{p2} \frac{C_{o2}}{d_2} |v_2| \cos \omega_o t + V_{p2}^2 \frac{C_{o2}}{d_2^2} |x| \sin \omega_o t$$

Adds to the quadrature term  $\rightarrow k_e$ 's add, no matter the electrode configuration!

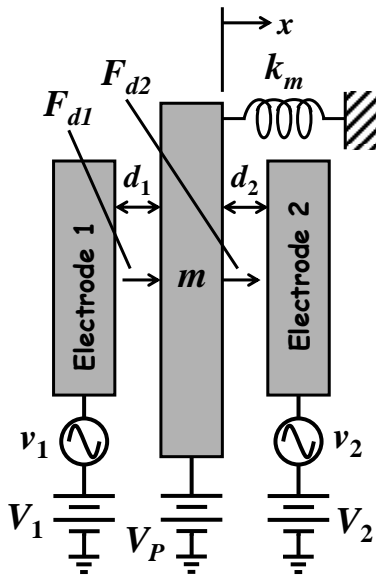
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## Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
  - ↪ Resonance frequency becomes dependent on parameters (e.g., bias voltage  $V_P$ )
  - ↪ Output current will also take on nonlinear characteristics as amplitude grows (i.e., as  $x$  approaches  $d_0$ )
  - ↪ Noise can alias due to nonlinearity
- Range of motion is small
  - ↪ For larger motion, need larger gap ... but larger gap weakens the electrostatic force
  - ↪ Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)



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## Electrostatic Comb Drive

# Electrostatic Comb Drive

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### Electrostatic Comb Drive

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- Use of comb-capacitive transducers brings many benefits
  - ↳ Linearizes voltage-generated input forces
  - ↳ (Ideally) eliminates dependence of frequency on dc-bias
  - ↳ Allows a large range of motion

**Comb-Driven Folded Beam Actuator**

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### Comb-Drive Force Equation (1<sup>st</sup> Pass)

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Top View

Side View

$$C(x) = \frac{2\epsilon_0 x h}{d} \rightarrow \left[ \frac{\partial C}{\partial x} = \frac{2\epsilon_0 h}{d} \right]$$

$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{\partial C}{\partial x} (V_p - V_i)^2 = \frac{2}{2} \frac{\epsilon_0 h}{d} (V_p^2 - 2V_p V_i + V_i^2) \approx -2V_p \frac{\epsilon_0 h}{d} V_i = F_d$$

When  $V_i = (+) \rightarrow F_d = (-)$  ✓

↳ But wait! This ignores other practical effects! (No dependence on  $\lambda$ ! LINEAR!)

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### Lateral Comb-Drive Electrical Stiffness

Top View

Side View

- Again:  $C(x) = \frac{2Nshx}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2Nsh}{d}$
- No  $(\partial C/\partial x)$  x-dependence  $\rightarrow$  no electrical stiffness:  $k_e = 0!$
- Frequency immune to changes in  $V_p$  or gap spacing!

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### Typical Drive & Sense Configuration

2-port Lateral Microresonator

$N_f$ : # shuttle fingers

Simple Analysis:

$$F_{d1} = \frac{1}{2} \frac{\partial C_1}{\partial x} (V_1 - V_{P1})^2 = \frac{1}{2} \left( -\frac{\epsilon_0 h}{d} \right) (N_1^2 - 2V_{P1}V_1 + V_{P1}^2) (2N_f)$$

$$F_{d2} = \frac{1}{2} \frac{\partial C_2}{\partial x} (V_2 - V_{P2})^2 = \frac{1}{2} \left( \frac{\epsilon_0 h}{d} \right) (N_2^2 - 2V_{P2}V_2 + V_{P2}^2) (2N_f)$$

$$\therefore F_{net} = F_{d1} + F_{d2} = \frac{1}{2} \left( \frac{\epsilon_0 h}{d} \right) (N_2^2 - N_1^2 - 2(V_{P2}V_2 - V_{P1}V_1) + V_{P2}^2 - V_{P1}^2) (2N_f)$$

For  $V_1 = V_2, V_1 = -V_2$

$$F_{net} = 2(2N_f) \left( \frac{\epsilon_0 h}{d} \right) V_{P1} V_1$$

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### Comb-Drive Force Equation (2<sup>nd</sup> Pass)

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- In our 1<sup>st</sup> pass, we accounted for
  - ↪ Parallel-plate capacitance between stator and rotor
- ... but neglected:
  - ↪ Fringing fields
  - ↪ Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!

The diagram illustrates a comb-drive actuator with stator and rotor fingers of length  $L$  and gap  $g$ . The rotor is displaced by a distance  $x$ . A ground plane is located at a distance  $z_0$  from the rotor. The electrical circuit shows a voltage source  $V_s$  connected to the stator and a voltage source  $V_r$  connected to the rotor and ground.

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### Comb-Drive Force With Ground Plane Correction

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- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

The equivalent circuit diagram shows three nodes: stator (s), rotor (r), and ground plane (p). The capacitances are:  $\frac{1}{2} C_{sp}$  between stator and ground plane,  $C_{rp}$  between rotor and ground plane,  $\frac{1}{2} C_{rs}$  between stator and rotor, and  $C_{rp}$  between rotor and ground plane. The rotor is connected to a voltage source  $V_r$  and the stator to a voltage source  $V_s$ .

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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### Capacitance Expressions

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- Case:  $V_r = V_p = 0V$
- $C_{sp}$  depends on whether or not fingers are engaged

$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

Capacitance per unit length

**Region 2**

**Region 3**

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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### Comb-Drive Force With Ground Plane Correction

UC Berkeley

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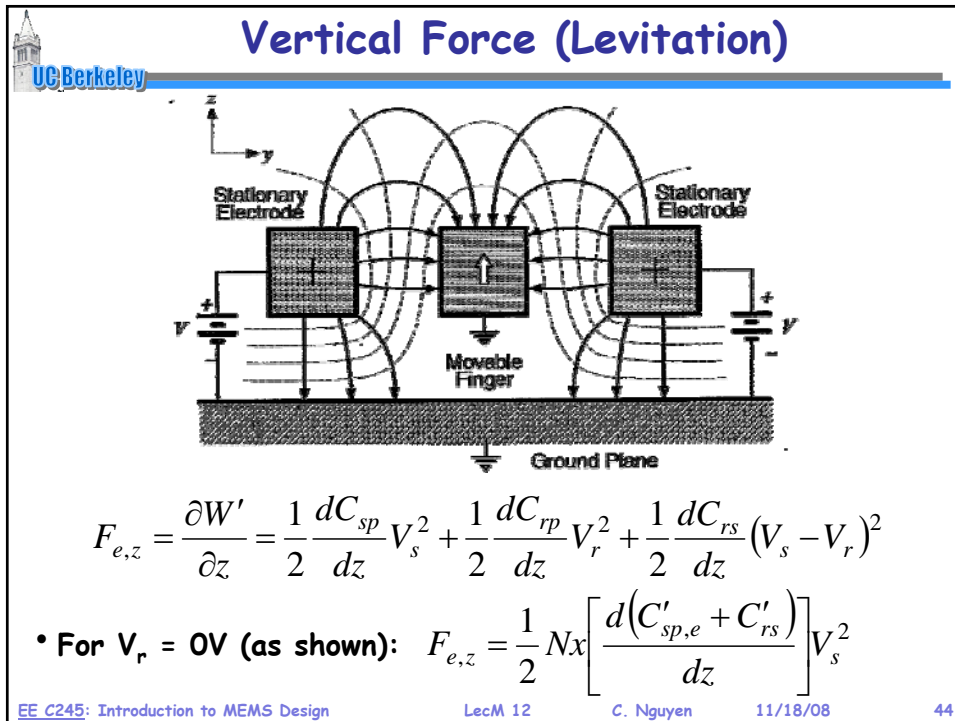
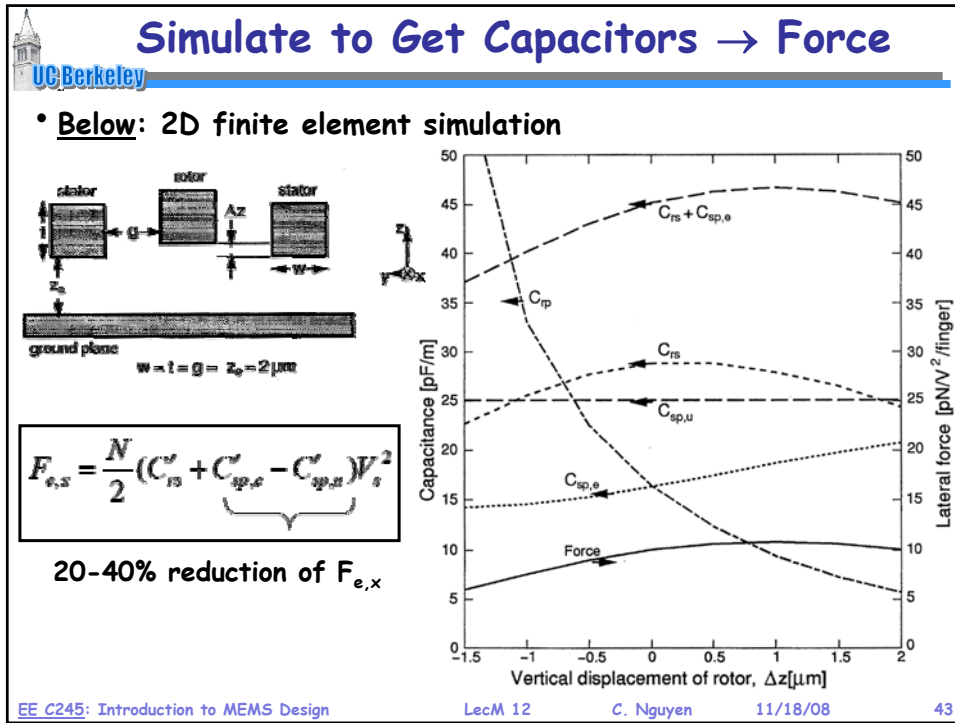
$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

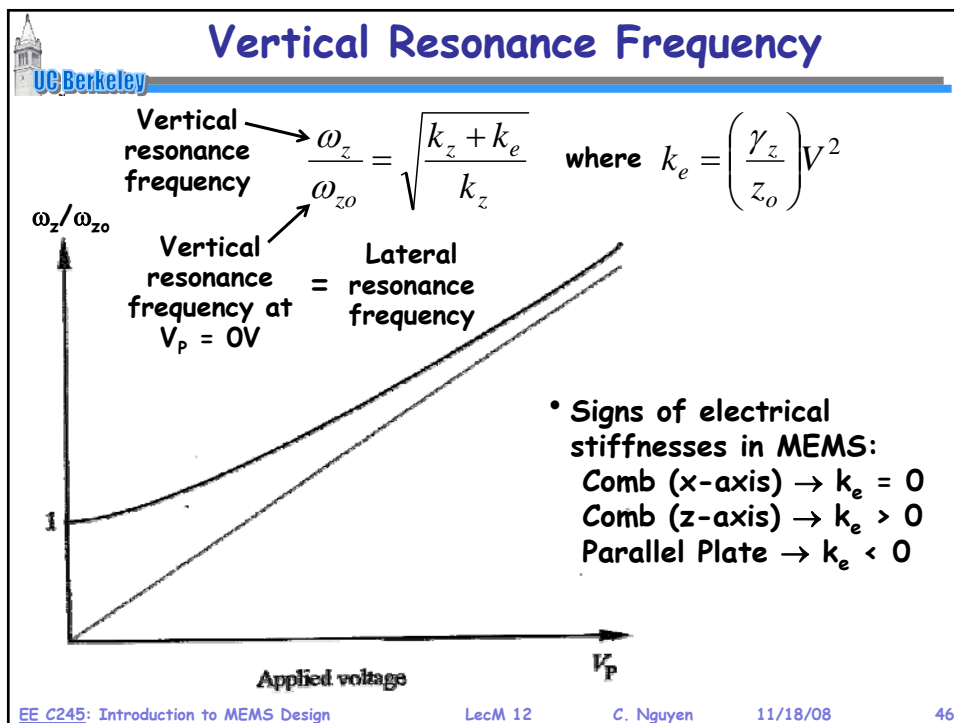
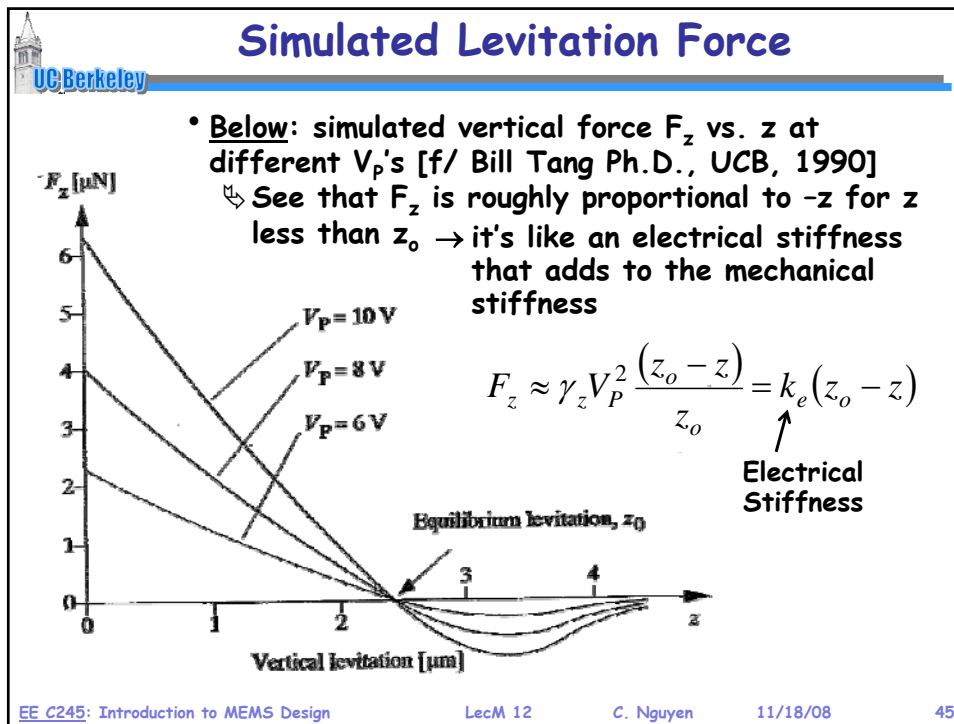
$$F_{e,x} = \frac{N}{2} (C'_{rs} + C'_{sp,e} - C'_{sp,u}) V_s^2$$

(for  $V_r = V_p = 0$ )

[Gary Fedder, Ph.D., UC Berkeley, 1994]

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### Suppressing Levitation

- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

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### Force of Comb-Drive vs. Parallel-Plate

gap =  $g = 1 \mu\text{m}$ ,  
 thickness =  $t = 2 \mu\text{m}$   
 finger length =  $L = 100 \mu\text{m}$   
 overlap length  $x = 75 \mu\text{m}$

- Comb drive (x-direction)  
 $\hookrightarrow V_1 = V_2 = V_s = 1V$   

$$F_{e,x} = \frac{\epsilon_0 t}{g} V_s^2$$
- Differential Parallel-Plate (y-direction)  
 $\hookrightarrow V_1 = 0V, V_2 = 1V$   

$$F_{e,y} = \frac{1}{2} \frac{\epsilon_0 t x}{g^2} V_2^2$$

$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\epsilon_0 t x}{g^2} V_2^2}{\frac{2 \epsilon_0 t}{g} V_s^2} = \frac{1}{2} \frac{x}{g}$$

Parallel-plate generates a much larger force; but at the cost of linearity

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