


EE C245 - ME C218
Introduction to MEMS Design
Fall 2011

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 14: Sensing Circuits

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 1



Lecture Outline

- Reading: Senturia, Chpt. 14
- Lecture Topics:
 - ↳ Detection Circuits
 - ↳ Velocity Sensing
 - ↳ Position Sensing

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 2

Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, w/ output shorted

Now, taking RL into account

Lx + Cx cancel @ resonance

Solve the problem @ resonance first, then multiply by H(s)

ω will alter @

$$\frac{x}{F_{dl}}(s) = \frac{\omega_0 Q}{k} H(s)$$

$$[F_{dl} = \eta_{e1} V_i] \Rightarrow \frac{x}{V_i}(s) = \eta_{e1} \frac{\omega_0 Q}{k} H(s)$$

$$[i_0 = \eta_{e2} x] \Rightarrow \frac{i_0}{V_i}(s) = \eta_{e1} \eta_{e2} \frac{\omega_0 Q}{k} H(s)$$

$$\frac{1}{R_{x12}} \Rightarrow \frac{\eta_{e1} \eta_{e2} Q}{m \omega} H(s)$$

$$\frac{i_0}{V_i}(s) = \frac{1}{R_{x12}} H(s)$$

$$\frac{V_o}{V_i}(s) = \frac{R_L}{R_{x12} + R_L} H(s)$$

$$Q' = Q \left(\frac{R_{x12}}{R_{x12} + R_L} \right)$$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 3

Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

Work @ resonance: (to simplify the analysis)

$$\frac{V_o}{V_i} = \frac{R_o}{R_x + R_o} \text{ (@ resonance)}$$

Then, generalize to off resonance:

$$\frac{V_o}{V_i} = \frac{R_o}{R_x + R_o} H(s, Q'), \text{ where } Q' = Q \left(\frac{R_x}{R_x + R_o} \right)$$

Cx & Lx cancel

Voltage Representing Velocity

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 4

Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

cantilever

Since this structure has completely symmetrical I/O ports:

R_x C_x L_x R_D

$Q = \frac{\omega_0 L_x}{R_x} \rightarrow Q' = \frac{\omega_0 L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_0}{Q'}$

Brute force approach:

$$\frac{N_o}{V_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_D C_x}{s^2 R_x C_x + 1 + s^2 L_x C_x + sR_D C_x} = \frac{s \frac{R_D}{L_x C_x}}{s^2 + s \frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$Q = Q' \left(\frac{R_x}{R_x + R_D} \right)$

$$\frac{N_o}{V_i}(s) = \frac{R_D}{R_x + R_D} \frac{s(\omega_0 Q')}{s^2 + s(\omega_0/Q') + \omega_0^2} = \frac{R_D}{R_x + R_D} (H)(s, Q')$$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 5

Position-to-Voltage Conversion

UC Berkeley

- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:

cantilever

R_x C_x L_x C_D

Brute force approach:


$$\frac{N_o}{V_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

$$\frac{N_o}{V_i}(s) = \frac{\frac{sC_x}{sC_D}}{sR_x C_x + 1 + s^2 L_x C_x + \frac{sC_x}{sC_D}} = \frac{C_x/C_D}{1 + C_x/C_D} \frac{1}{1 + \frac{sR_x C_x}{1 + C_x/C_D} + s^2 \frac{L_x C_x}{1 + C_x/C_D}}$$

$$= \frac{C_x/C_D}{1 + C_x/C_D} \frac{1 + C_D/C_D}{s^2 + s \left(\frac{R_x}{L_x} + \frac{1 + C_D/C_D}{L_x C_x} \right) + \frac{1 + C_D/C_D}{L_x C_x}}$$

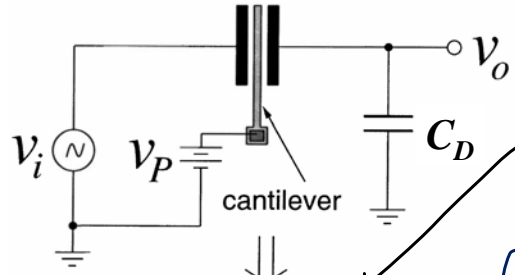
$\left[\omega_0^2 = \frac{1}{L_x C_x} \rightarrow (\omega_0')^2 = \omega_0^2 (1 + C_D/C_D) \right]$
 $\left[Q' = \frac{\omega_0' L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_0'}{Q'}, Q' = Q \sqrt{1 + C_D/C_D} \right]$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 6



Position-to-Voltage Conversion

• To sense position (i.e., displacement), use a capacitive load



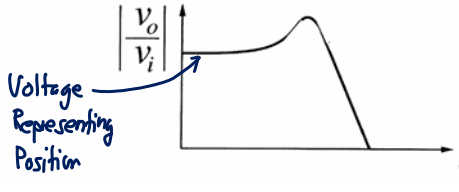
$$\frac{V_o}{V_i}(s) = \underbrace{\frac{C_x/C_0}{1 + C_x/C_0}}_{\text{DC Gain Term}} \underbrace{\frac{(\omega_0)^2}{s^2 + (\frac{\omega_0}{Q})s + (\omega_0)^2}}_{\text{Low-Pass BiQuad}}$$

→ To maximize gain → 1, need $C_0 \ll C_x$.
 (must minimize C_0)

Note: Can use similar short-cut to the R case.


- ① Get DC response → C's dominate.
- ② Then:

$$\frac{V_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \text{TP}(s, \omega_0, Q) \cdot \omega_0 Q$$



Voltage Representing Position

EE C245: Introduction to MEMS Design
LecM 14
C. Nguyen
11/18/08
7



Velocity Sensing Circuits

EE C245: Introduction to MEMS Design
LecM 14
C. Nguyen
11/18/08
8

Velocity-to-Voltage Conversion

UC Berkeley

- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

Work @ resonance: (to simplify the analysis)

$$\frac{V_o}{V_i} = \frac{R_o}{R_x + R_o} \text{ (@ resonance)}$$

Then, generalize to off resonance:

$$\frac{V_o}{V_i} = \frac{R_o}{R_x + R_o} \mathcal{H}(s, Q'), \text{ where } Q' = Q \left(\frac{R_x}{R_x + R_o} \right)$$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 9

Problems With Purely Resistive Sensing

UC Berkeley

Now, we get: (approximately)

$$\frac{V_o}{V_i}(s) \sim \frac{R_o}{R_x + R_o} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot \mathcal{H}(s, \omega_b, Q')$$

Depend on both R_o & C_p .

$$\omega_p = \frac{1}{(R_x || R_o) C_p}$$

Impact depends on where ω_p is relative to ω_b .

Includes C_o , line C , bond pad C , and next stage C

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 10

Problems With Purely Resistive Sensing

• In general, the sensor output must be connected to the inputs of further signal conditioning circuits \rightarrow input R_i of these circuits can load R_D

These change w/ hook-up \rightarrow not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

\rightarrow

Soln: use op amps.

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 11

The TransR Amplifier Advantage

• The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p and R_i

• The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground \Rightarrow No voltage across C_p
 $\hookrightarrow C_p$ effectively isn't there!

$$V_0 = \frac{R_2}{R_x} \oplus(s) V_i$$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 12

UC Berkeley

Position Sensing Circuits

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 13

UC Berkeley

Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load

$$\frac{V_o}{V_i}(s) = \frac{C_x/C_0}{1 + C_x/C_D} \cdot \frac{1}{s} \cdot (H(s, \omega_0, Q)) \cdot \omega_0^2 \theta'$$

Integration yields displacement.
 To maximize gain, minimize f_L .
 ⇒ Problem: parasitic capacitance
 $C_D \rightarrow C_D + C_{pi} + C_{pb}$
 ⇒ DC Gain: $\frac{C_x / (C_0 (C_{pi} + C_{pb}))}{1 + C_x / (C_0 + C_{pi} + C_{pb})}$
 Remedy: Suppress C_p via use of op amps.

Output will get smaller!

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 14

The Op Amp Integrator Advantage

• The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p

$N_o = -i_o (R_2 \parallel \frac{1}{sC_2})$
 $\approx -\frac{N_i}{R_x} \cdot \frac{1}{sC_2} \Rightarrow \frac{N_o}{N_i}(s) = -\frac{1}{R_x C_2} \frac{1}{s}$

well defined → good!
Can drive next stage's R_i w/o interference to transfer function!
 $R_2 \gg \frac{1}{sC_2}$ (for biasing)
 $R_o = 0\Omega$

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 15

Differential Position Sensing

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 16

Differential Position Sensing

UC Berkeley

• **Example: ADXL-50**

Tethers with fixed ends

Fixed Electrodes

Proof Mass

Sense Finger

Applied Acceleration

Capacitive divider

$$V_0 = -V_p + (2V_p) \frac{C_1}{C_1 + C_2}$$

$$= \frac{-V_p C_1 - V_p C_2 + 2V_p C_1}{C_1 + C_2} = V_p \left(\frac{C_1 - C_2}{C_1 + C_2} \right) = V_0$$

Issue: Parasitic Capacitance

$$V_0 = \frac{C_1 - C_2}{C_1 + C_2 + C_p} V_p$$

As before, C_p reduces gain \rightarrow Soln: Use op amp!

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 17

Buffer-Bootstrapped Position Sensing

UC Berkeley

$+V_P$

$-V_P$

Includes capacitance from interconnects, bond pads, and C_{gs} of the op amp

Unity Gain Buffer

$C_{gd} = \text{gate-to-drain capacitance of the input MOS transistor}$

• Bootstrap the ground lines around the interconnect and bond pads

- ↪ No voltage across C_p
- ↪ It's effectively not there!

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 18

Effect of Finite Op Amp Gain

UC Berkeley

Total ADXL-50 Sense C ~ 100fF

Unity Gain Buffer

V_0

$N_0 = A_0(V_+ - V_-) = A_0(N_i - N_0) \rightarrow N_0(1 + A_0) = A_0 N_i \rightarrow \frac{N_0}{N_i} = \frac{A_0}{1 + A_0}$
 Get $Z_i = \frac{V_i}{i_i}$: $i_i = (N_i - N_0) s C_p = N_i \left(1 - \frac{A_0}{1 + A_0}\right) s C_p = N_i \frac{1}{1 + A_0} s C_p$
 $\therefore \frac{N_i}{i_i} = Z_i = \frac{1}{s \left[\frac{C_p}{1 + A_0} \right]} \rightarrow C_{eff} = \frac{C_p}{1 + A_0}$
 No longer zero!
 Ex: $A_0 = 100, C_p = 2\text{pF} \Rightarrow C_{eff} = \frac{2\text{pF}}{101} = 20\text{fF}$
 Not negligible compared w/ ADXL-50 $C_{tot} \sim 100\text{fF}$!

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 19

Integrator-Based Diff. Position Sensing

UC Berkeley

$R_2 \gg \frac{1}{sC_2}$ (for biasing)

$R_0 = 0\Omega$

Can drive next stage's R_1 w/o interference to transfer function!

$i_0 = i_1 + i_2 = N_p(sC_1) - N_p(sC_2) = N_p s(C_1 - C_2)$
 $\therefore N_0 = -i_0 \left(\frac{1}{sC_F}\right) = -N_p \left(\frac{C_1 - C_2}{C_F}\right)$
 $\frac{N_0}{V_p} = -\frac{C_1 - C_2}{C_F} \Rightarrow$ A seemingly perfect differential sensor/amplifier output! ... but only when the op amp is ideal...

EE C245: Introduction to MEMS Design LecM 14 C. Nguyen 11/18/08 20