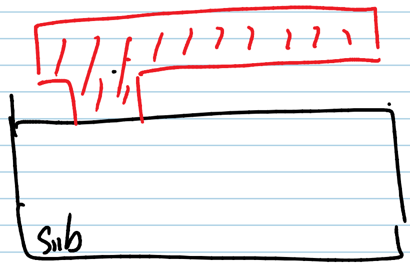
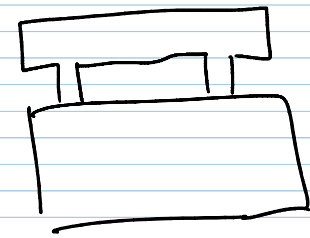
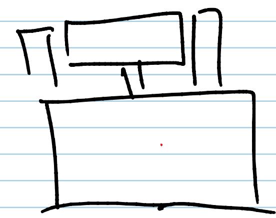
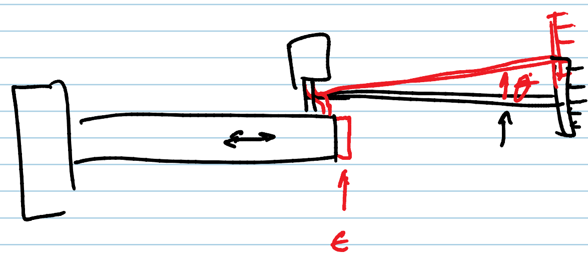
# Dis 7- Mon Oct 15

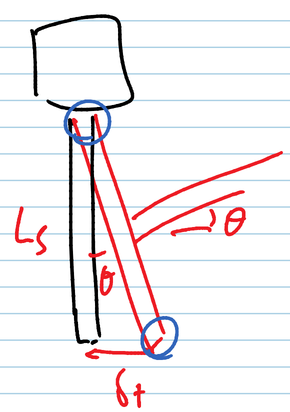
Monday, October 15, 2012
4:08 PM



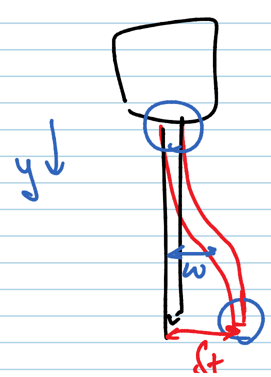








$$\tan\theta = \frac{\delta_t}{L_s} \simeq \theta$$




$$w(0) = 0$$

$$\frac{dw(0)}{dy} = 0 \qquad \frac{dw}{dy}(L_s) = 0$$

$$w(L_s) = \delta_t$$

$$\begin{cases} w \propto \text{bending profile} \\ \dfrac{dw}{dy} \propto \text{slope} \\ \dfrac{d^2w}{dy^2} \propto \text{moment} \\ \dfrac{d^3w}{dy^3} \propto F \\ \dfrac{d^4w}{dy^4} \propto q \end{cases}$$

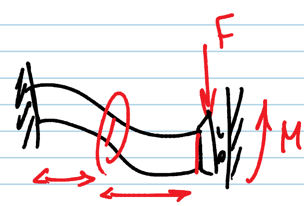
$$EI\frac{d^3w}{dy^3} = F$$

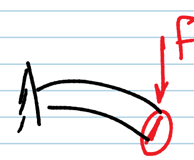
4 B.C.

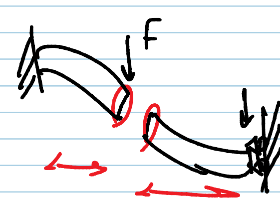
$$EI\frac{d^2w}{dy^2} = -F(L-x) + M$$

$\downarrow$

$$M = \frac{FL}{2}$$

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$$I = \iint \rho x^2 \, ds$$

$ds \downarrow dx\,dy$

$\leftarrow$ kinetik energy

$\widetilde{EI} = \iint E x^2 \, d\xi$ ← Potential energy

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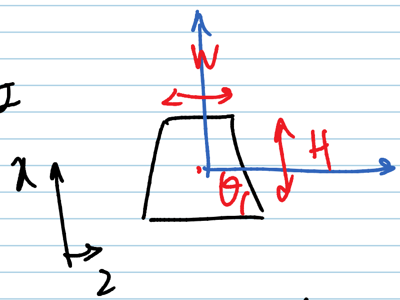
$$\underbrace{EI}\frac{d^2w}{dx^2} = \underbrace{\cdots}$$

↑ B.C., force, ...

↓

EI




$$EI = \iint E x^2 \, dx \, dz \sim EI(y)$$

$w = f(y)$

$H = g(y)$

$$EI(y)\frac{d^2w}{dy^2} = -F(L-y)$$

$$\int_0^L \left( \frac{d^2w}{dy^2} = \frac{-F(L-y)}{EI(y)} \right)$$

$W(y)$

$$\frac{1}{y+a} \sim \ln(y+a)$$

$$\frac{1}{y^2+ay+b} = \frac{\sim}{y+c} + \frac{\sim}{y+d}$$

$\ln(y+c) + \ln(y+d)$

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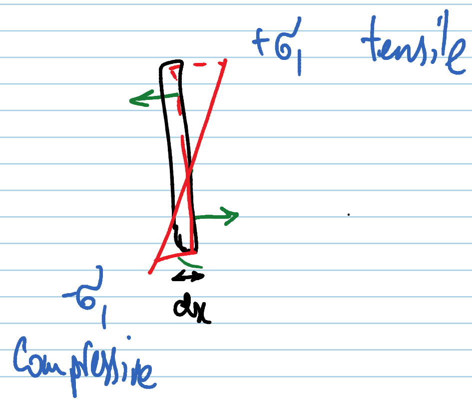
$$EI \frac{d^2w}{dx^2} = M$$

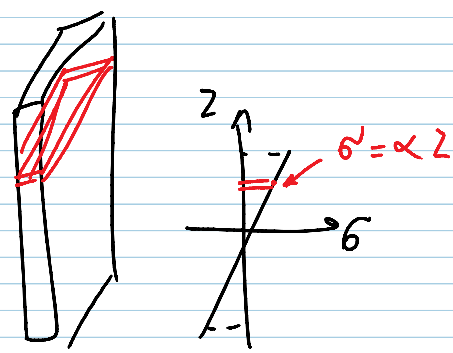



$F = 0$

$M \neq 0$

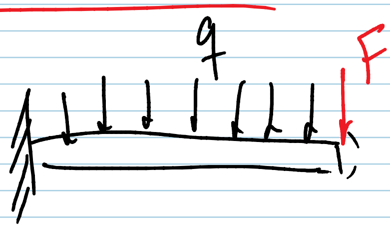
$$\int F . z = \int \alpha W z \, dz . z$$

$$\underbrace{\underbrace{(\alpha z)}_{\text{stress}} \underbrace{(w dz)}_{\text{area}}}_{\text{force}} . \underbrace{z}_{\text{arm}}$$




$$M = \int \alpha(x) \, z \, w dz \, z = \int \alpha(x) \, z^2 w dz$$

$$EI \frac{d^2w}{dx^2} = M(x)$$




$$EI \frac{d^4w}{dy^4} = q$$

$w(0) = 0$

$\frac{dw(0)}{dy} = 0$

$\frac{d^2w}{dy^2}(L) = 0$

$$EI \frac{d^3w}{dy^3} = qy + A$$

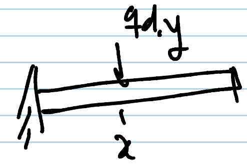
$dy$

$$EI \frac{d^2w}{dy^2} = \frac{qy^2}{2} + Ay + B \qquad \boxed{\frac{qL^2}{2} + AL + B = 0}$$

$$EI \frac{dw}{dy} = \frac{qy^3}{6} + \frac{Ay^2}{2} + By + \underbrace{\cancel{C}}_{0}$$

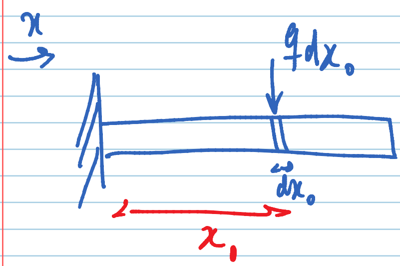
$$M_{@\text{anchor}} = \int_0^L q\,dy \cdot y = \frac{qL^2}{2}$$

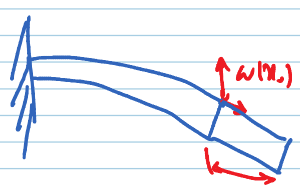
$$EI\, w(y) = \frac{qy^4}{24} + \frac{Ay^3}{6} + \frac{B}{2}y^2 + \underbrace{\cancel{D}}_{0}$$




$$w(y) = \frac{qL^4}{EI}\left(\frac{1}{24}(y/L)^4 - \frac{1}{6}(y/L)^3 + 1/4\,(y/L)^2\right) \qquad q - (N/m)$$

$$\text{effective stiffness at tip} = \frac{(qL)}{w(L)}$$




$$w(x) = \begin{cases} \dfrac{q\,dx_0}{EI}\left(x_0\dfrac{x^2}{2} - \dfrac{x^3}{6}\right) & x \le x_0 \\[2ex] \dfrac{q\,dx_0}{EI}\left[\dfrac{x_0^2}{2}(x - x_0) + \dfrac{x_0^3}{3}\right] & x_0 \le x \le L \end{cases}$$

$$w(x) = \int_0^x \left[\frac{x_0^2}{2}(x - x_0) + \frac{x_0^3}{3}\right]\frac{q}{EI}\,dx + \int_x^L \frac{q\,dx_0}{EI}\left[x_0\frac{x^2}{2} - \frac{x^3}{6}\right]$$

$$w(x) = \frac{qL^4}{EI}\left[\frac{1}{24}(x/L)^4 - \frac{1}{6}(x/L)^3 + \frac{1}{4}(x/L)^2\right]$$