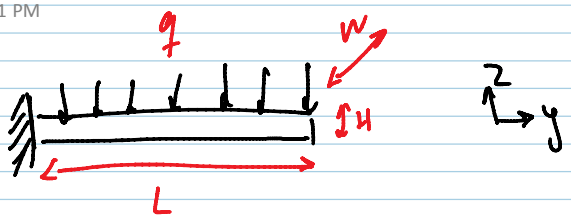
Dis 9, Mon Nov 5

Monday, November 05, 2012
4:11 PM



$$W = \iiint \frac{1}{2} E \epsilon^2 \, dx\,dy\,dz \qquad \leftarrow \sigma d\epsilon$$

$$\epsilon = \left(-\frac{z}{R}\right) = -z\frac{d^2w}{dy^2}$$

$$W = \frac{1}{2}E \int_{-\frac{H}{2}}^{\frac{H}{2}} \int_0^L \int_0^W z^2 \left(\frac{d^2w}{dy^2}\right)^2 dx\,dy\,dz$$

$$W = \frac{1}{2}EW \int_{-H/2}^{H/2} z^2 dz \int_0^L \left(\frac{d^2w}{dy^2}\right)^2 dy$$

$$W = \frac{1}{2}EW\,\frac{H^3}{12} \int_0^L \left(\frac{d^2w}{dy^2}\right)^2 dy = \frac{EWH^3}{24} \int_0^L \left(\frac{d^2w}{dy^2}\right)^2 dy$$

" $w(y) = Ay^3 + By^2 + Cy + D$ " Point force

$$\ddot{w}(y) = 6Ay + 2B$$

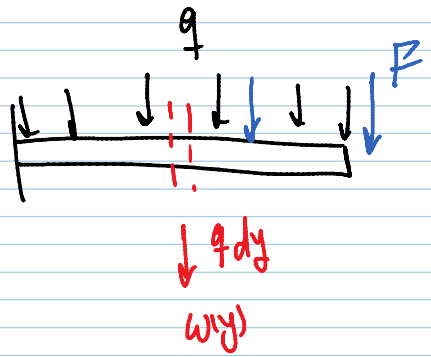
distributed force

$$w(y) = Ay^4 + By^3 + Cy^2$$

$$W = \frac{WEH^3}{24} \int^L \left(36A^2y^2 + 4B^2 + 24ABy\right) dy = \frac{1}{2}EI\left(12A^2L^3 + 4B^2L + 12ABL^2\right)$$

$$W = \frac{WEH^3}{24}\int_0^L \left(36A^2y^2 + 4B^2 + 24ABy\right)dy = \frac{1}{2}EI\left(12A^2L^3 + 4B^2L + 12ABL^2\right)$$

$$I = \frac{1}{12}WH^3$$



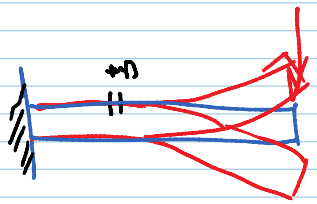
$$U = \int_0^L \underbrace{q\,dy}_{\text{Force}}\ \underbrace{w(y)}_{\text{displacement}}$$

$$U = \int_0^L q\left(Ay^3 + By^2\right)dy = q\left(\frac{A}{4}L^4 + \frac{B}{3}L^3\right)$$

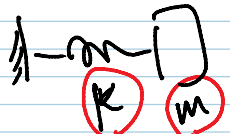
$$U = F\,w(L)$$

$$\mathcal{T} = W - U = \frac{1}{2}EI\left(12A^2L^3 + 4B^2L + 12ABL^2\right) - q\left(\frac{A}{4}L^4 + \frac{B}{3}L^3\right)$$

$$\begin{cases} \dfrac{\partial \mathcal{T}}{\partial A} = 0 \\ \dfrac{\partial \mathcal{T}}{\partial B} = 0 \end{cases}$$

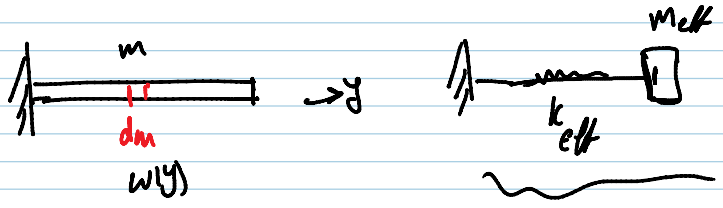


"Distributed structure"



$$\omega_n = \sqrt{k/m}$$

"$EI\dfrac{d^2w}{dx^2} = M$"



$x(t) = a \cos \omega t$

$\ddot{x}(t) = -(a\omega) \sin \omega t$

Velocity $= 2\pi f\, w(y)$

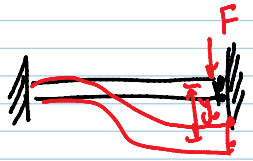
$$dKE = \frac{1}{2} dm\, v^2 = \frac{1}{2} dm \left(2\pi f\, w(y)\right)^2$$

$$dm = \rho H W\, dy = \frac{M_s}{L} dy$$

$$KE = \int_0^L \frac{1}{2} dm \left[2\pi f\, w(y)\right]^2 = \int_0^L \frac{1}{2} \frac{M_s}{L} \left[2\pi f\, w(y)\right]^2 dy$$

$$KE = \frac{1}{2} \frac{M_s}{L} (2\pi f)^2 \cdots$$

$$KE = \frac{1}{2} M_{eff} \left[2\pi f\, w(L)\right]^2$$



$$w(y) = y_0 \left[3\left(\frac{y}{L}\right)^2 - 2\left(\frac{y}{L}\right)^3\right]$$

$$KE = \frac{1}{2} \frac{M_s}{L} (2\pi f)^2 y_0^2 \int_0^L \left[3\left(\frac{y}{L}\right)^2 - 2\left(\frac{y}{L}\right)^3\right]^2 dy \qquad \frac{y}{L} = t$$

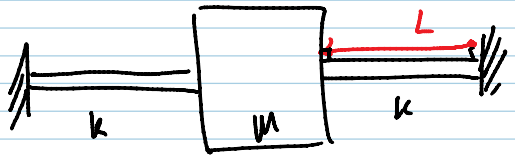
$$KE = \frac{1}{2} M_s (2\pi f)^2 y_0^2 \int_0^1 \left[3t^2 - 2t^3\right]^2 dt$$

$$KE = \frac{1}{2} M_{eff} (2\pi f)^2 y_0^2$$

$M_{eff} = \frac{33}{140} M_s$

$$M_{eff} = M_s \int_0^1 \left[3t^2 - 2t^3\right]^2 dt$$

$$KE = \frac{1}{2} M_{eff} (2\pi f)^2 \left(\frac{y_0}{2}\right)^2$$



$$\omega_n = \sqrt{\frac{2k}{m}}$$

$$k = EW\left(H/L\right)^3 = \frac{1}{2}\,\frac{1}{4} EW\left(H/L/2\right)^3 =$$

$$M_{eff} = m + 2 * M_s \int_0^1 \left[3t^2 - 2t^3\right]^2 dt$$