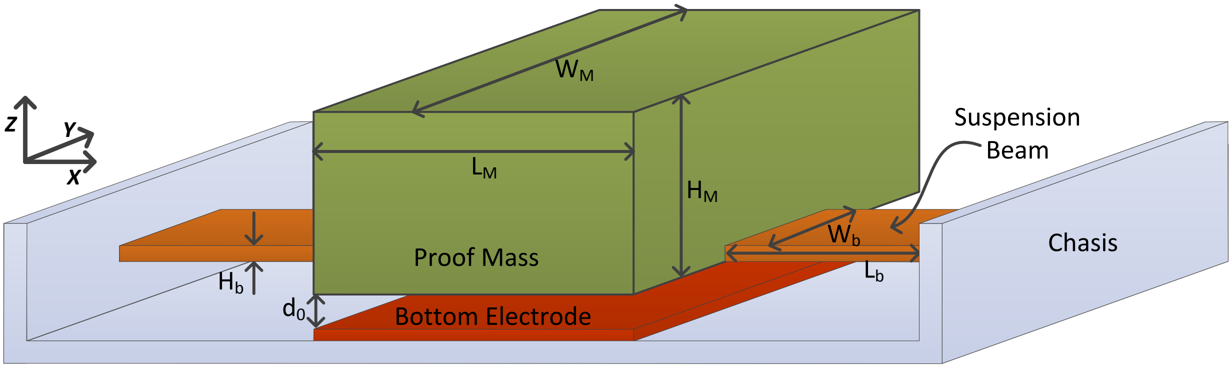
PROBLEM SET #1

Issued: Tuesday, Aug. 28, 2012

Due (at 7 p.m.): Tuesday Sept. 11, 2012, in the EE C245 HW box near 125 Cory.

This homework assignment is intended to give you some early practice playing with dimensions and exploring how scaling can greatly improve or degrade certain performance characteristics of mechanical systems. Don't worry at this point if you do not understand fully some of the physical expressions used. They will be revisited later in the semester.

1. Scaling to microscopic dimensions often provides benefits in some performance characteristics, but can also degrade others. To investigate, this problem explores the behavior of the simple accelerometer shown in Figure PS1-1 as all its dimensions are scaled by a factor of (1/2)×, where the scaling factor indicates the number that multiplies the original value. (In other words, L scaled by a factor of (1/2)× means the new $L_{new} = (1/2)\times L$.)



$E=150$ GPa, $\rho=2300$ kg/m^3
$L_m=300$ µm, $H_m=50$ µm, $W_m=200$ µm
$L_b=260$ µm, $H_b=4$ µm, $W_b=50$ µm
$d_0=1$ µm

Figure PS1-1

(a) Assuming $L_b >> H_b$, and assuming each suspension beam is rigidly attached to the chassis, the z-direction stiffness at the "proof mass end" of suspension beams can be approximated by the expression

$$K_z = 2EW_b\left(\frac{H_b}{L_b}\right)^3$$

where E is the Young's modulus of the structural material, and dimensions are given in Figure PS1-1. What is the total z-direction stiffness at the proof mass location before scaling? What is the stiffness after all dimensions are scaled by (1/2)×?

(b) The resonance frequency for the structure is given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

where K is the stiffness in the direction of resonance, and M is the mass of the system, which for simplicity, should be assumed to be the mass of the proof mass, i.e., neglect the mass of the suspension beams. What is the resonance frequency of the structure before scaling? What is it after scaling by (1/2)×?

(c) Provide an expression and a numerical value for the overlap capacitance C_o between the proof mass and the bottom electrode. By what factor does it scale if all dimensions are scaled by (1/2)×?

(d) Write an expression for the z-directed displacement Δz that ensues after the chassis experiences a constant z-directed acceleration of magnitude a. What is Δz for a 10g acceleration? By what factor does Δz change if all dimensions are scaled by (1/2)×?

(e) Write an expression for the change in bottom electrode-to-proof mass capacitance ΔC that ensues after the chassis experiences a constant z-directed acceleration of magnitude a. What is ΔC for a 10g acceleration? By what factor does ΔC change if all dimensions (including the bottom electrode-to-proof mass gap spacing) are scaled by (1/2)×?

(f) Many MEMS-based accelerometers sense ΔC as a measure of the magnitude of acceleration experienced. Figure PS1-2 presents one possible circuit that converts a change in capacitance to a readable output voltage. Here, the fixed capacitor C_o has the same value as the bottom electrode-to-proof mass capacitance of the accelerometer at rest. (The accelerometers electrode-to-proof mass capacitance is represented by C(z) in the figure.) For this circuit, write an expression for the output voltage V_{out} as a function of acceleration a, assuming the op amp is ideal. How does the output voltage scale as all dimensions are scaled by (1/2)×? (The circuit is biased with V_p=10V)

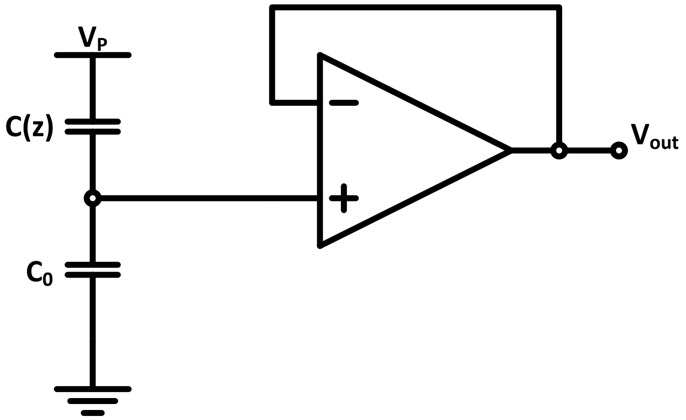


Figure PS1-2

(g) The resolution of a sensor is the smallest input it can detect. Assuming the smallest change in capacitance that the electronic circuit in Fig. PS1-2 can detect is 1fF, what is the resolution of the overall accelerometer (in volts)? Can the unscaled accelerometer measure the force of gravity, i.e., 1g? If not, how would you change the accelerometer to allow measurement of 1g? [Hint: Does scaling improve the minimum detectable acceleration?]

(h) Assume that a 150 mm (6″) diameter wafer has a useful area of 100 mm × 100 mm over which accelerometers can be fabricated. (Here, the edges of the wafer are for handling, so do not yield working devices.) A dicing saw is used to cut the wafer into individual dies and the width of each cut is 50µm. The cost per sensor is given by $C(n,d) = (\$3000 + \$1 \times n + \$2 \times d)/d$, where *n* is the number of cuts through the wafer and *d* is the number of dies. Here, the fixed \$2 cost per sensor is due to post processing, packaging, and testing costs. Assuming that the minimum die size that can be reliably handled is 1 mm × 1 mm, what is the lowest achievable fabrication cost per sensor (to the nearest cent)? [Hint: it would be helpful to define *d(n)* and to find *n*.]

2. The general equation for the deflection of a thin bar, such as shown in Figure PS1-3, can be expressed as

$$\frac{\partial^4 u(x)}{\partial x^4} = \omega^2 \frac{\rho A}{EI} u(x)$$

where *u* is the displacement function, and *x* is the planar spatial coordinate. For now, just treat this problem as a math problem, designed to jog your memory on how to solve differential equations.

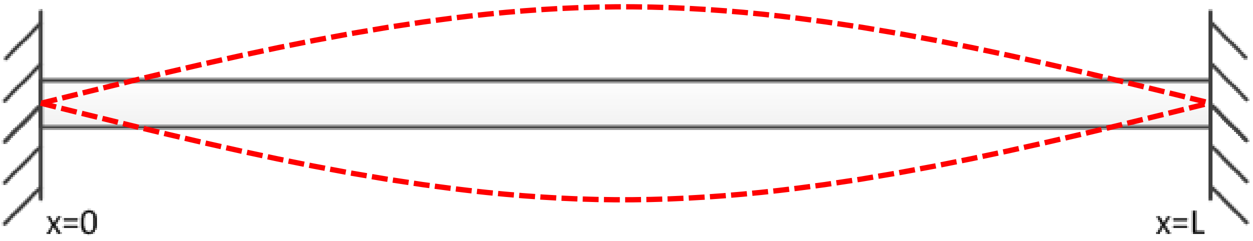


Figure PS1-3

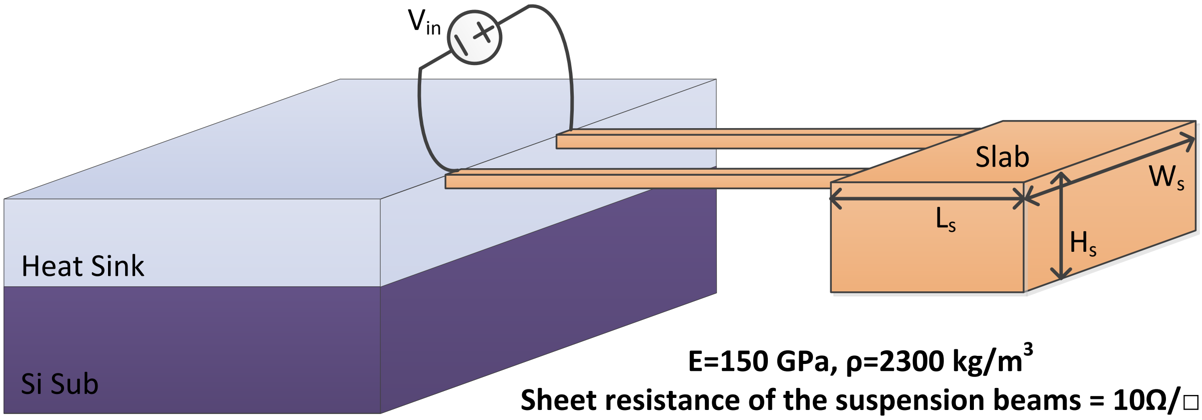
(a) Derive the general solution of the differential equation.

(b) For the boundary conditions given below, find the first three solutions of the differential equation and plot them on the same graph.

$$u(0) = 0, u(L) = 0$$

$$\frac{\partial u}{\partial x}(0) = 0, \frac{\partial u}{\partial x}(L) = 0$$

3. Figure PS1-4 presents a slab suspended by two beams fixed rigidly to a large substrate which is kept at room temperature i.e. 25°C. The slab and suspending beams are made out of the same material with given properties and are all conductive, which allows an input voltage V_{in} to send a current through the structure when applied across the anchor points of the suspension beams.



E=150 GPa, ρ=2300 kg/m^3
Sheet resistance of the suspension beams = 10Ω/□
Specific heat=0.77 j/g.K
Thermal conductivity=30 W/m.K
Suspension Beams: L=150 µm, H=2 µm, W=2 µm
Si Slab: L_s=50 µm, H_s=60 µm, W_s=100 µm

Figure PS1-4

(a) If the input voltage is a step function, with what time constant will the slab reach its steady-state temperature? How long does it take for its temperature to reach 90% of the final value assuming the initial temperature is zero?

(b) If the final step function value of V_{in} is 1V, what is the steady-state temperature of the slab?

(c) What input voltage is required to maintain the slab temperature at 100°C?

(d) If all the dimensions were scaled by a factor of (1/2)×, by what factors do each of your answers to the parts above change?