PROBLEM SET #4

Issued: Thursday, Oct. 4, 2012

Due (at 7 p.m.): Tuesday Oct. 16, 2012, in the EE C245 HW box near 125 Cory.

1. Diagnostic structures are essential to any MEMS layout. This fact is perhaps most evident when a process fails and requires debugging. If inplane stress is the problem, one useful diagnostic structure is the vernier stress gauge shown in Figure PS4.1-1. This structure mechanically amplifies the strain caused by residual stress in a film and is capable of measuring both tensile and compressive stress. In particular, when there is stress in the film, the length of the test beam changes and the slope beam bends, thereby changing the angle of the indicator beam with respect to the horizontal axis. By comparing the position of the fingers on the indicator beam with respect to the fixed fingers via optical microscope, one can estimate the stress.

Suppose the vernier gauge is constructed from a polysilicon layer $H = 2\mu\text{m}$ thick.

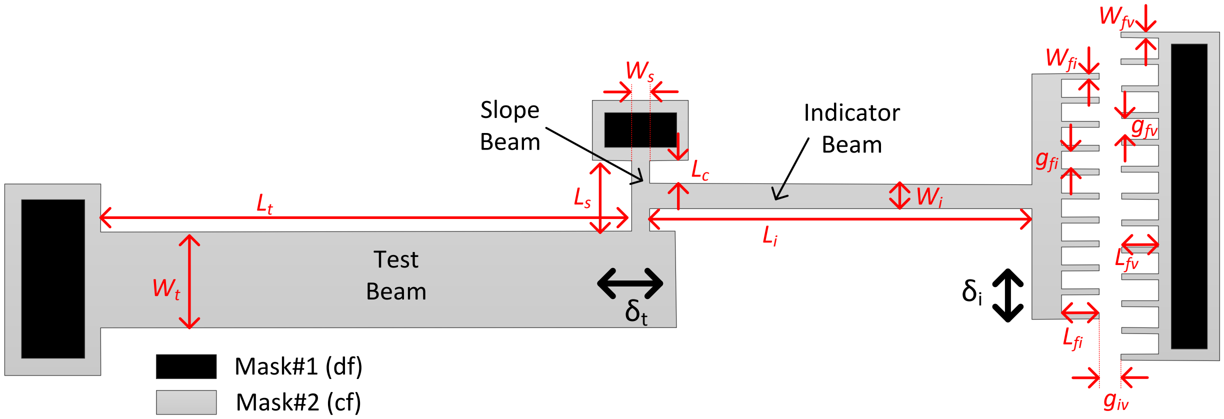


Figure PS4.1-1

(a) If the inplane residual stress in the film is isotropic with a magnitude σ, what is the change in the test beam length (δ_t)? Derive an expression for the bending profile of the slope beam as a function of δ_t and then as a function of residual stress σ.

(b) Write an expression for displacement at the vernier (δ_i) as a function of residual stress (σ) in the film.

(c) What value of L_c maximizes the vernier displacement (δ_i) for a given residual stress (σ)?

(d) The vernier gauge dimensions for a particular design are given in table PS4.1. If the vernier moves 18μm upward, indicate whether the residual stress in the film is tensile or compressive and find its numeric value.

Table PS4.1

ρ_m	E	ν	α_T	L_t	W_t	L_s	W_s	L_i	W_i
2330 kg/m^3	150 GPa	0.2	2.8 μstrain/K	500 μm	30 μm	20 μm	1 μm	500 μm	2 μm

L_c	W_{fi}	g_{fi}	L_{fi}	W_{fv}	g_{fv}	L_{fv}	g_{iv}
10 μm	1 μm	2 μm	4 μm	1 μm	2.1 μm	4 μm	2 μm

2. You are given a polysilicon cantilever with side-view, top-view and cross-section shown in Figure PS4.2-1. Suppose the beam is subjected to a transverse point force at its tip, as shown. Assume material properties of polysilicon as given in Table PS4.2-1 and beam dimensions as given in Table PS4.2-2.

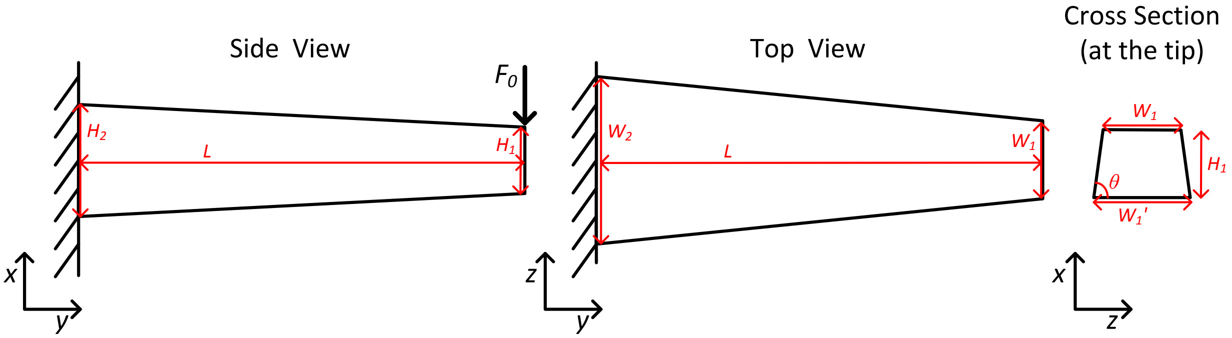


Figure PS4.2-1

Table PS4.2-1

ρ_m	E	ν	α_T
2330 kg/m^3	150 GPa	0.2	2.8 μstrain/K

Table PS4.2-2

L	H_1	H_2	W_1	W_2	θ
100 μm	2 μm	3 μm	5 μm	10 μm	80^o

(a) Derive an expression for the moment of inertia at a distance *y* from the anchor. Assume the angle θ in the cross-section is constant throughout the whole length of the cantilever and also assume that θ is close to 90° so the neutral line remains at the center of the beam.

(b) Derive an expression for the beam deflection due to a transverse x-directed point force F_0 at the tip. What is the equivalent stiffness at the tip in x-direction under point force F_0?

(c) Derive an expression for the max stress in the beam as a function of tip deflection δt. What is the max stress when the tip deflects 1μm?

3. One effective method to measure the stress gradient in a thin film entails measuring the radius of curvature of a cantilever beam. In some popular measurement systems, this is often done by directing a laser beam onto the surface of the beam and measuring the angle θ between the point where you expect to detect the beam and where you actually detect it.

For this problem, you are given a cantilever fabricated via a surface micromachined polysilicon process. Assume the cantilever width is W and the deposited polysilicon has the same properties as those given in Table PS4.2-1.

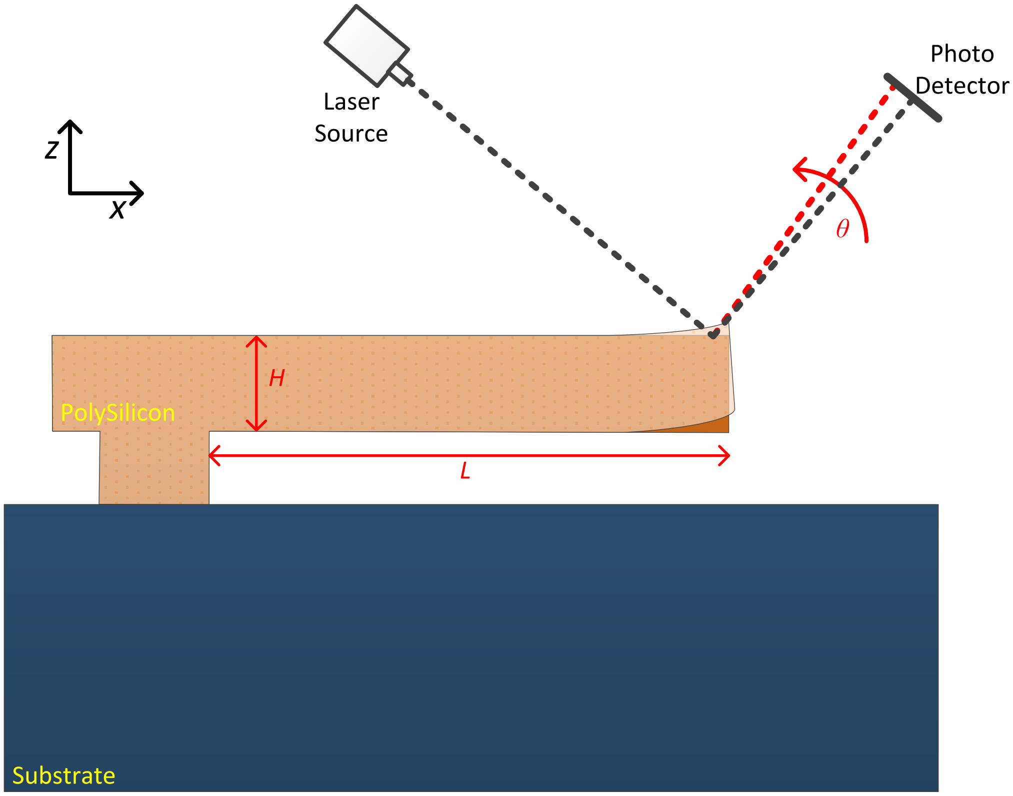


Figure PS4.3-1

(a) The stress gradient in the polysilicon film is shown in Figure PS4.3-2. Derive an expression for the moment due to this stress gradient.

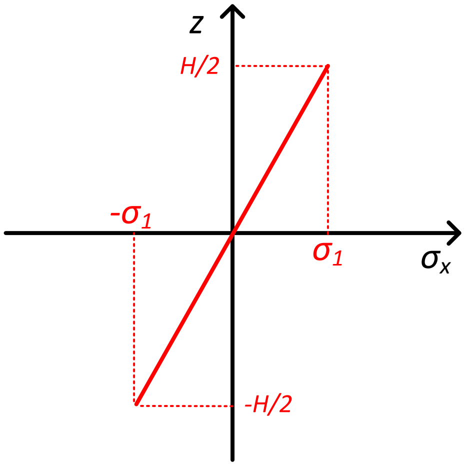


Figure PS4.3-2

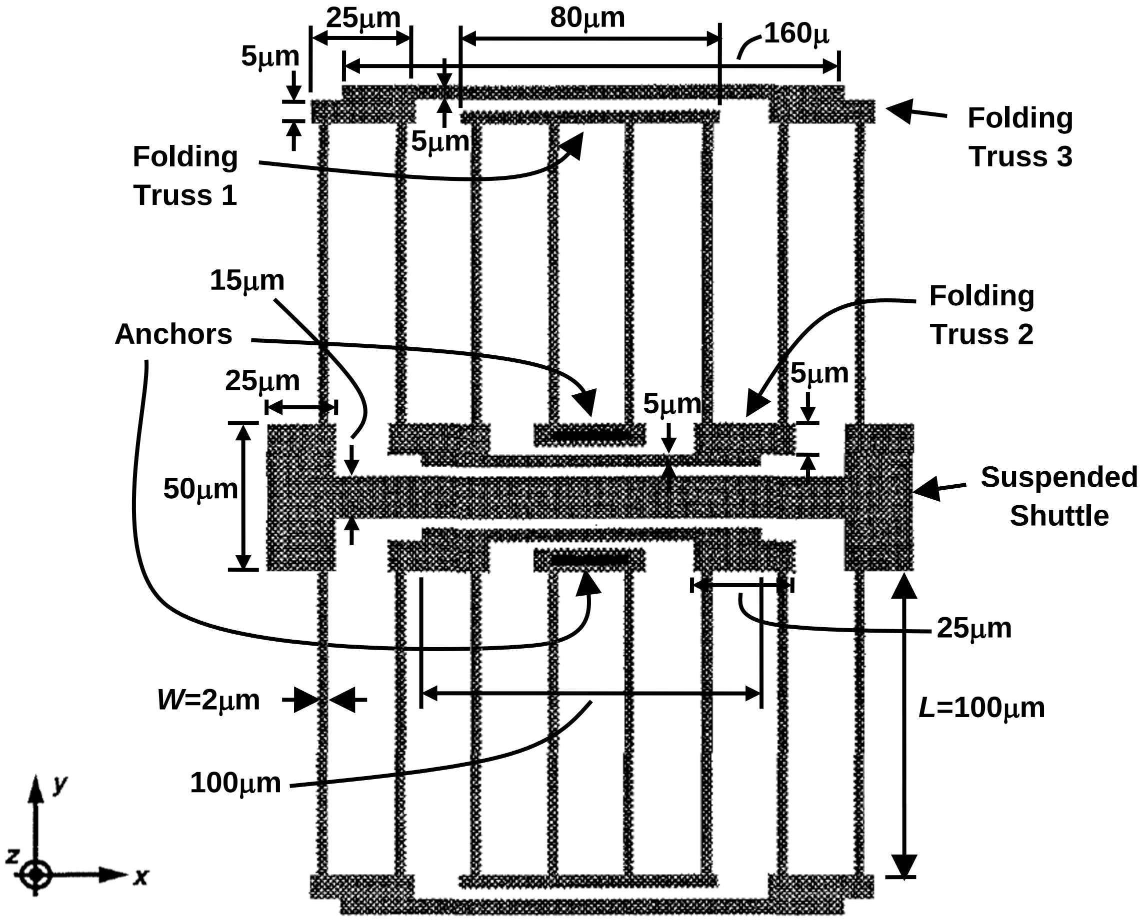
(b) Find the cantilever beam deflection due to the stress gradient shown in Figure PS4.3-2. What is the tip deflection?

(c) Find the angle θ as a function of the maximum stress in the film, σ_1.

(d) How much force must be applied at the tip to counteract the stress so that $\theta = 0$? Is the cantilever perfectly flat now?

(e) Now consider the case when the stress gradient is a function of *x*, as well. Derive an expression for the angle θ assuming $\sigma_1 = \sigma_0 - \frac{\sigma_L}{L}x$. How much force is needed to make θ zero in this case?

4. The figure below presents the top view of a shuttle mass suspended 3μm above a substrate by a triple-folded beam suspension and achieved via a surface micromachining process with a 2μm-thick structural layer and using a 3μm-thick oxide as a sacrificial layer that etches in hydrofluoric acid at the rate of 1μm/min. Data on the structural material used in this problem is given in the box below the figure. Also, assume that all folding trusses are rigid in all directions, including the vertical (i.e., *z*) direction.



Structural Material Properties:

Young's Modulus, E = 150 GPa; Density, ρ = 2,300 kg/m^3; Poisson ratio, ν = 0.226

DI Water Contact Angle for Structural and Substrate Materials: 85^o

Water-Air Interface Surface Tension: 72.75×10^{-3} N/m

(a) Write an expression for the static spring constant in the *x*-direction at a location on the shuttle and calculate its numerical value (with units).

(b) Write an expression for the static spring constant in the *z*-direction at a location on the shuttle and calculate its numerical value (with units).