# PROBLEM SET #7

*Issued: Tuesday, Nov. 20, 2012*

*Due (at 7 p.m.): Thursday Dec. 6, 2012,* in the EE C245 HW box near 125 Cory.

Gyroscopes are inertial sensors that measure rotation rate, which is an extremely important variable to know when navigating. One must know rotation rate (as well as other parameters, e.g., time, linear acceleration, etc.) in order to determine position accurately (without the aid of GPS). Among the applications that use gyroscopes are airplanes (for navigation), boats (again, for navigation), automobiles (for skid control, among other applications), GPS receivers (to allow position determination during periods when the GPS signal cannot be received), cell phones, and game controllers (e.g., the Wii). Of these applications, the last four already use MEMS-based gyroscopes, and the first two are presently targeted by MEMS realizations.

Gyroscopes operate by taking advantage of the conservation of momentum, where an object moving in a given direction with a certain momentum will tend to continue moving in that direction even if its frame of reference is rotated about an axis. This is perhaps best further explained via example.

This problem involves the MEMS-based micro-gyroscope shown in Fig. PS7.1. In this device, momentum is generated by driving the proof mass into resonance vibration using the capacitive comb fingers along the *x*-axis. When the device is rotated about the *z*-axis (indicated in Fig. PS7.1), the vibrating mass will attempt to preserve its momentum in the original *x*-direction,

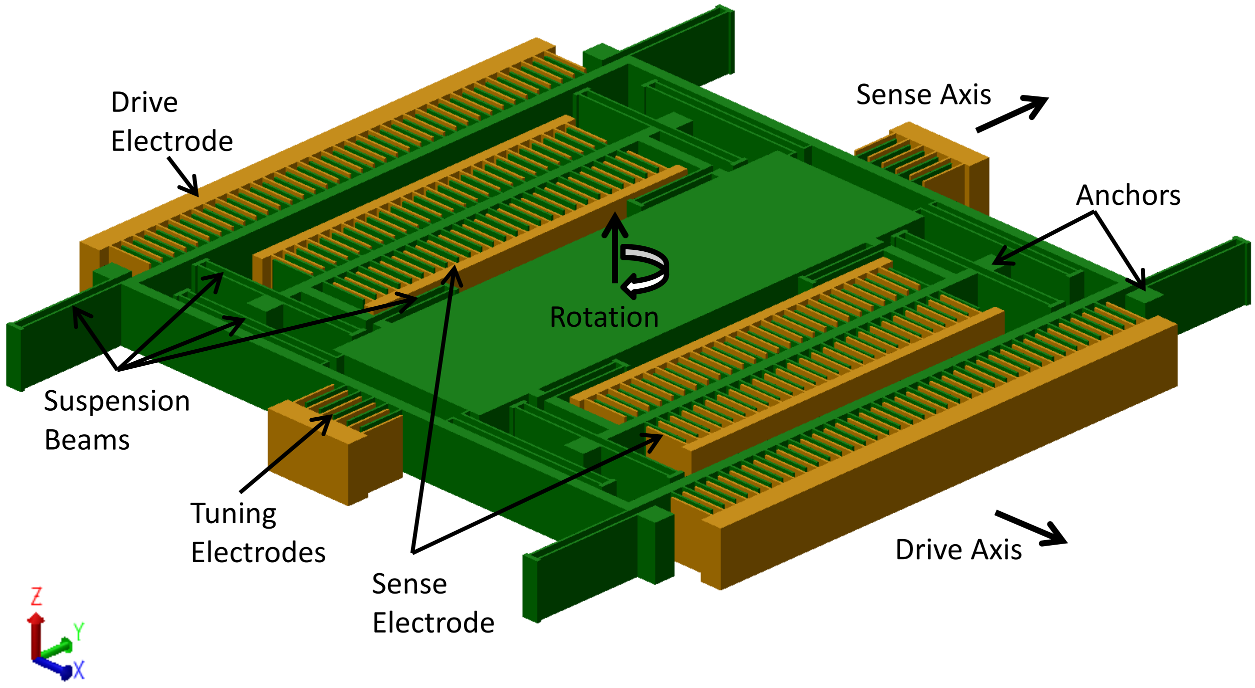

**Figure PS7.1 – Perspective view of the gyroscope.**

which will then make the mass appear to deflect in the $y$-direction. This $y$-directed motion is then sensed by parallel-plate capacitors to determine rotation rate. In quantitative terms, the angular velocity $\Omega$ is sensed about $z$-axis, which then generates a *Coriolis Force* ($F = 2m_s\dot{x}_d \times \Omega$ where $m_s$ is the equivalent mass in the sense mode, and $\dot{x}_d$ is the structure velocity in the drive mode) that acts on the proof mass along the $y$-direction, which is then picked up by the varying gap capacitances.

The following table and figures support the questions at the end of this assignment. In particular, Table PS7.1 provides the properties of the materials used in the structure. Figs. PS7.2-4 then identify different parts of the structure, indicating which portions are freely suspended and which are anchored, and providing key dimensions. As indicated, comb finger gaps are all 4μm, and parallel-plate capacitive finger gaps in the sense mode are all 2μm. All structures are 5μm thick.

**Table PS7.1**

| Material | $\rho_m$ kg/m$^3$ | $E$ GPa | $\nu$ | $\alpha_T$ μstrain/K | $\sigma_0$ MPa | Comment |
|---|---|---|---|---|---|---|
| Polysilicon | 2300 | 150 | 0.2 | 2.8 | Varies | Random Grains |
| Silicon Dioxide | 2200 | 69 | 0.17 | 0.7 | -300 | Amorphous |
| Silicon Nitride | 3170 | 270 | 0.27 | 2.3 | 1100 | Stoichiometric |

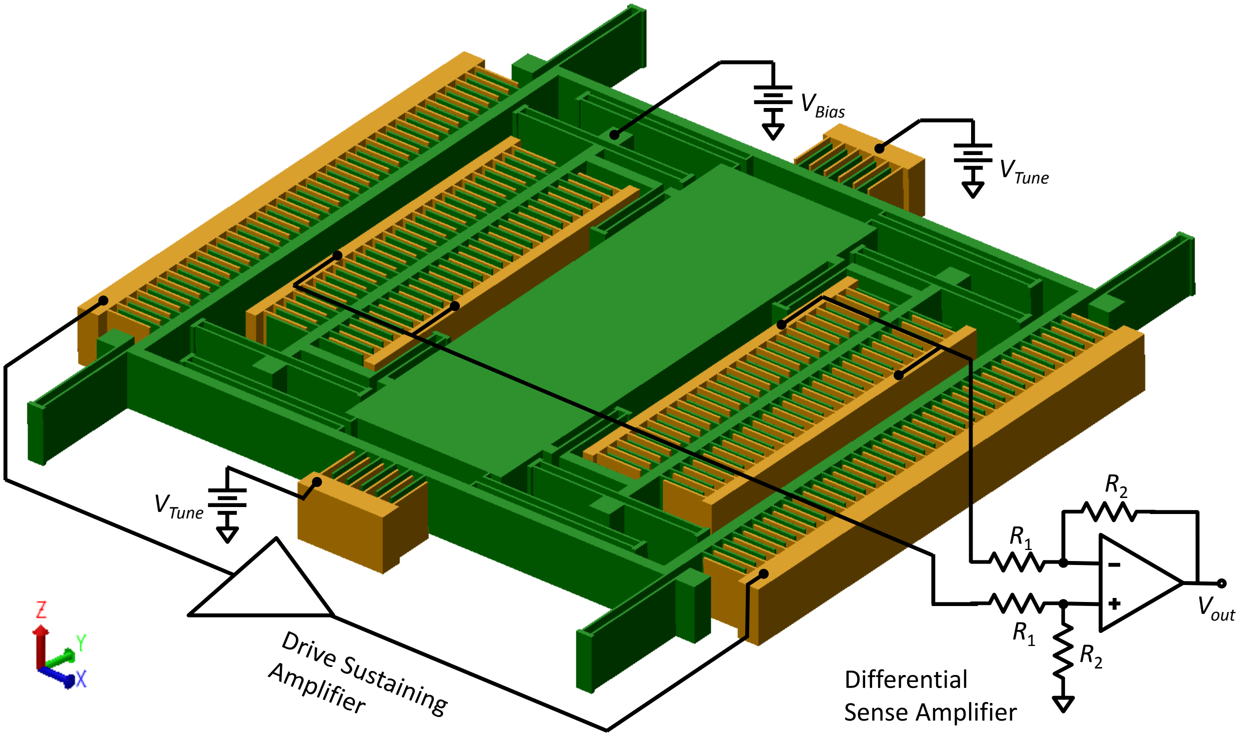

**Figure PS7.2 – Perspective view of the gyroscope showing the actuating and sensing circuits.**

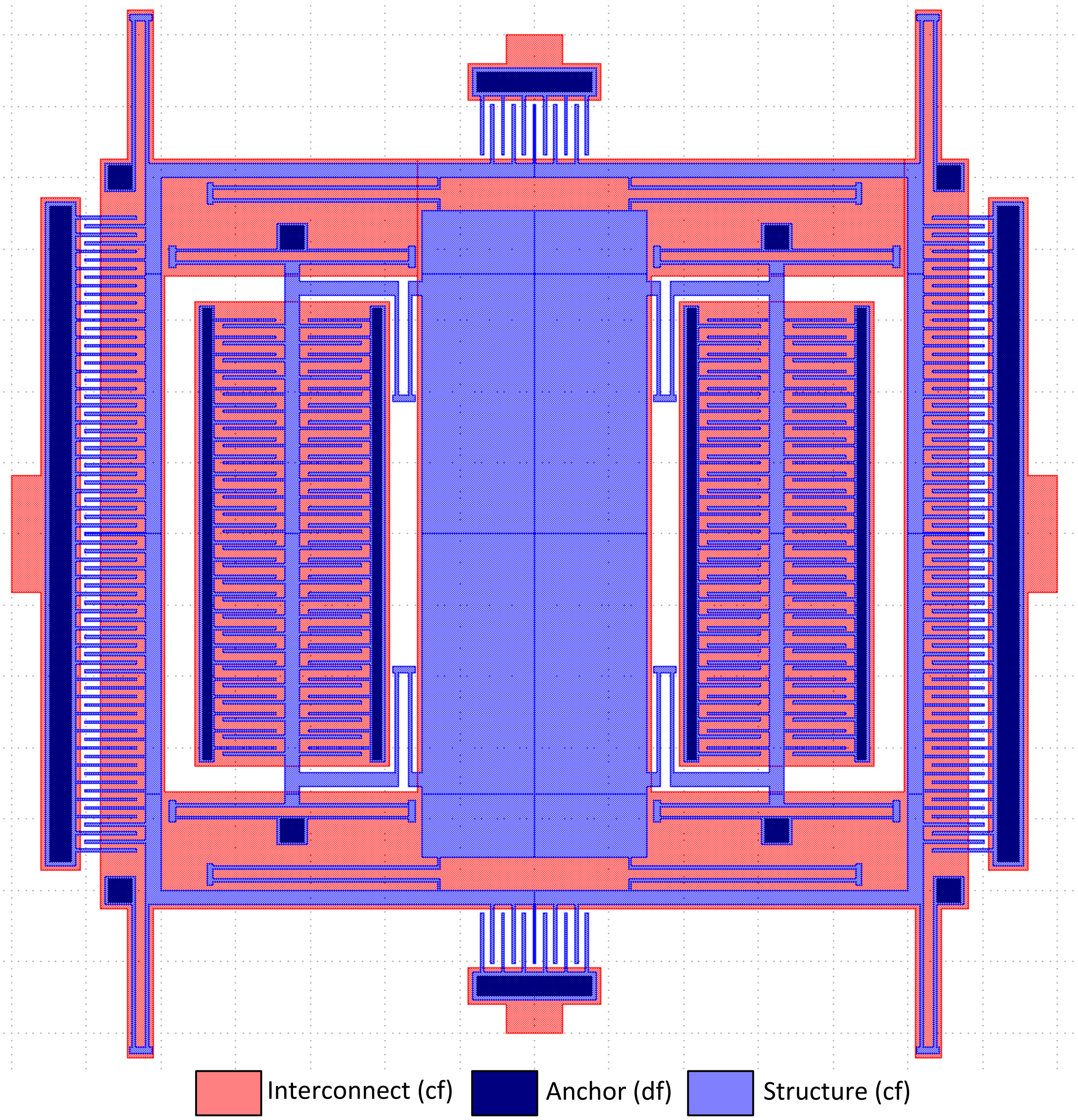

**Figure PS7.3 – Top view of the gyroscope.**

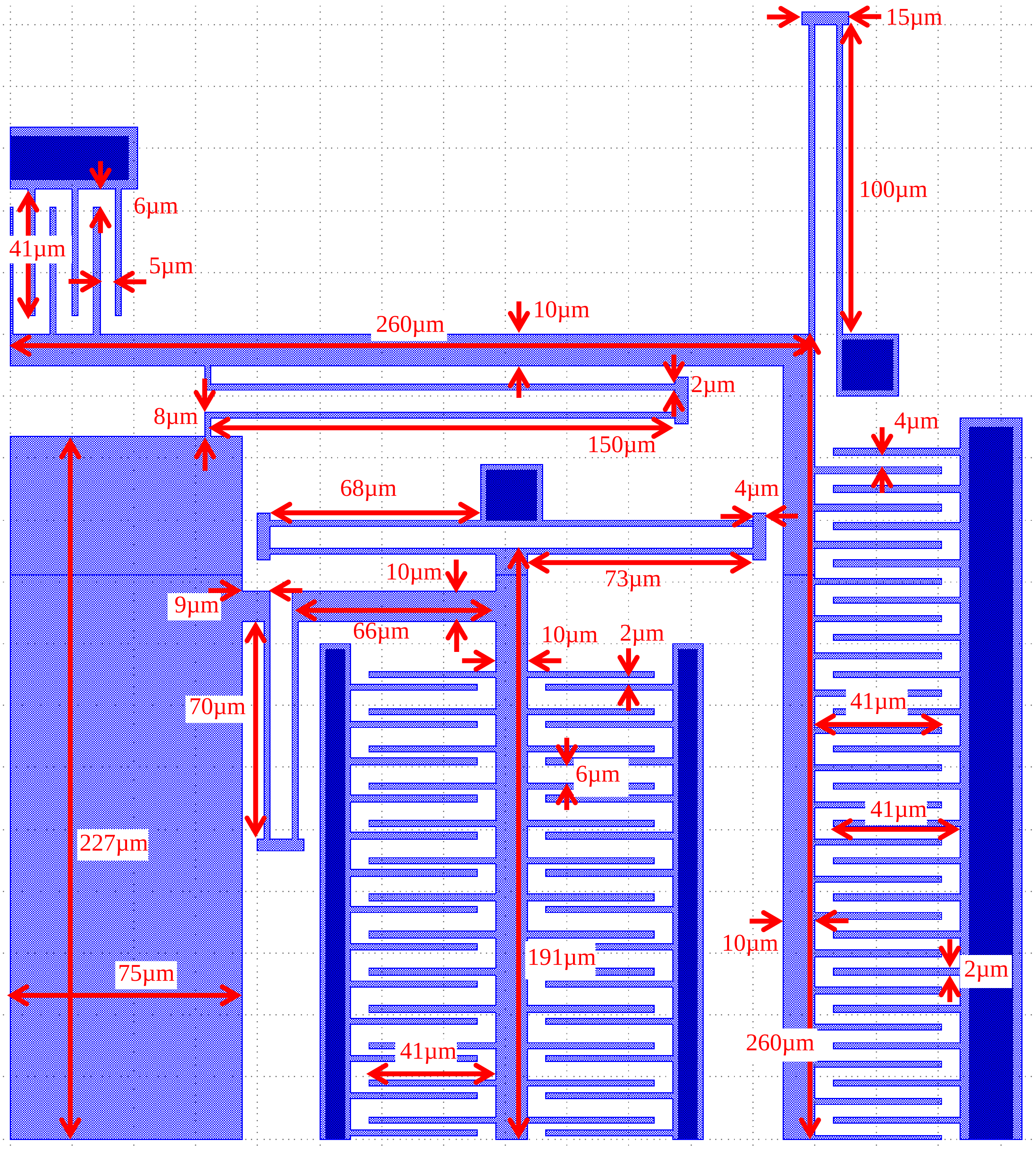

**Figure PS7.4 – Zoom-in view of the gyroscope.**

**1.** Calculate the $x$- and $y$-directed resonance frequencies of the gyroscope structure when all the ports are grounded. Do NOT ignore the beam masses in these calculations.

**2.** Determine the capacitance and change in capacitance per unit displacement for the drive and sense electrodes.

**3.** Calculate the $x$- and $y$-directed resonance frequencies of the gyroscope structure when the structure is biased at 20V (i.e. $V_{Bias}$ = 20V and $V_{Tune}$ = 20V).

**4.** How much frequency tuning do the tuning electrodes provide for $x$-direction operation, if $V_{Tune}$ can be increased up to 100V?

**5.** Suppose the measured quality-factors of the structure the in $x$- and $y$-directions are 150 and 100, respectively. Draw and specify (numerically) all element values in the equivalent circuits (transformer + *LCR*) modeling the (i) drive mode and (ii) sense mode when $V_{Tune}$ = 20V.

**6.** Code the equivalent circuits of problem 5 into SPICE netlists, add the necessary elements (e.g., a voltage source, a resistor, a capacitor, or an inductor) to drive at one end and detect (i) velocity and (ii) displacement at the other, and simulate Bode plots for the transfer functions using SPICE.

**7.** Assume that during steady-state oscillation along the drive axis, the drive amplifier delivers an ac voltage $v_d$ with an amplitude of 1.5V and a frequency equal to the resonance frequency of the drive mode. Also, assume that the input of the drive amplifier detects velocity and that its input resistance is very small. Determine the rotation rate-to-output current scale factor for this gyroscope. Give an expression and calculate its numerical value.

**8.** If the device is subjected to a rotation rate of $\Omega = 1$ radian/second along the axis indicated in Fig. PS7.1, what is the magnitude of the output voltage $V_{out}$ if $R_1$ is set to 0 Ω and $R_2$ is set to 10 kΩ. Assume the differential amplifier employs an ideal op-amp.

**9.** Repeat problem 8 with $R_1$ and $R_2$ set to 1 kΩ and 20 kΩ, respectively.

**10.** Assuming you drove the device along its sense axis with an input force, sketch the sense axis input force-to-output voltage Bode plots for problems 8 and 9 on one graph. Give numerical values for important markers, like resonance frequency, $Q$, and the intersection with the $y$-axis.