Lecture 21: Capacitive Transducers

- Announcements:
- HW#6 is due Tuesday, Nov. 20
  - Jalal has posted numerous documents to help you get started with Cadence
  - Information on getting an EECS account was given on the first day of class → refer to documents from that lecture to get EECS computer access
- First project slide due 11/9/11 (email it)
  - Subject & 3 key references
- No lecture Tuesday, next week, Nov. 13
  - As before, we will make it up by going longer in some lectures, starting this Thursday
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- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1^{st} Order Analysis
    - 2^{nd} Order Analysis
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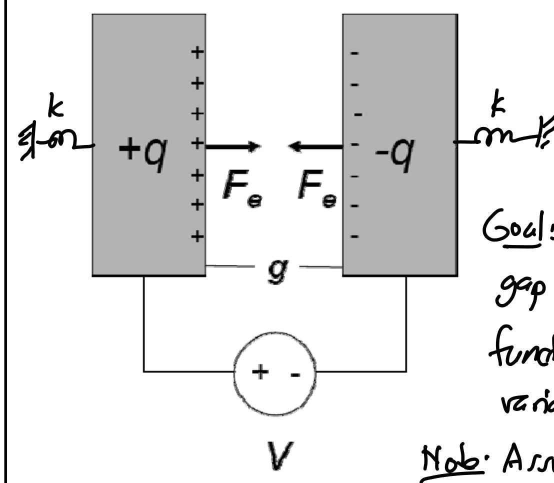
- Last Time: Energy Conserving Transducers
- Now, continue with this

Basic Physics of Electrostatic Actuation



Goal: Determine the gap spacing g as a function of input variable.

Note: Assume the plates are supported elastically.

1st: Determine the energy of the system.

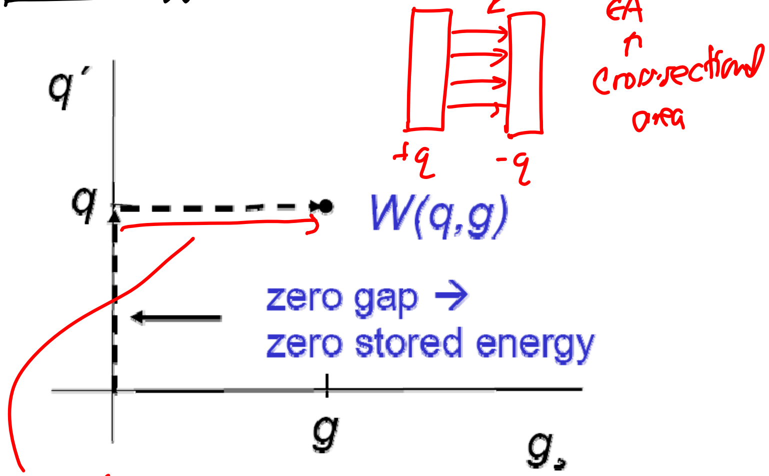
2nd: Ask: What can I do to Δ the energy of the system?

1. change the charge q
2. change the separation g

$$\Delta W(q,g) = V\Delta q + F_e \Delta g$$

$$\boxed{dW = V\,dq + F_e\,dg}$$

Stored Energy



No change in charge: $dq = 0$

$$W = 0 + \int_0^g F_e \, dg'$$

$$F_e = \left(\frac{q}{2}\right)\mathcal{E} = \frac{1}{2}\frac{q^2}{\epsilon A} \qquad \text{(independent of } g\text{)}$$

$$\therefore W = \int_0^g F_e \, dg' = F_e g' \Big|_0^g = F_e g$$

$$\boxed{W(g) = \frac{1}{2}\frac{q^2}{\epsilon A} g}$$

*

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over

* — Work done to charge a C to q at fixed gap g: $dW = V dq + F_e \cancel{dg}^{\,0}$

For a capacitor:

$q = CV \rightarrow V = \frac{q}{C}$

$$\therefore W(q) = \int_0^q V dq = \int_0^q \left(\frac{q}{C}\right) dq = \frac{1}{2}\frac{q^2}{C}$$

$$= \frac{1}{2}\frac{q^2}{\epsilon A} g = W(q)$$

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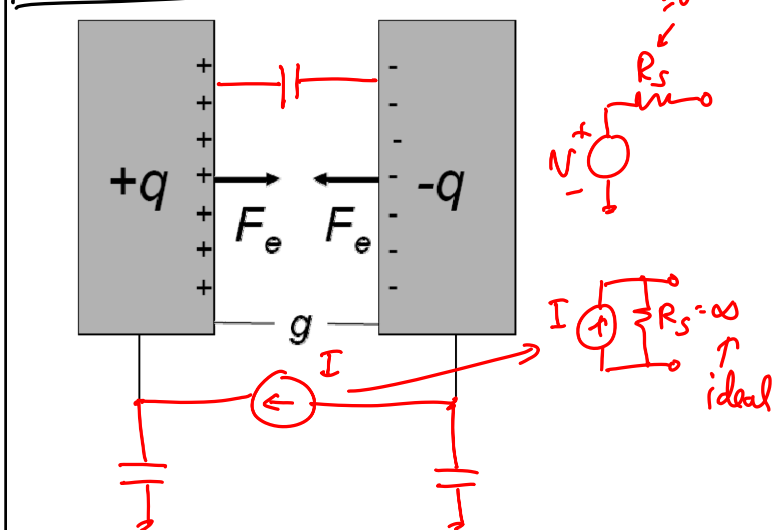
Charge-Control Case



From $dW = Vdq + F_e dg$

hold q = constant → $Vdq \to 0$

$dW = F_e dg \to F_e = \frac{\partial W}{\partial g}\Big|_{q=\text{const.}}$

⇒ Force is given by:

$$F_e = \frac{\partial W}{\partial g}\Big|_{q=\text{const.}} = \frac{\partial}{\partial g}\left(\frac{1}{2}\frac{q^2}{\epsilon A}g\right)$$

$$\therefore \boxed{F_e = \frac{1}{2}\frac{q^2}{\epsilon A}}$$ ⇒ indep. of gap spacing!

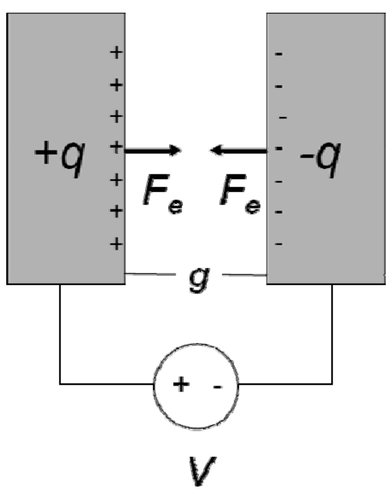
⇒ Voltage is given by:

$$V = \frac{\partial W(q,g)}{\partial q}\Big|_{g=\text{const}} = \frac{\partial}{\partial q}\left(\frac{1}{2}\frac{q^2}{\epsilon A}g\right) = \frac{qg}{\epsilon A}$$

$$\boxed{V = \frac{q}{C}}$$ ✓

(consistent w/ what we already know)

Voltage Control



Want to write $F_e = f(V)$

We know this:

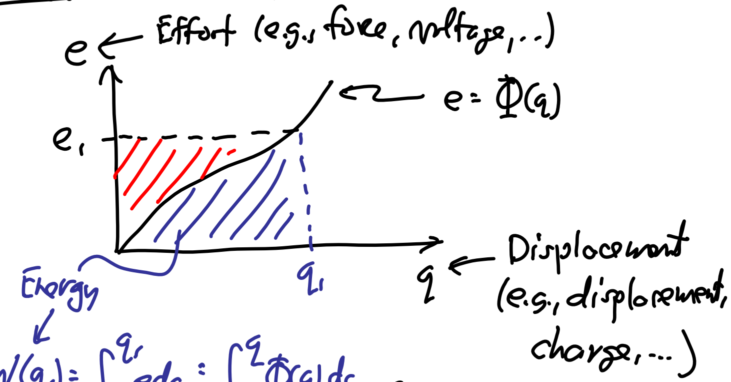
$dW = Vdq + F_e dg$

$W = W(q,g)$

Need: $W'(V,g)$ ← replace charge q w/ V

Can get this w/ a Legendre transformation.

Energy & Co-Energy



$W(q_1) = \int_0^{q_1} e\,dq = \int_0^{q} \Phi(q)\,dq$

Co-Energy:

$W'(e_1) = \int_0^{e_1} q\,de = \int_0^{e} \Phi^{-1}(e)\,de$

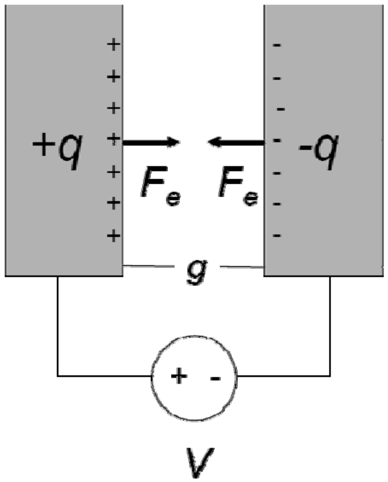
For a linear system, these will be equal.

Can define co-energy as:

$W'(e) = eq - W(q)$ (from the plot) *

↑ co-energy ↑ energy

Co-Energy Formulation for Voltage-Control



* ← $W'(V,g) = Vq - W(q,g)$

Differentially, this becomes:

$$dW'(V,g) = (q\,dV + \cancel{V\,dq}) - dW(q,g)$$

$$\left[dW(q,g) = F_e\,dg + \cancel{V\,dq}\right]$$

$$\boxed{dW'(V,g) = q\,dV - F_e\,dg}$$ ← working Co-Energy expression

Find Co-Energy in terms of voltage, V:

$$W' = \int_0^V q(g,V')\,dV' = \int_0^V \left(\frac{\epsilon A}{g}\right)V'\,dV'$$

$$= \frac{1}{2}\left(\frac{\epsilon A}{g}\right)V^2 = \frac{1}{2}CV^2 \checkmark \quad \text{(as expected)}$$

Electrostatic (a Voltage-Controlled) Force:

$$F_e = -\frac{\partial W'(V,g)}{\partial g}\bigg|_{V=\text{const.}}$$

$$= -\frac{1}{2}\left(\frac{\epsilon A}{g^2}\right)V^2 = \boxed{\frac{1}{2}\frac{C}{g}V^2 = F_e}$$

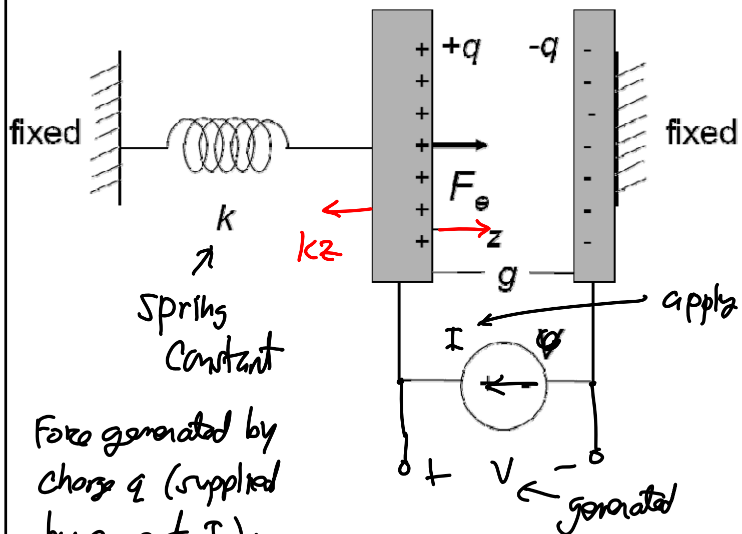
depends on gap!

Charge:

$$q = \frac{\partial W'(V,g)}{\partial V}\bigg|_{g=\text{const.}} = \frac{\epsilon A}{g}V = CV \checkmark$$

(as expected)

Charge-Control of a Spring-Suspended C



Force generated by charge q (supplied by current I):

$$F_e = \frac{\partial W(q,g)}{\partial g}\Big|_q = \frac{q^2}{2\epsilon A}$$

Restoring force of spring: $F_{spring} = kz = F_e$ ← equilibrium

The gap:

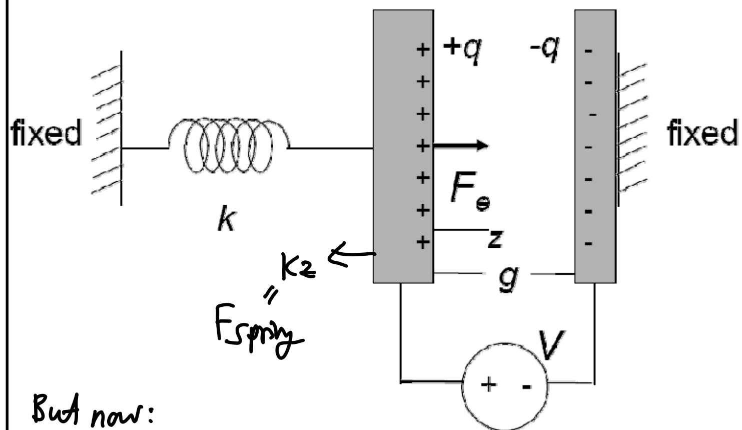
$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2}\frac{q^2}{\epsilon A}\frac{1}{k} = g}$$

↑ initial gap

$q\uparrow$ can drive $g \to 0$

$$V = \frac{q}{C} = \frac{q}{\epsilon A} g = \frac{q}{\epsilon A}\left(g_0 - \frac{1}{2}\frac{q^2}{\epsilon A}\frac{1}{k}\right) \rightarrow V\downarrow \text{ as } g\downarrow$$

Voltage-Control of a Suspended C



But now:

$$F_e = \frac{\partial W'(V,g)}{\partial g}\Big|_q \rightarrow F_e = \frac{1}{2}\frac{\epsilon A}{g^2}V^2$$

And the gap:

$$g = g_0 - z = g_0 - \frac{F_e}{k} = \boxed{g_0 - \frac{1}{2}\frac{\epsilon A}{g^2}\frac{V^2}{k} = g}$$

↑ initial gap spacing

g shows up on both sides!

If $V\uparrow \rightarrow g\downarrow \rightarrow F_e\uparrow$

(+) Feedback!

If loop gain > 1 ⇒ this will go unstable!

plate will collapse (catastrophic!)

Charge: (for a stable gap)

$$q = \frac{\partial W'(V,g)}{\partial V}\bigg|_g = CV \quad ✓$$

Stability Analysis

⇒ determine under what conditions voltage-control will cause collapse of the plates:

$$F_{net} = F_e - F_{spring} = \underbrace{\frac{\epsilon A V^2}{2g^2}}_{F_e} - \underbrace{k(g_o - g)}_{F_{spring}}$$

What happens when g changes by increment dg?

↳ get an increment in the net attractive force dF_{net}

$$\underset{(-)}{dF_{net}} = \frac{\partial F_{net}}{\partial g}dg = \left[-\frac{\epsilon A V^2}{g^3} + k\right]\underset{(-)}{dg}$$

If $g\downarrow \rightarrow dg = (-)$ and

For stability, need $F_{net}\downarrow \rightarrow dF_{net} = (-)$

then need this to be (+) → otherwise plates will collapse!

Thus: $\boxed{k > \frac{\epsilon A V^2}{g^3}}$ (for a stable uncollapsed state)