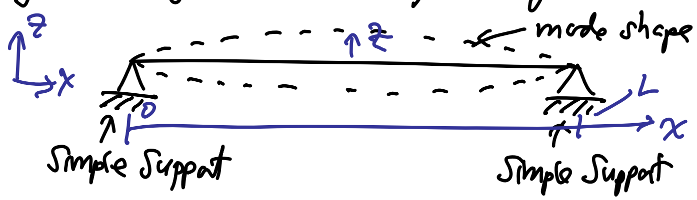
Lecture 2: Benefits of Scaling I

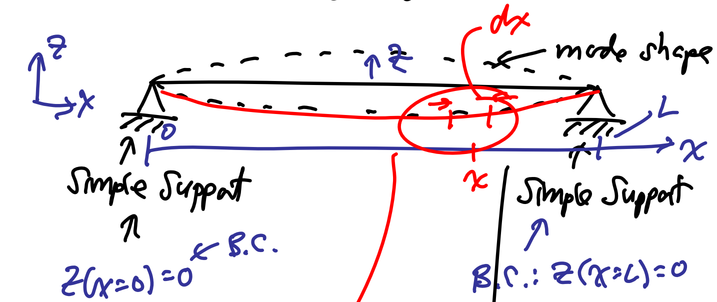
- Announcements:
- The notes from last time are online
- Modules 1 & 2 are online
- HW#1 will soon be online
  - Due in two weeks
- Change Discussion Section time?
  - M 2-3? → 1 conflict M 1-2: → 1 conflict
- I will be gone this coming Thursday → no lecture
- Make-up lecture will probably be next Friday, 9/7, in the afternoon
  - Possible afternoon times? → ✓
- -----------------------------------
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
- -----------------------------------
- Start going through module 2

Scaling of Guitar Strings

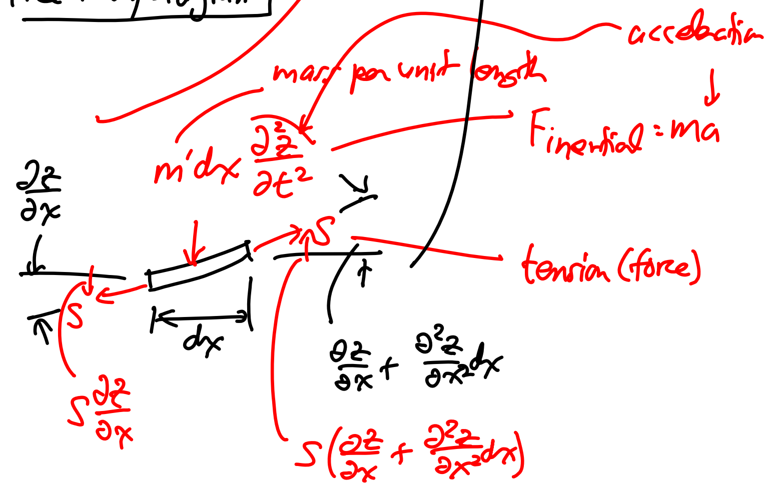
guitar string ≡ transversely vibrating stretched wire



Find the resonance frequency of the string:



Free Body Diagram



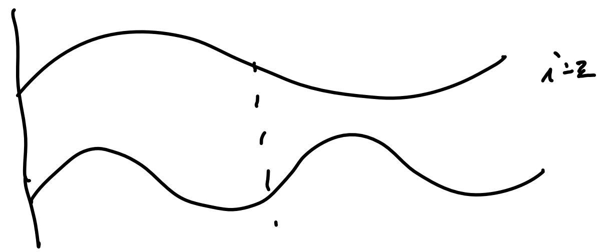
→ condition for dynamic equilibrium:

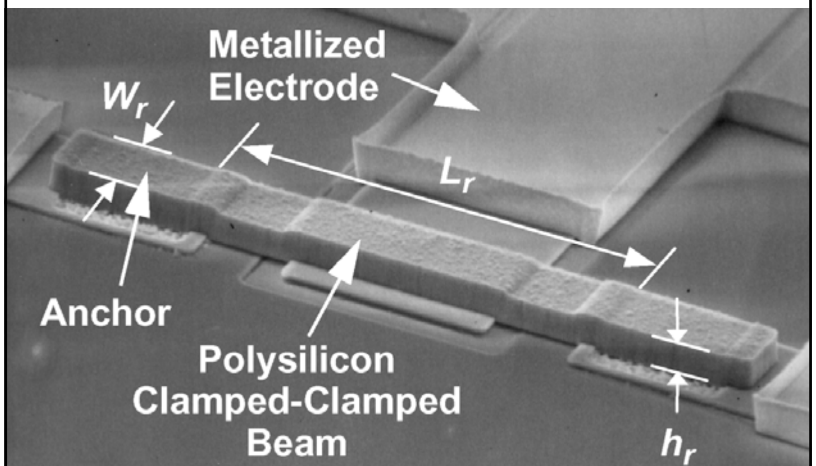
$$S\left(\frac{\partial z}{\partial x}+\frac{\partial^2 z}{\partial x^2}\right)-S\frac{\partial z}{\partial x}-m'dx\frac{\partial^2 z}{\partial x^2}=0$$

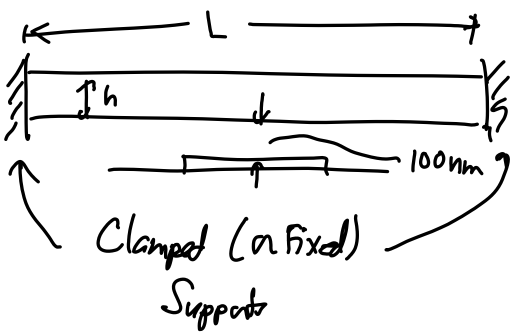
Solve

$$\boxed{f_i = \frac{i}{2L}\sqrt{\frac{S}{m'}}}$$

tension (force)
mass per unit length
freq.
i: mode number







⇒ Eq. for Resonance Freq:

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \qquad (1)$$

acoustic velocity

"giga"

where $E \triangleq$ Young's modulus [GPa]

$\rho \triangleq$ density [kg/m^3]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

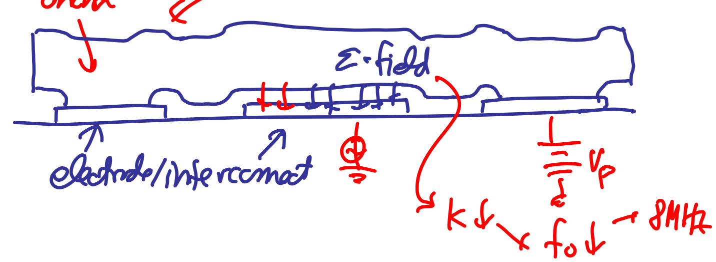
Example: $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150\ GPa$, $\rho = 2300\ kg/m^3$

$$\therefore f_o = (1.03)\underbrace{\sqrt{\frac{150G}{2300}}}_{\text{acoustic velocity} = 8{,}076\ m/s}\frac{2\mu}{(40\mu)^2} = 10.4\ MHz$$

→ In reality, don't quite get this...

→ actual beams   topography → $k\downarrow$ → $f_o\downarrow$ → ~9.5MHz



Scaling

⇒ If we scale all dimensions equally by a factor S

$f_o \sim \frac{S}{S^2} = \frac{1}{S}$ → $f_o\uparrow$ as all dimensions are scaled

⇒ If we scale just L:

$f_o \sim \frac{1}{S^2}$ → even faster rise in $f_o\uparrow$

Example. $L = 4\mu$ → $\boxed{f_o = 1.04\text{GHz}!}$

↳ GHz freq. possible!

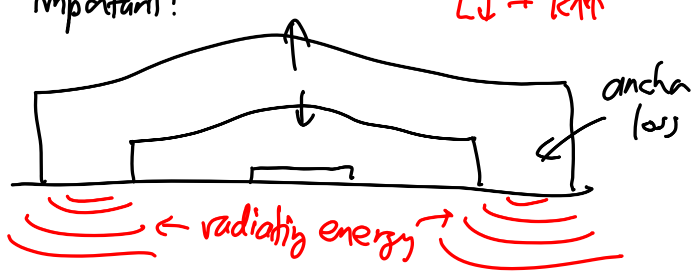
smaller → faster!

Remarks:

① Eq.(1) is not accurate when $L \approx h \approx W$ (width)

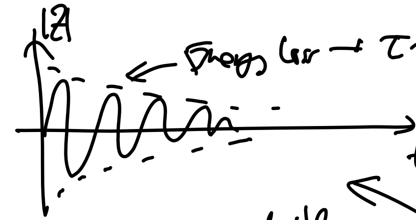
② When $L \approx h$, then anchor losses become more important!

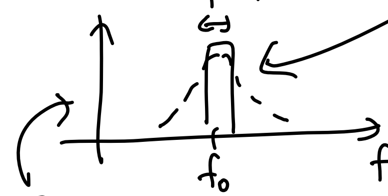
$L\downarrow \rightarrow k\Uparrow$

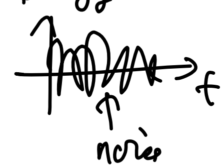


$$Q = \frac{\text{Energy per cycle}}{\text{Energy Dissipated per cycle}} \begin{matrix} \rightarrow \downarrow \\ \nearrow \end{matrix}$$

Problem!







③ Soln: nanodimensions → keeps K small

↘ less anchor radiation

↘ Q stays high

↙ Problem: power handling!

↓

④ Soln: use an array & sum the outputs

↘ *

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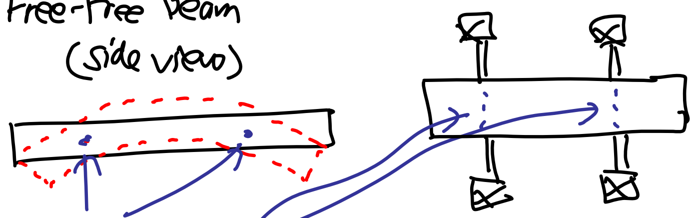
⑤ Better Soln: other geometries

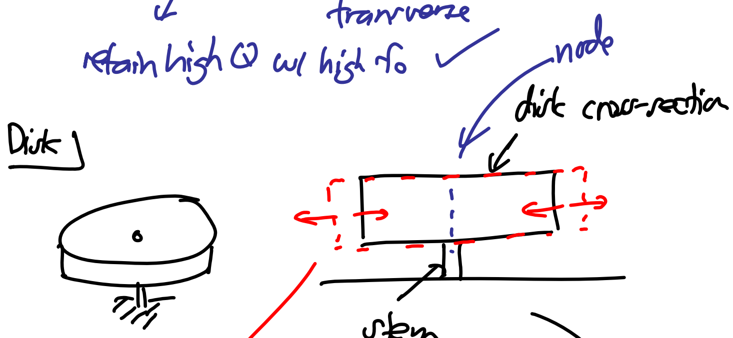
Free-Free Beam (side view)

(top view)



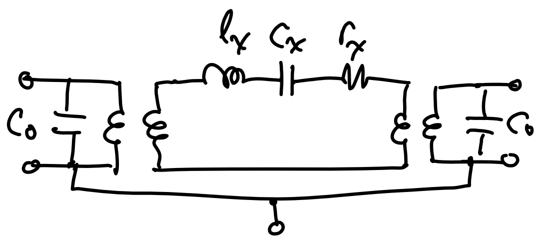
nodal locations → no motion (transverse)

retain high Q w/ high fo



2GHz → Q's ~ 70,000

⑥ This device is a glorified LCR ckt.



Resonator Equiv. Ckt.