EECS 290S: Network Information Flow	Fall 2008

Lecture 14 — October 14

Lecturer: David Tse — *Scribe: Igor Ganichev*

Before we consentrated on the case of single source multiple destinations. We first looked at the wireline model then at linear deterministic model. Then, we generalized to nonlinear deterministic model and finally showed some results for the Gausian network. Today we will start looking into several extensions beyond the single source multiple destinations case.

14.1 Natural Extensions of the Single Source Multiple Destinations Case

1. **Many-to-One:** Multiple senders sending independent information to the same destination t. The destination t is interested in all of the informations. Common practical scenarios with many-to-one communication pattern include cases when a central node is gathering information from multiple nodes in the network. Note that the solution in this case is categorized not by a single rate but by a set of achievable rate tuples. For each tuple $(r_1, r_2, \ldots r_q)$ in the set, r_i is the rate of sender s_i. This extension is the relatively easy one.

2. **One-to-Many:** Single source is sending independent as well as common information to multiple destinations. The destinction from the case that we studied is that not all destinations are interested in all of the messages. For example, let s be the source node and t_1, t_2 be the destination nodes. Assume that s has three messages m_1, m_2, m_3 to send and that t_1 is interested in messages m_1 and m_2, while t_2 is interested in messages m_1 and m_3. In this case, m_1 is the common information in which all destinations are interested and m_2, m_3 is the independent information. In this case, the solution is also a collection of achievable tuples of rates, but rates are the rates of independent information that the sender can send to each of the destinations and one rate for the common information. This case is a somewhat strange case, because there is a solution for the case of two destinations but no general solution is known.

3. **Many-to-Many:** Multiple sources are sending independent information to multiple destinations. Each source is sending to a single destination. This problem is also known as multiple unicast problem. The solution would also take the form of achievable tuples, but no proof is known even for the case of two sources and two destinations.

14.1.1 Many-to-One

We first consider the wire-line model. For the ease of explanation we will consider a special case with 2 senders s_1 and s_2. The proof should be easily extensible to the general case.

Let r_i be the rate of sender s_i. We can immediately write a set of constraints on the rates using the minimum cut upper bound. For any cut separating a set of senders from the destination, the cumulative rate of all these senders cannot exceed the capacity of the cut. For the case of two senders, we have

$$\begin{aligned} r_1 &\leq mincut(s_1, t) \\ r_2 &\leq mincut(s_2, t) \\ r_1 + r_2 &\leq mincut\,(\{s_1, s_2\}, t) \end{aligned}$$

In general, there will be $2^q - 1$ constraints of the form

$$\sum_{i \in A} r_i \leq mincut\,(\{s_i : i \in A\}, t)$$

for all $A \subset \{1, 2, \ldots, q\}$, where q is the number of senders.

Each constraint restricts the set of possible solutions to lie below the hyperplane defined by the constraint in N^q. Thus, a q-tuple of rates that satisfies all of these constraints will lie in a convex region that is the intersection of all half-spaces defined by the hyperplanes.

We next show that all tuples that satisfy all of the constraits are in fact achievable.