EECS 290S: Network Information Flow	**Fall 2008**

Lecture 25 — November 25

Lecturer: David Tse — *Scribe: Alistair N. Boettiger*

25.1 Interference

Relay Channel: superposition but no interference. The destination is interested in all information sources.

Interference Channel (IC): Different destinations each want information only from some sources, but broadcast nature means all sources contribute and interfere. "Contention of Resources" – sharing the net.

25.1.1 Two user Gaussian Interference Channel

Goal: Find characterize the capacity region, $\mathfrak{C}_{IC} = \{R_1, R_2\}$ i.e. the set of simultaneously achievable rates.

Proposed in the 70s, still an open problem.

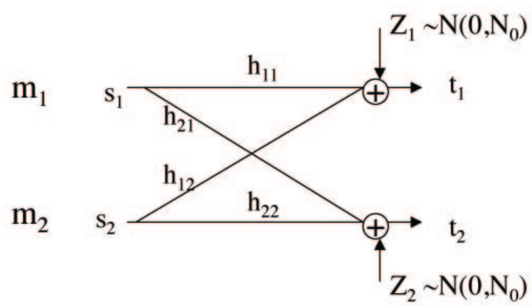

Figure 25.1.

Recall multiple access problem capacity region.

MAC

Gaussian solved – pentagons. Broadcast

We won't solve the capacity region but will prove tight bounds on the region separated by a 1 bit gap.

Without loss of generality, we can renomalize this problem by putting everything into the signal gains, and chose the input power and noise power equal. This leaves us with:

4 parameters

$$SNR_1 = \frac{p_1|h_{11}|^2}{N_0} = |h_{11}|^2$$

$$SNR_2 = \frac{p_2|h_{22}|^2}{N_0} = |h_{22}|^2$$

$$INR_{12} = \frac{p_1|h_{21}|^2}{N_0} = |h_{21}|^2$$

$$INR_{12} = \frac{p_2|h_{12}|^2}{N_0} = |h_{12}|^2$$

25.1.2 Strong Interference Channel (SIC)

$$INR_{12} > SNR_1$$

$$INR_{21} > SNR_2$$

Interference created by signal is stronger for the unintended user than the intended user.

Claim:

In any feasible system, each decoder can decode both messages.

intuition: if t_1 can decode m_1, he could cancel his own signal, and then have a clear view of m_2. Since the weaker m_2 was decodable at t_2 when it had a weaker channel and the interference from m_1, m_2 must be decodable at t_2. By the same argument t_2 can always decode both messages if he can decode his own message.

Observation: the capacity region does not depend on correlation in noise. Introduce correlation such that one channel is just a degraded version of the other.

Observation: The capacity region of the SIC must be in the intersection of the two corresponding MACs (multiple access channels) ,

$$\mathfrak{C}_{SIC} \subset \mathfrak{C}_{MAC_1} \cap \mathfrak{C}_{MAC_2}$$

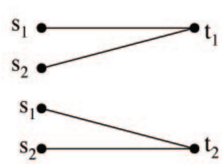

Figure 25.2. Decomposing the strong interference channel as two MACs

Next, recall that for any MAC, the optimal input distribution is Gaussian. Therefore we can satisfy both MACs simultaneously by choosing scalar Gaussian inputs. This gives us the following rate region from Gaussian channels, (log of one plus the sum of the input powers over the noise power),

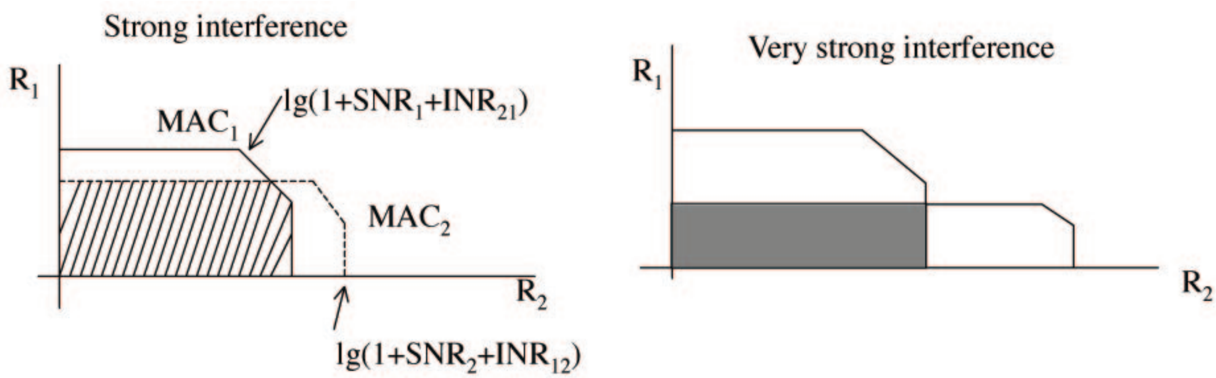

Figure 25.3. Capacity region intersections of MACs

$$R_1 + R_2 \leq \log(1 + SNR_1 + INR_{21}) \tag{25.1}$$

Very strong interference limit

Note at very high interference the effects of interference vanish. The interfering signal clear enough to be completely reconstructable and canceled from the signal.

25.1.3 Weak Interference Channels

MAC Strategy

- MAC strategy optimal for strong INR.
- MAC strategy will be no good for weak INR. We'll sacrifice a lot of rate trying to decode the weak interference signal that the destination doesn't actually care about.

Treat Interference is Gaussian Noise Strategy

- If the interference is very small, treating it as another additive Gaussian noise source we can proceed to decode just the signal of interest to the destination
- As INR approaches the signal power the rate achieved by this strategy goes to zero,

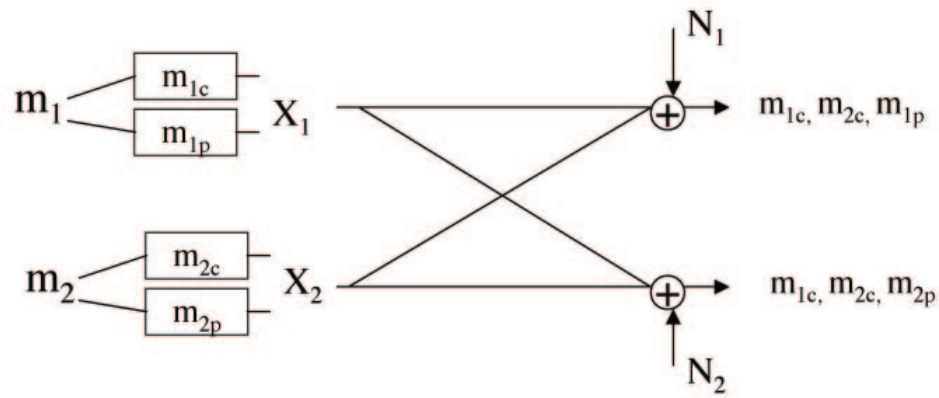

Figure 25.4. Han-Kobayashi scheme employs separate treatment of high power and low power messages as either private (m_p) or common data (m_c)

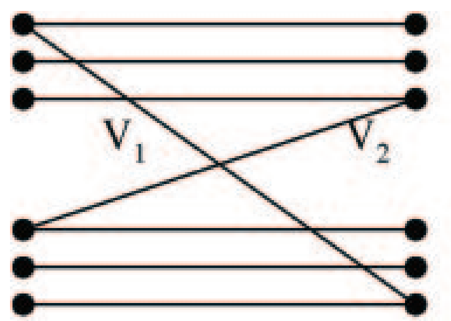

Figure 25.5. The deterministic weak interference channel. Only the most significant bits are strong enough to create interference at the off target destinations. The rest of the bits are below the noise threshold and create no interference.

The Gaussian Tradeoff

Gaussian is signals maximize capacity for a fixed power constraint. Therefore Gaussian signals are optimal for decoding ones own message. However they are optimally bad for creating interference at the other destinations. It is no longer clear then that Gaussian distributions are optimal. This however makes it much harder to prove capacity claims.

25.1.4 Partial Deocode: Han-Kobayashi

Regions are uncomputable, optimality is unproven.

Split messages into common and private information.

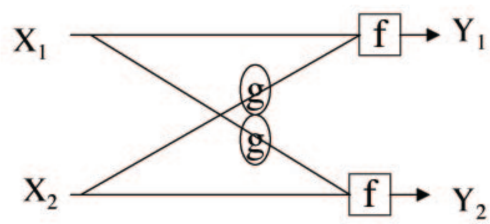

Figure 25.6. El Grunnel and Costa's deterministic model for which the capacity has been solved.

25.1.5 Deterministic Channel Insights

El Grunnel and Costa 1982

Consider the deterministic channel where source one sends message X_1 to Y_1 through transfer function f and interference from source 2 which is filtered through transfer function g. Define $V_2 = g(X_2)$. Similarly $Y_2 = f(g(X_1), X_2))$.

Low Interference Assumptions: Given X_1, Y_1 V_2 can be reconstructed Given X_2, Y_2 V_1 can be reconstructed

Natural Split: encode m_1 into two parts, V_1 and V_1^C. V_1 is the common message that we will try to decode everywhere. V_1^C will is the private message we will only try to decode at destination t_1.

No Tension: The private message m_1 is invisible to t_2, since it's effectively below his noise threshold. The other destinations don't need to try to decode this.

Proposal

Choose fraction of message below the noise floor of the unintended destinations as the private message. These effectively don't interfere at the wrong destination, so we can choose them to be Gaussian to make them decodable at their intended sources. The public messages we decode at every source, so it is also optimal to make them Gaussian as well.