EE290T Lecture Notes: 9/30/09

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Topics:

- Hierarchy of transformations
- Actions of affinities and projectivities on line at infinity
- Decomposition of projective transformations
- Recovering metric and affine properties from images

Notes:

I. Hierarchy of Transformations
   - Geometric Transformations can be classified into 3 groups: similarity, affine, and projective.
   - Each group has certain elements that are invariant under their transformation.
   - Class I: Isometries - preserve Euclidean distance (3 DOF).

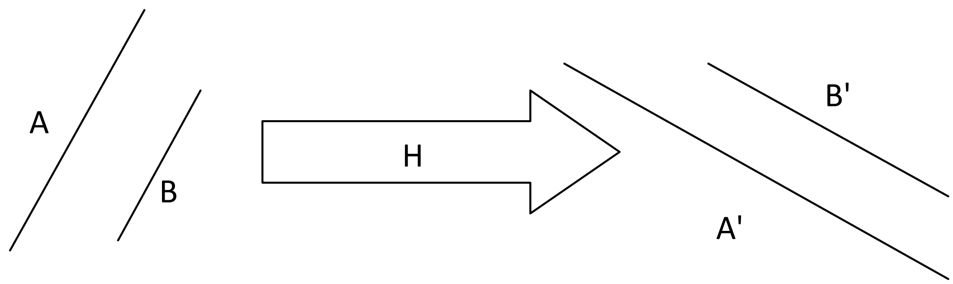
$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} \varepsilon\cos\theta & -\sin\theta & t_x \\ \varepsilon\sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \qquad \varepsilon = \pm 1$$

   - Class II: Similarities - Preserve shape, but alter Euclidean distance by scaling factor (4 DOF).

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

   - Class III: Affine Transformations - preserve parallel lines, ratio of parallel lengths, and ratio of areas (6 DOF).

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$



$$\frac{A}{B} = \frac{A'}{B'}$$

- The ratio of lengths are preserved under an affine transformation.

- Class IV: General Non-singular projective transformations - the only invariant characteristic is the cross ratio of four points on a line

II. Actions of affinities and projectivities on line at infinity

- Under affine transformation, lines at infinity remain at infinity.
- Under projectivities, lines at infinity become finite, making it possible to observe points on horizon.

III. Decomposition of projective transformations

- Any general projective transformation can be decomposed into a product of projective, affine, and similarity transformations.

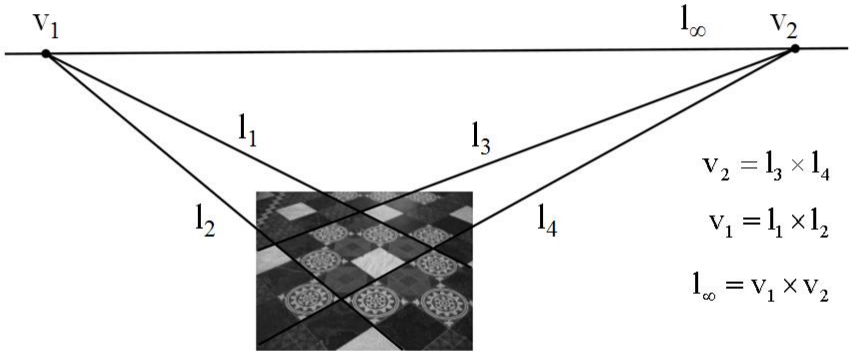
$$\mathbf{H} = \mathbf{H}_S \mathbf{H}_A \mathbf{H}_P = \begin{bmatrix} s\mathbf{R} & \mathrm{t} \\ 0^\mathsf{T} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{K} & 0 \\ 0^\mathsf{T} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I} & 0 \\ \mathrm{v}^\mathsf{T} & v \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathrm{t} \\ \mathrm{v}^\mathsf{T} & v \end{bmatrix}$$

IV. Recovering metric and affine properties from images

- The line at infinity l_∞ is a fixed line under a projective transformation H if and only if H is an affinity
- If we captured a projective image, but wanted an affine representation of the scene, how do we remove the projective transformation?
  1. Remove projective distortion
     a. Specify image of l_∞.
     b. Transform identified images l_∞ line to its canonical position of $l_\infty = (0,0,1)^T$. This transformation sets the projective matrix.
     d. Apply projective matrix to all points in image, which results in the affine rectified image.

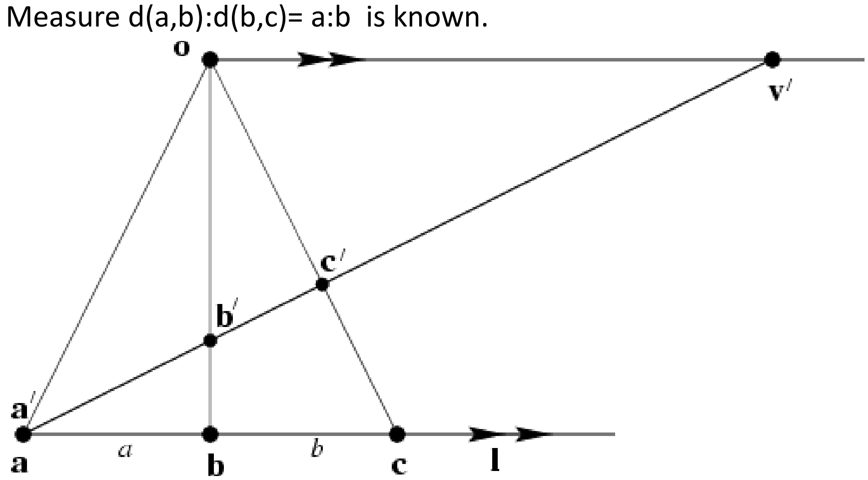
$$\mathbf{H}_{PA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix} \mathbf{H}_A \qquad \mathbf{l}_\infty = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^\mathsf{T}, l_3 \neq 0$$

- Can solve for $\mathbf{l}_\infty$ by connecting the parallel lines from the real world in their projected image.



$$v_2 = l_3 \times l_4$$
$$v_1 = l_1 \times l_2$$
$$l_\infty = v_1 \times v_2$$

- Distance Ratios
    - Suppose we are given 2 intervals on a line with a known ratio in the real world.
    - We want to determine the point at infinity on the line.
    - Assume that a',b',c' are on a line in the image.
    - Let a,b,c be corresponding collinear points on the world line.
    - Measure d(a,b):d(b,c)= a:b is known.



    - We can solve for the point on the line at infinity using:

$$d(\mathrm{a}', \mathrm{b}') : d(\mathrm{b}', \mathrm{c}') = a' : b'$$

$$(0,1)^\mathsf{T}, (a,1)^\mathsf{T}, (a+b,1)^\mathsf{T} \xrightarrow{H} (\mathrm{a}', \mathrm{b}', \mathrm{c}')$$

$$\mathrm{v}' = \mathbf{H}(1,0)^\mathsf{T}$$