Cameras, lenses and sensors

Marc Pollefeys

COMP 256

Cameras, lenses and sensors

- Camera Models
  - Pinhole Perspective Projection
  - Affine Projection
- Camera with Lenses
- Sensing
- The Human Eye

Reading: Chapter 1.

Images are two-dimensional patterns of brightness values.

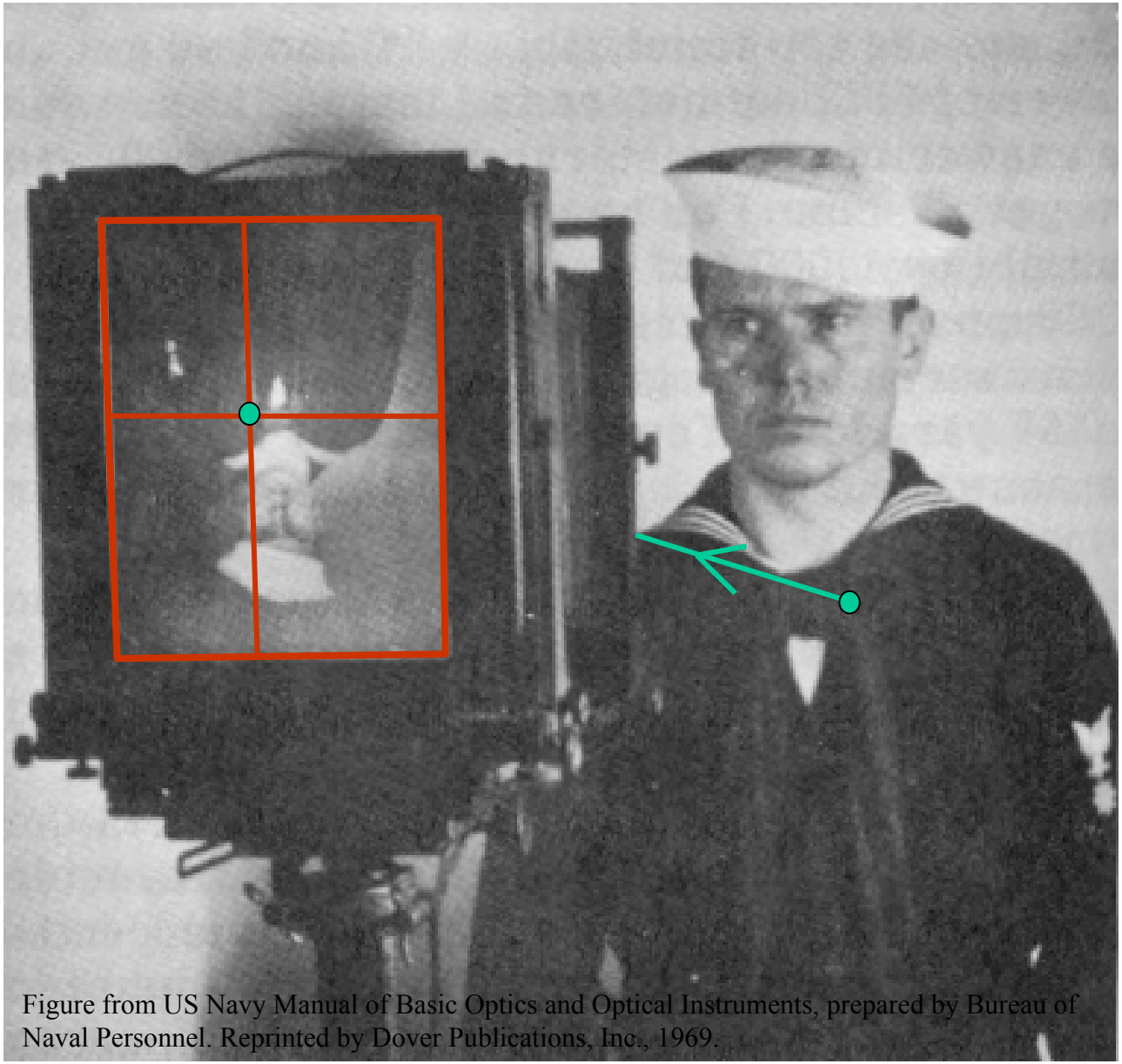
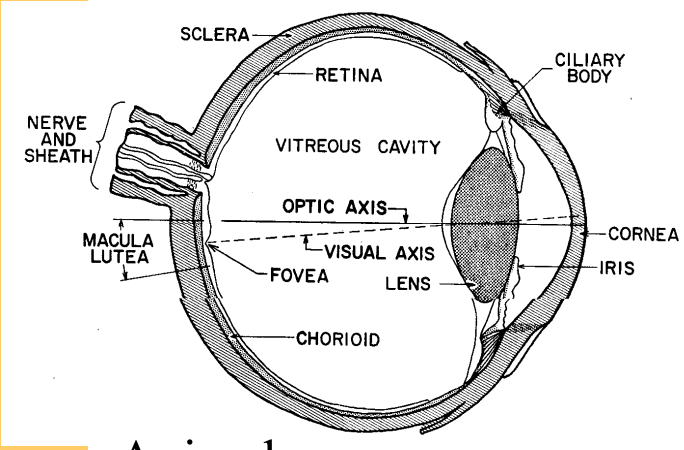


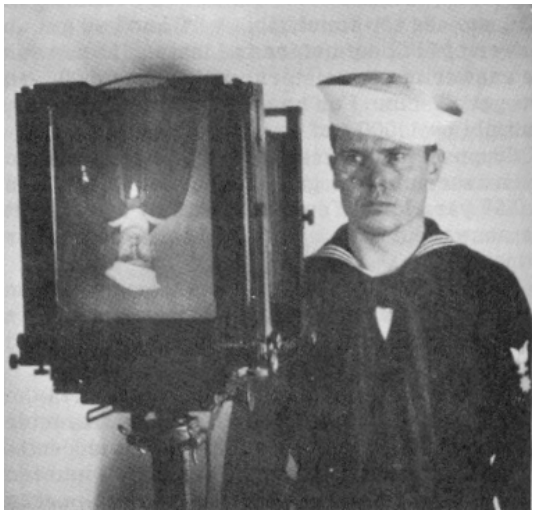
Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

They are formed by the projection of 3D objects.

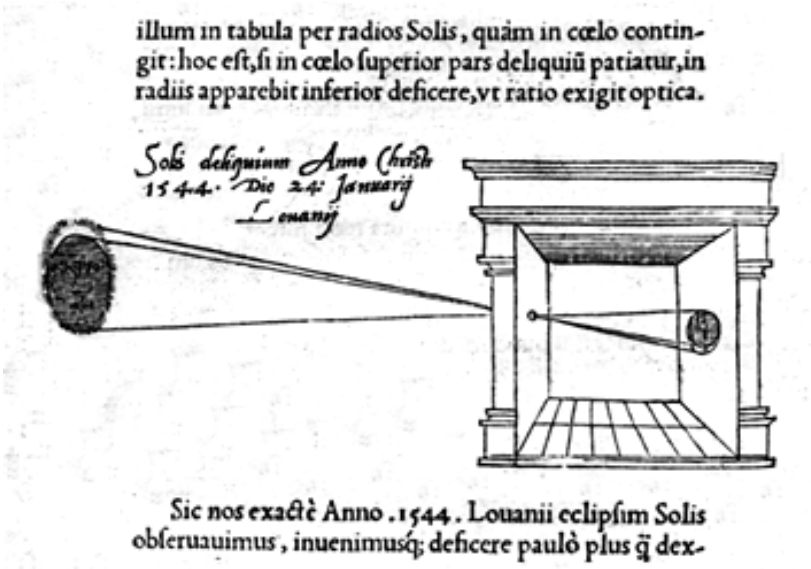


Animal eye:

a looonnng time ago.

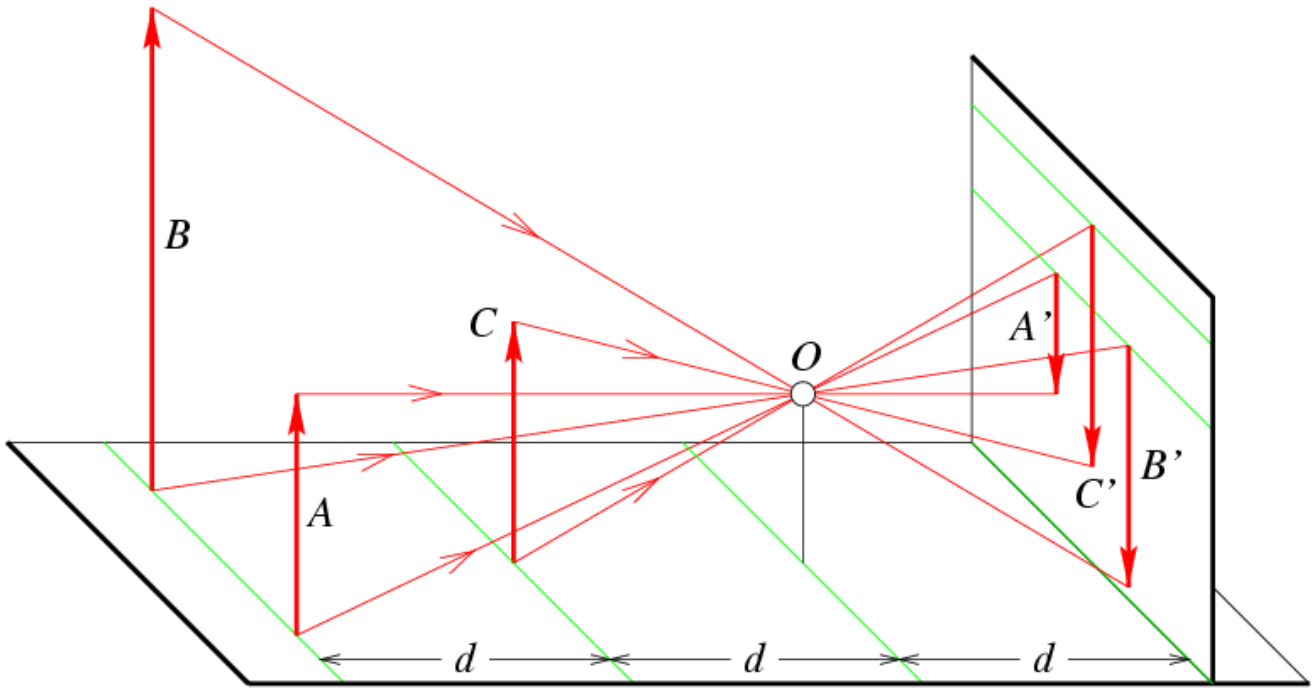


Photographic camera:
Niepce, 1816.



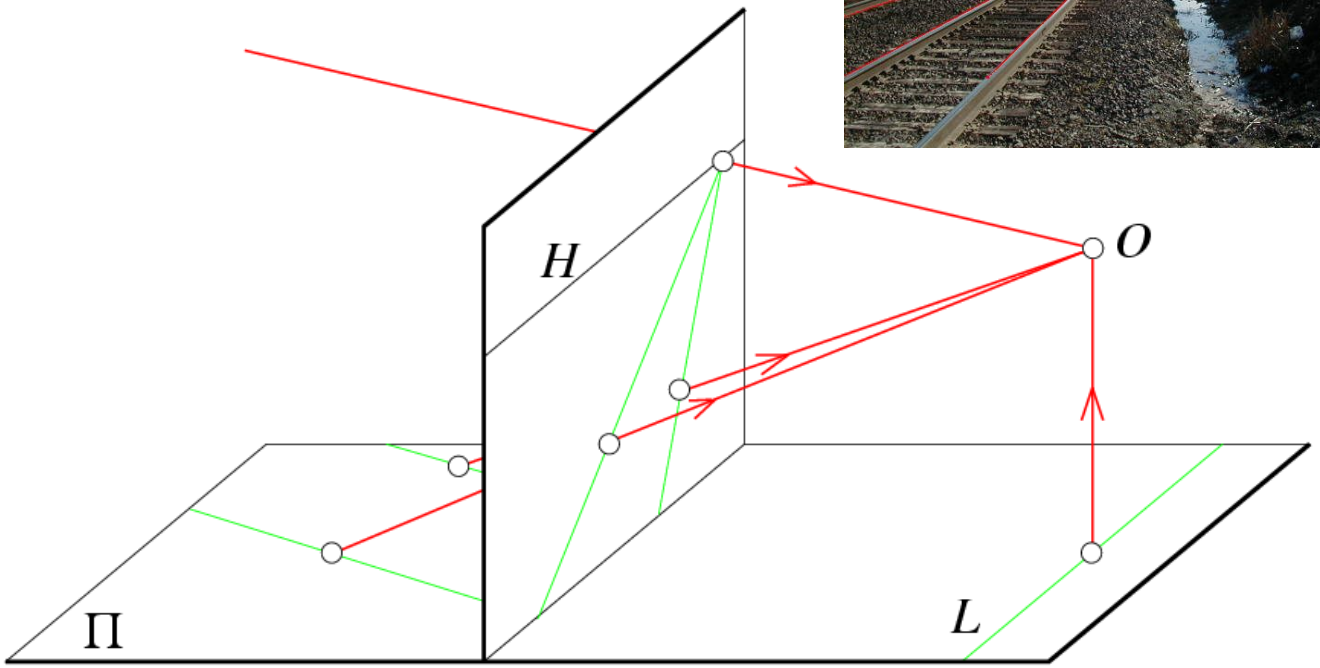
Pinhole perspective projection: Brunelleschi, XVth Century.
Camera obscura: XVIth Century.

Distant objects appear smaller

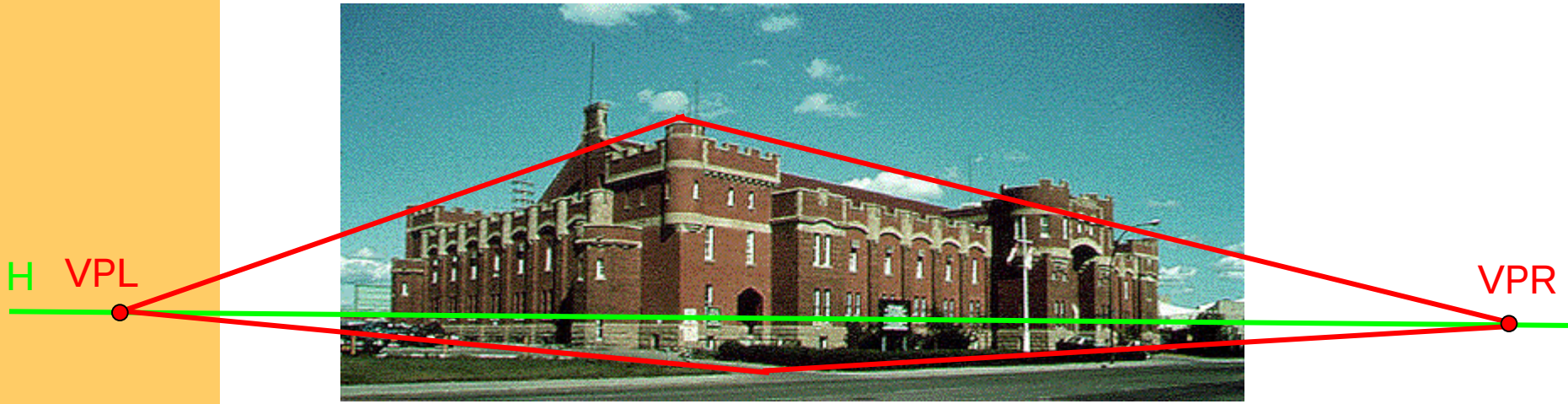


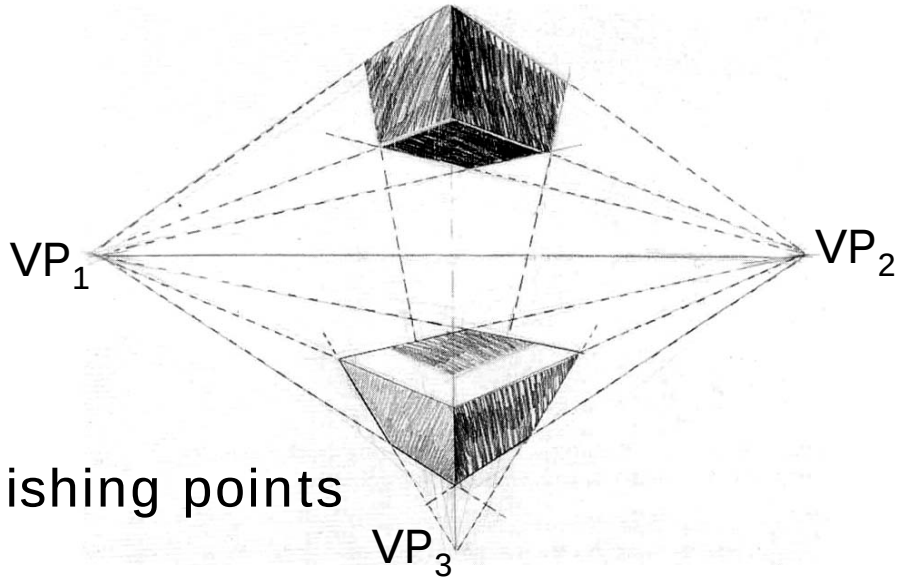
Parallel lines meet

- vanishing point



Vanishing points

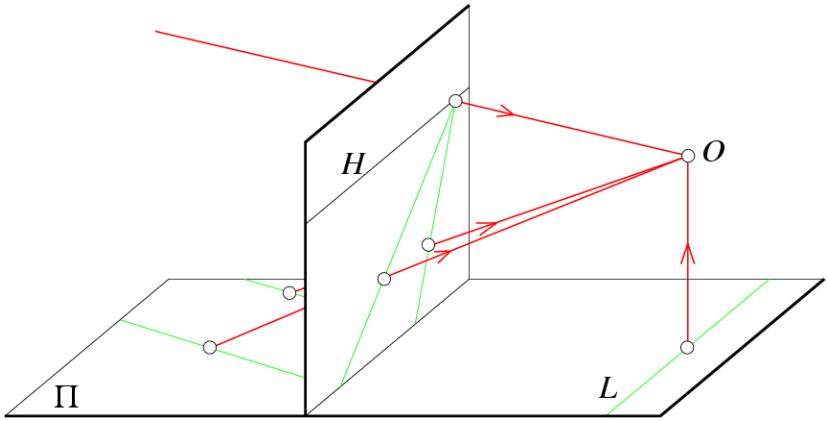




To different directions
correspond different vanishing points

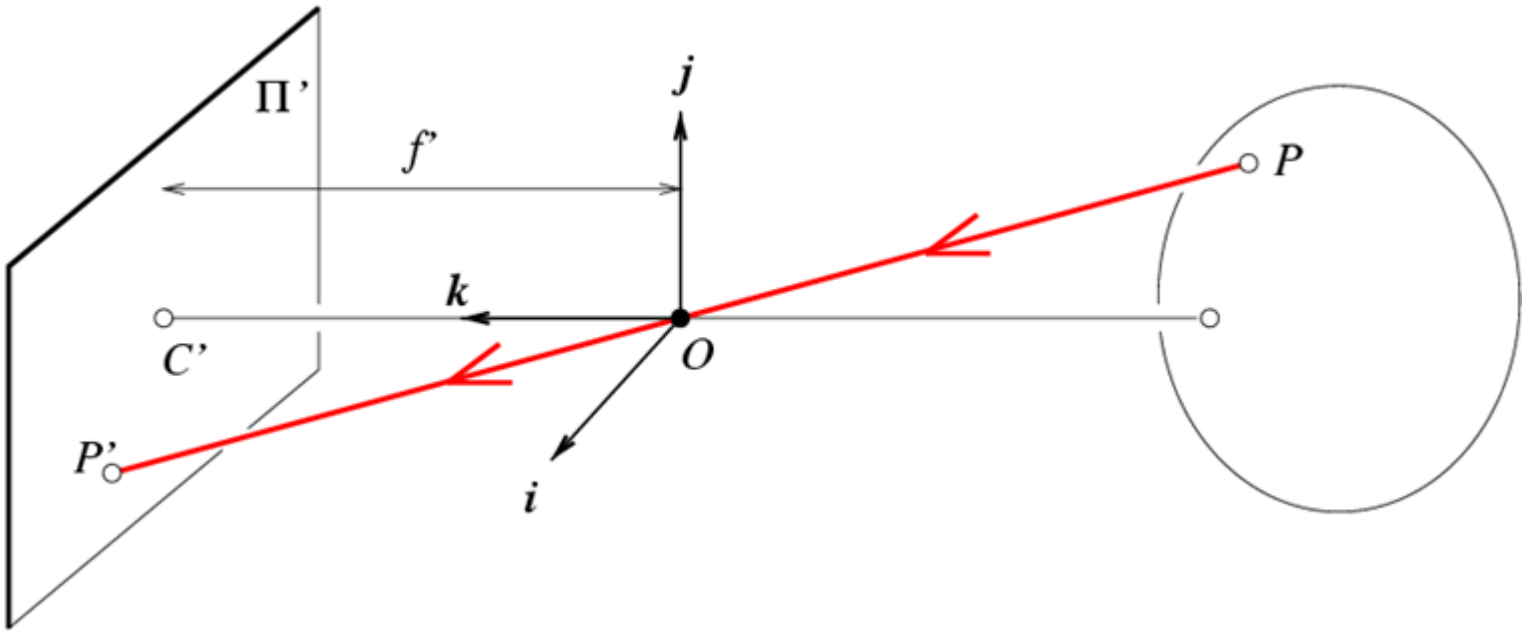
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image
  or half-plane
- Polygons go to polygons



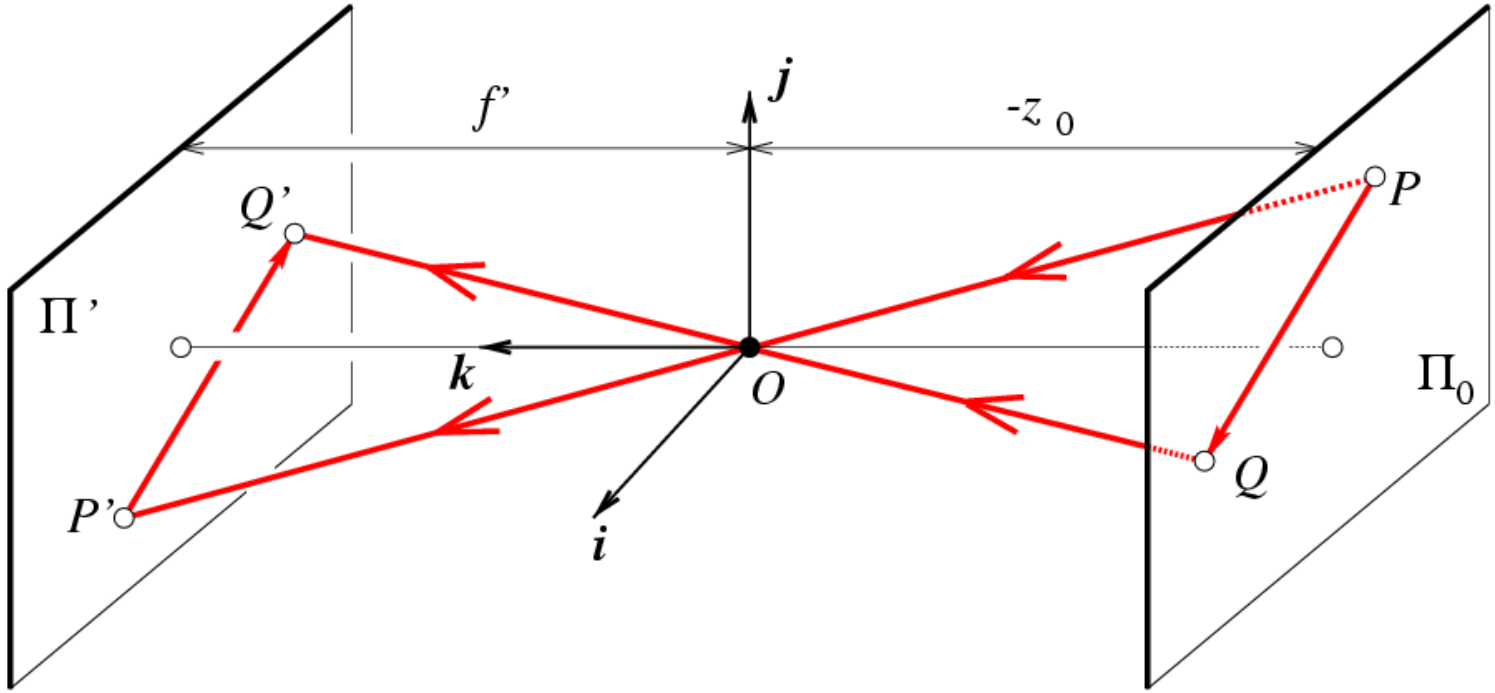
- Degenerate cases:
  - line through focal point yields point
  - plane through focal point yields line

Pinhole Perspective Equation



$$\begin{cases} x' = f'\dfrac{x}{z} \\ y' = f'\dfrac{y}{z} \end{cases}$$

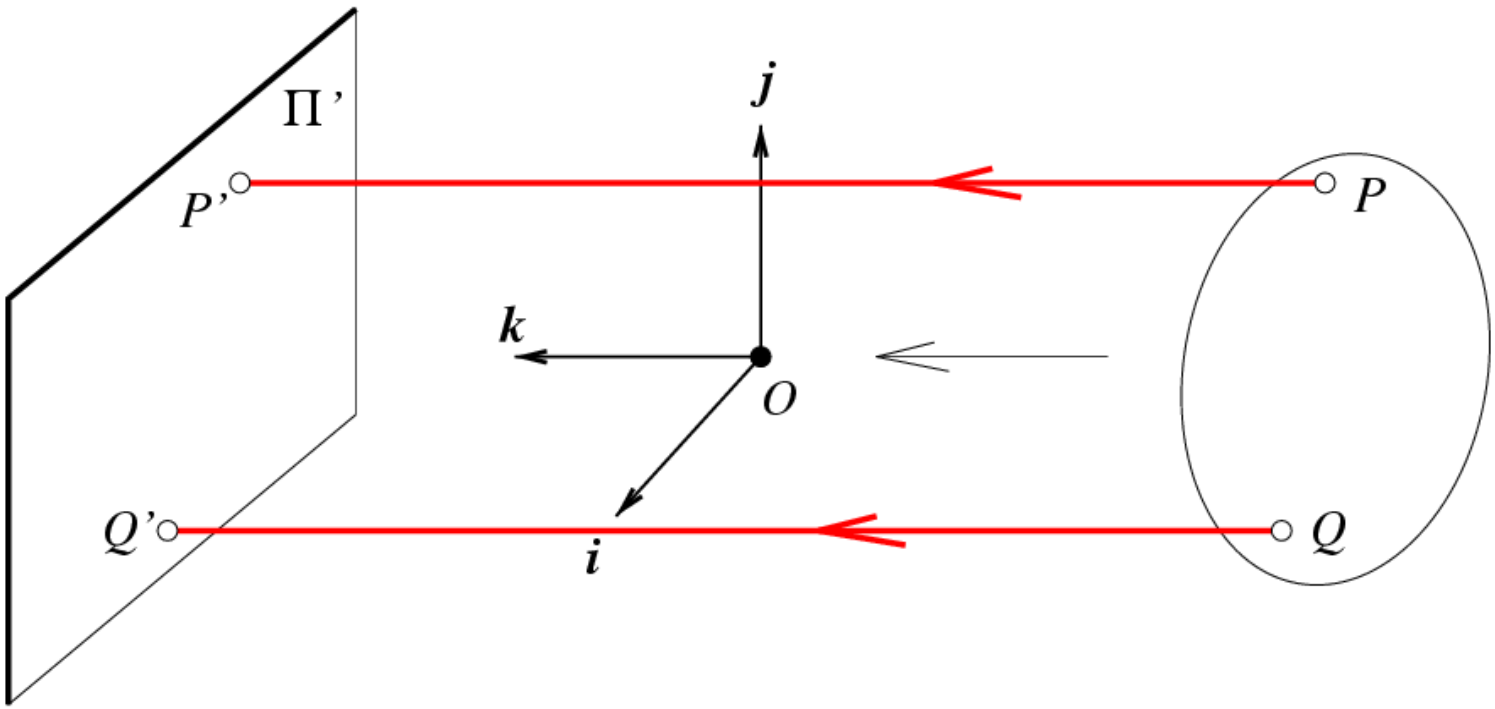
Affine projection models: Weak perspective projection



$$\boxed{\begin{cases} x' = -mx \\ y' = -my \end{cases}}$$ where $m = -\dfrac{f'}{z_0}$ is the magnification.

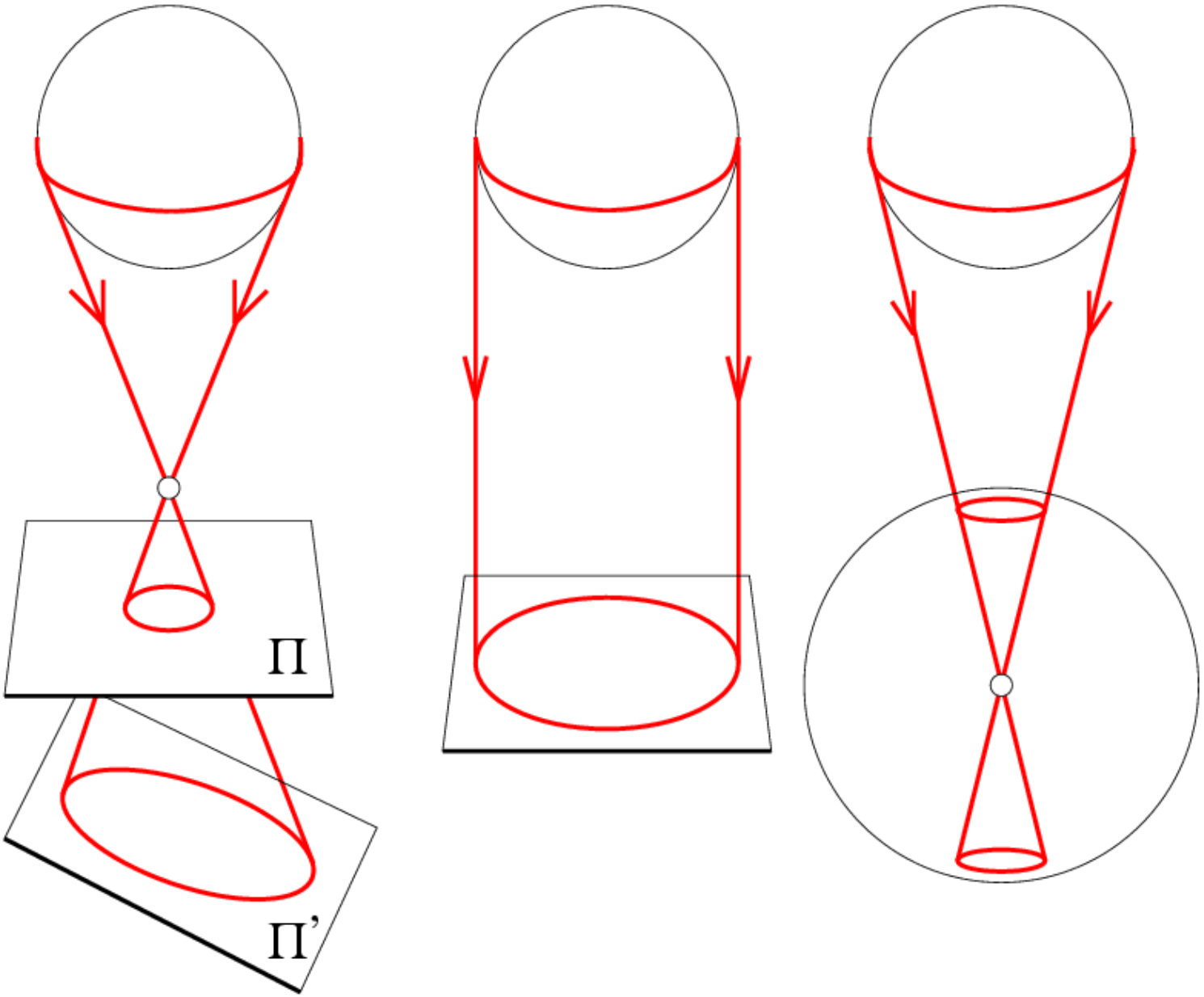
When the scene relief is small compared its distance from the Camera, m can be taken constant: weak perspective projection.

Affine projection models: Orthographic projection



$$\boxed{\begin{cases} x' = x \\ y' = y \end{cases}}$$

When the camera is at a (roughly constant) distance from the scene, take m=1.

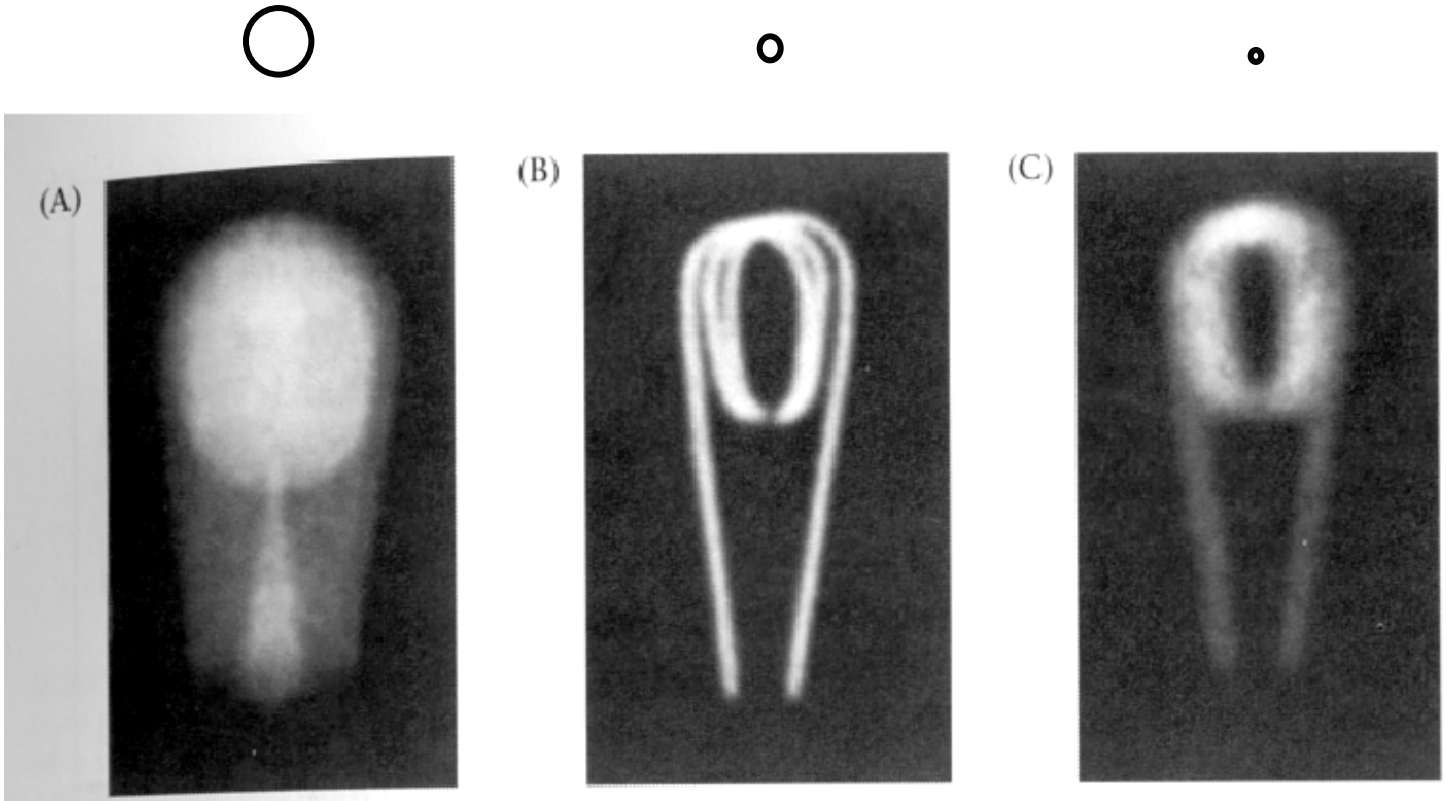


Planar pinhole perspective

Orthographic projection

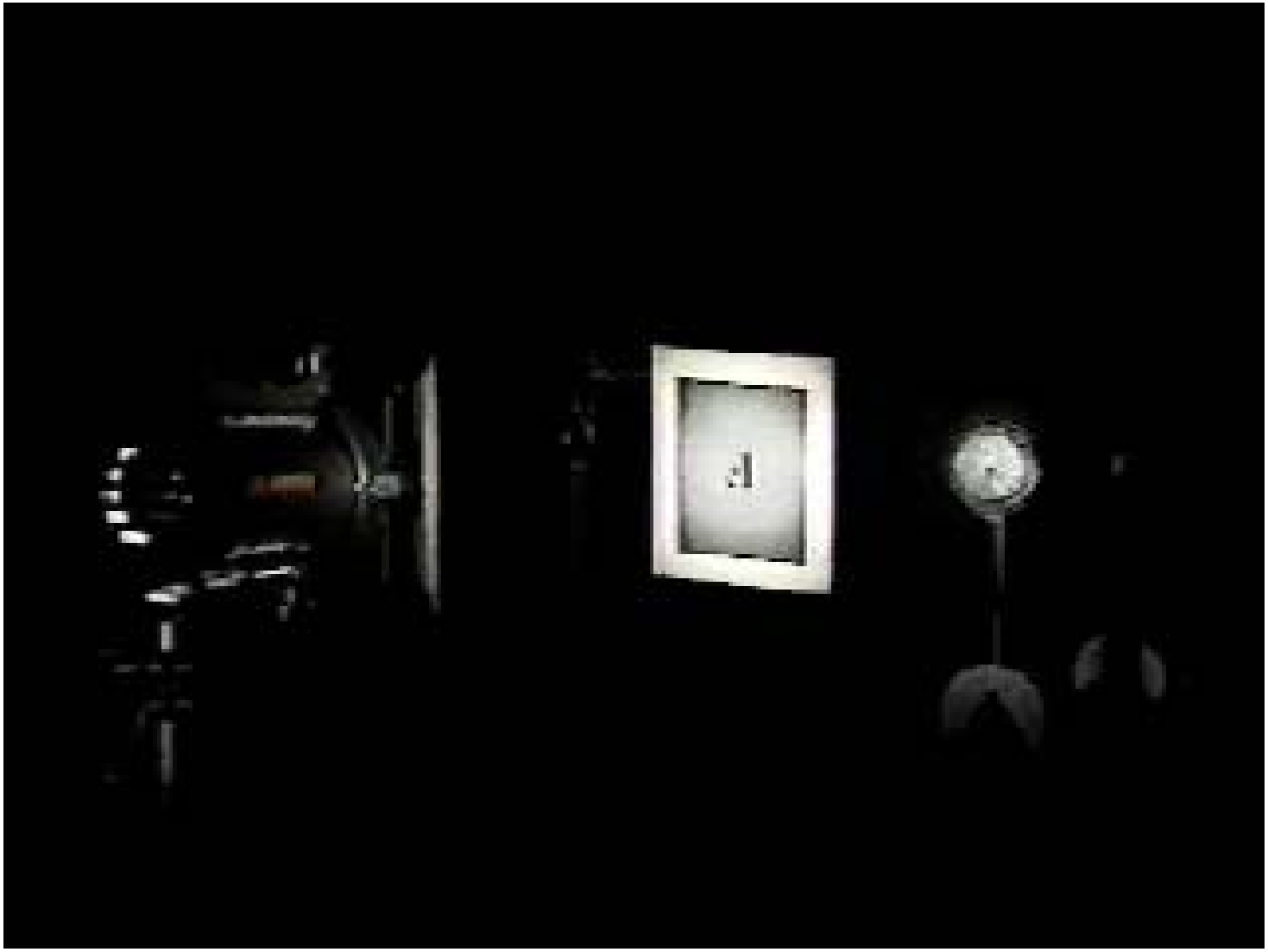
Spherical pinhole perspective

Limits for pinhole cameras

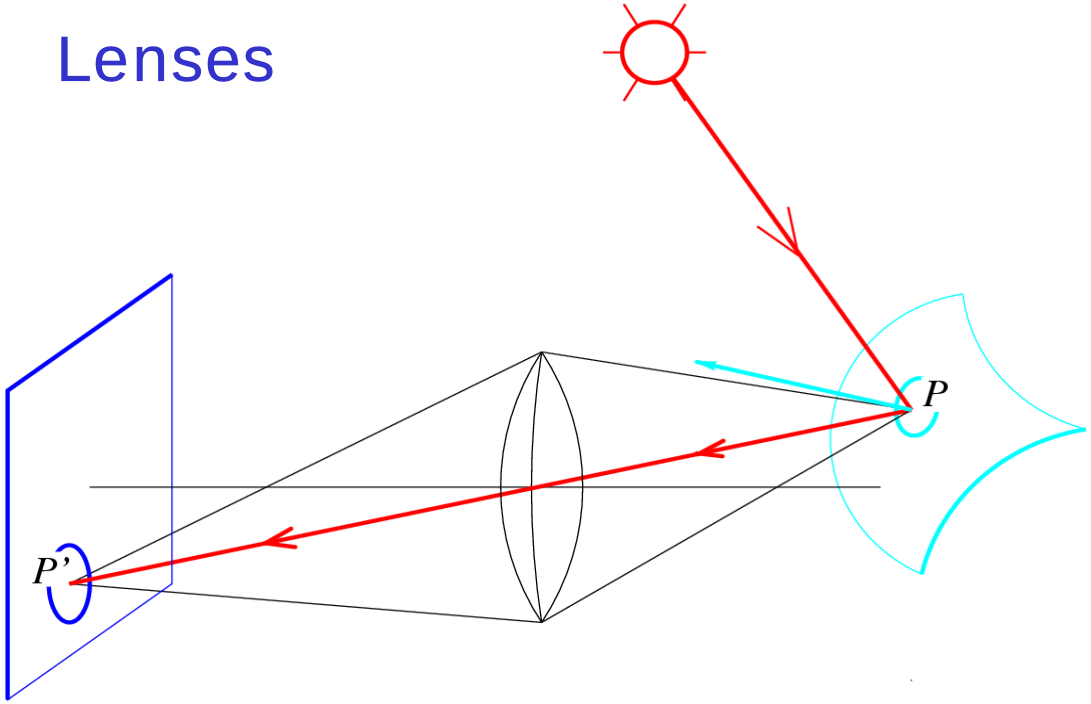


2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

Camera obscura + lens

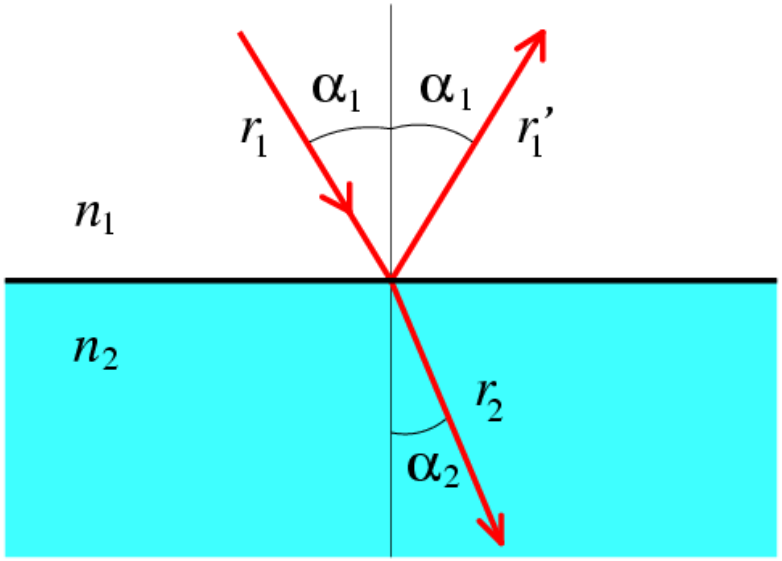


Lenses



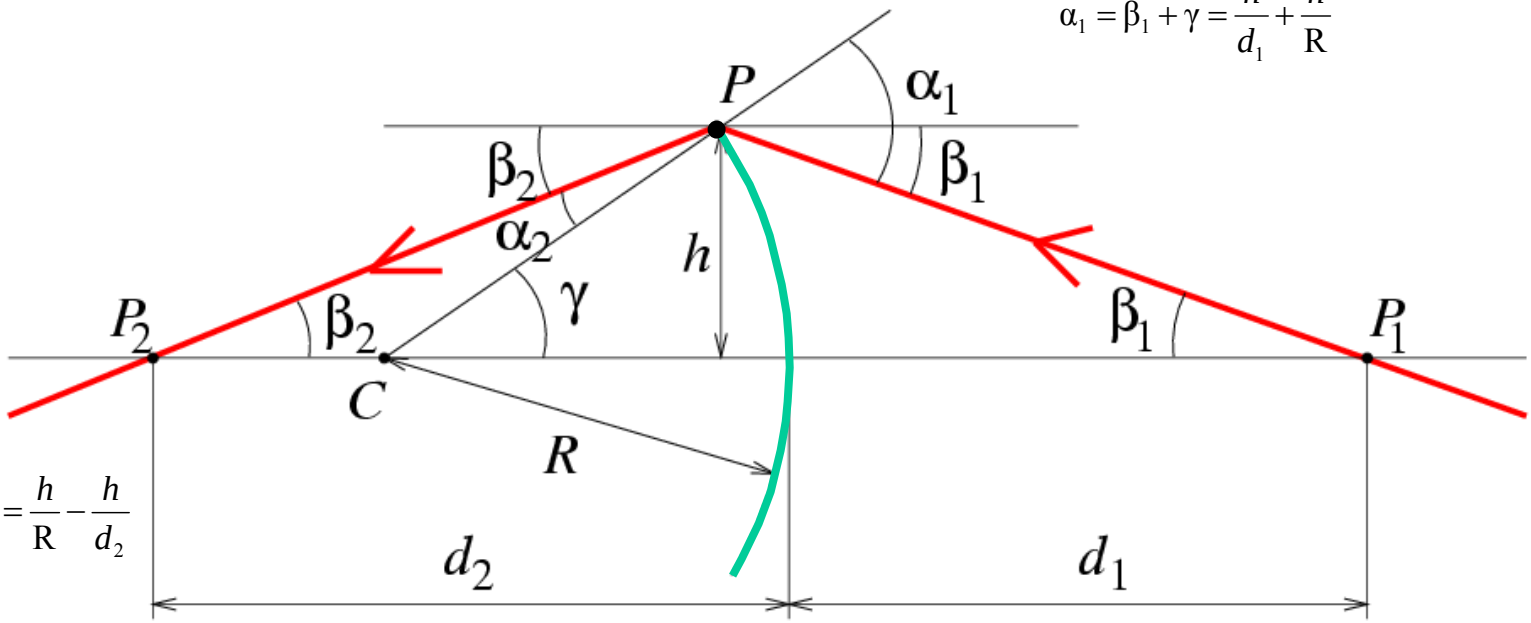
Snell's law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$



Paraxial (or first-order) optics

$$\alpha_1 = \beta_1 + \gamma = \frac{h}{d_1} + \frac{h}{R}$$



$$\alpha_2 = \gamma - \beta_2 = \frac{h}{R} - \frac{h}{d_2}$$

Snell's law:

$$n_1 \sin\alpha_1 = n_2 \sin\alpha_2$$

Small angles:

$$n_1\left(\frac{h}{d_1} + \frac{h}{R}\right) = n_2\left(\frac{h}{R} - \frac{h}{d_2}\right)$$

$$n_1 \alpha_1 \approx n_2 \alpha_2$$

$$\boxed{\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}}$$

Thin Lenses

$$\frac{n_1}{d_1}+\frac{n_2}{d_2}=\frac{n_2-n_1}{R}$$

$$\frac{1}{Z}+\frac{n}{Z^*}=\frac{n-1}{R}$$

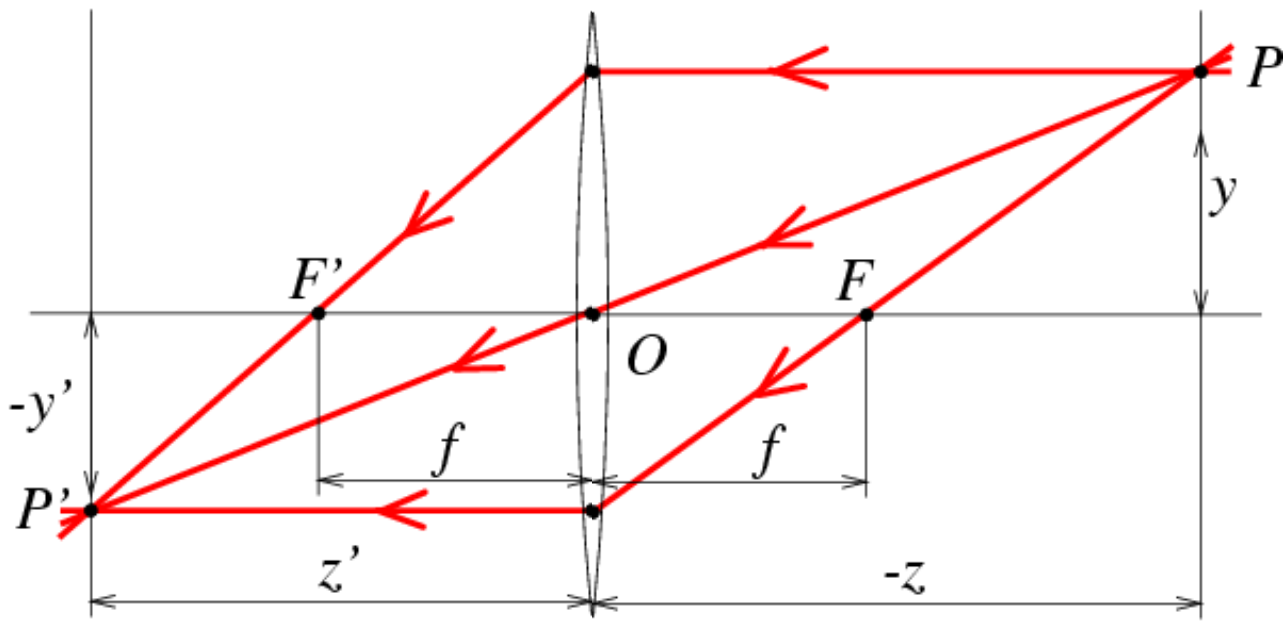
$$\frac{n}{Z^*}+\frac{1}{Z'}=\frac{1-n}{R}$$

$$\frac{n}{Z^*}=\frac{n-1}{R}-\frac{1}{Z}$$

$$\frac{n}{Z^*}=\frac{1-n}{R}-\frac{1}{Z'}$$

$$\frac{n-1}{R}-\frac{1-n}{R}=\frac{1}{Z}-\frac{1}{Z'}$$

spherical lens surfaces; incoming light ± parallel to axis; thickness << radii; same refractive index on both sides

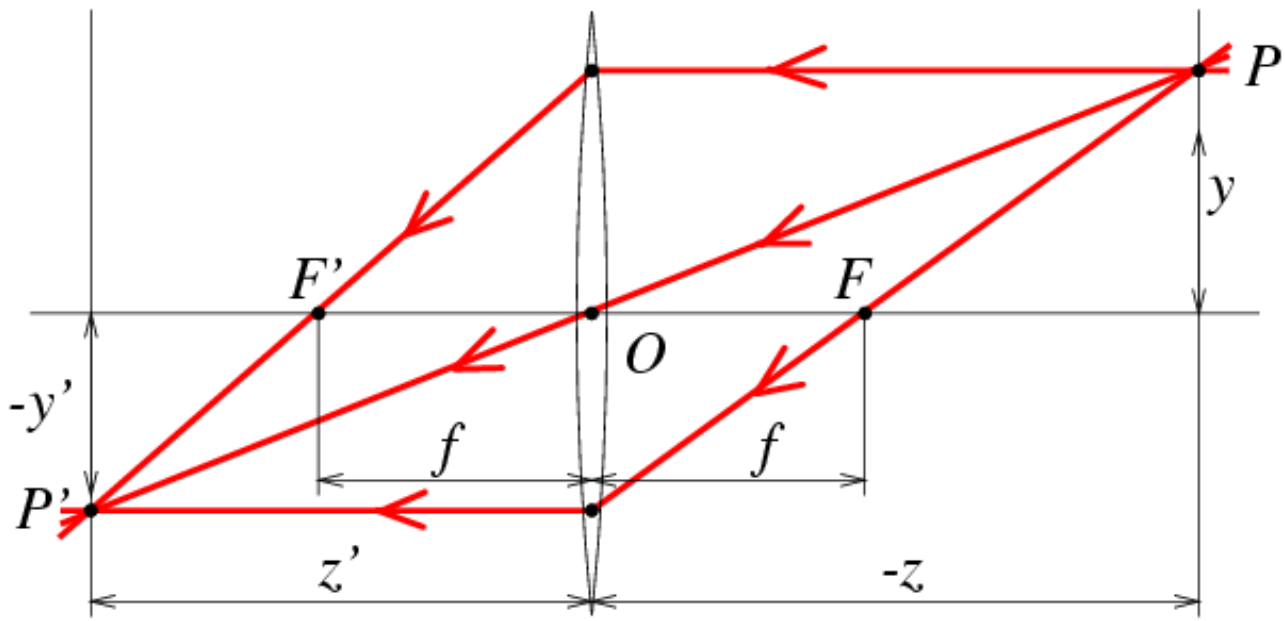


$$\frac{1}{z'}-\frac{1}{z}=\frac{1}{f}$$

and

$$f=\frac{R}{2(n-1)}$$

Thin Lenses

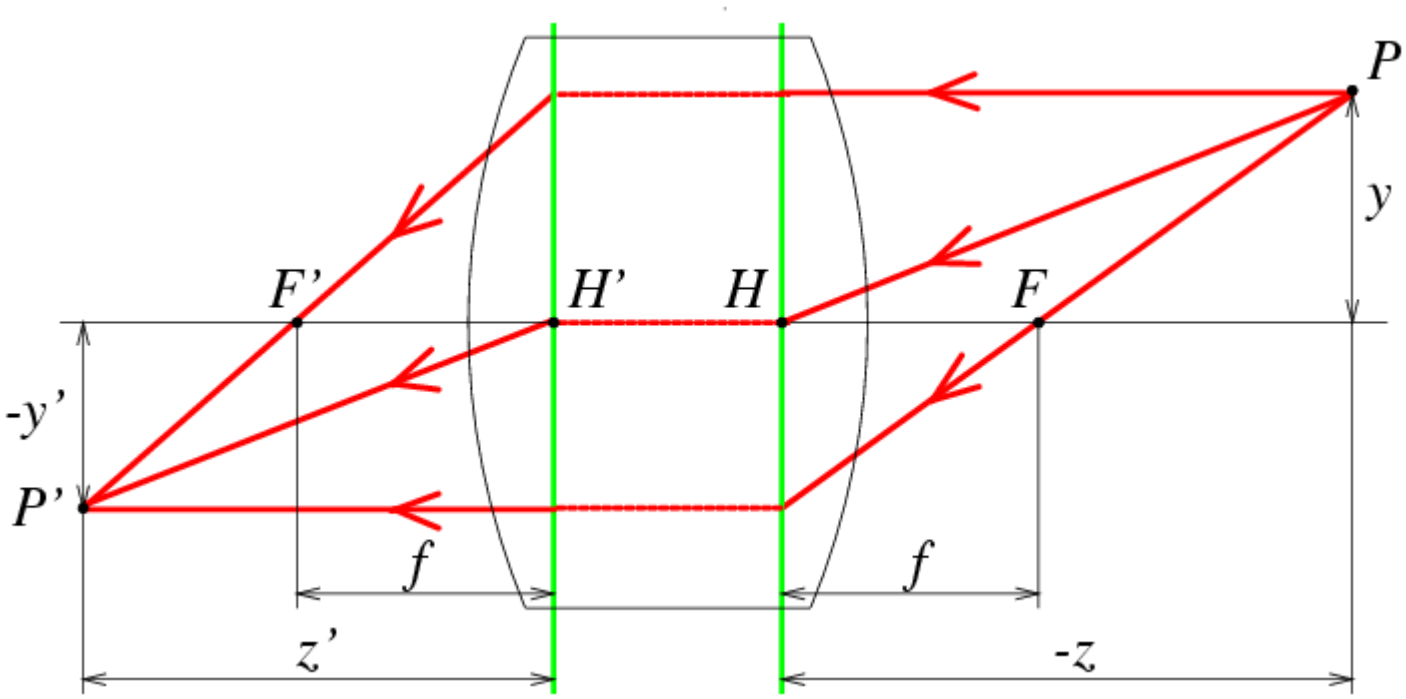


$$\begin{cases} x' = z'\dfrac{x}{z} \\ y' = z'\dfrac{y}{z} \end{cases}$$

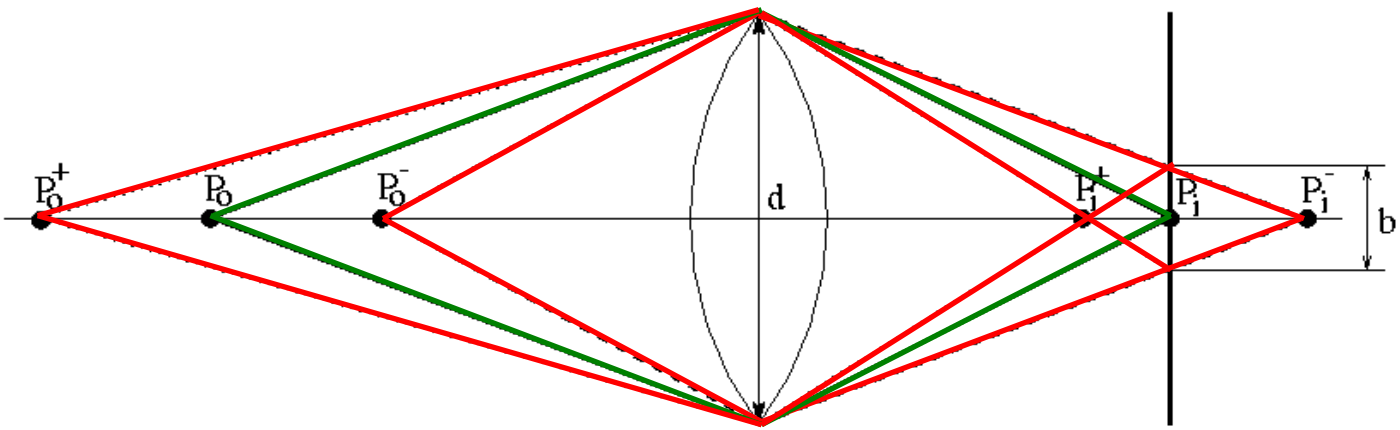
where $\dfrac{1}{z'} - \dfrac{1}{z} = \dfrac{1}{f}$ and $f = \dfrac{R}{2(n-1)}$

http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html

Thick Lens



The depth-of-field



The depth-of-field

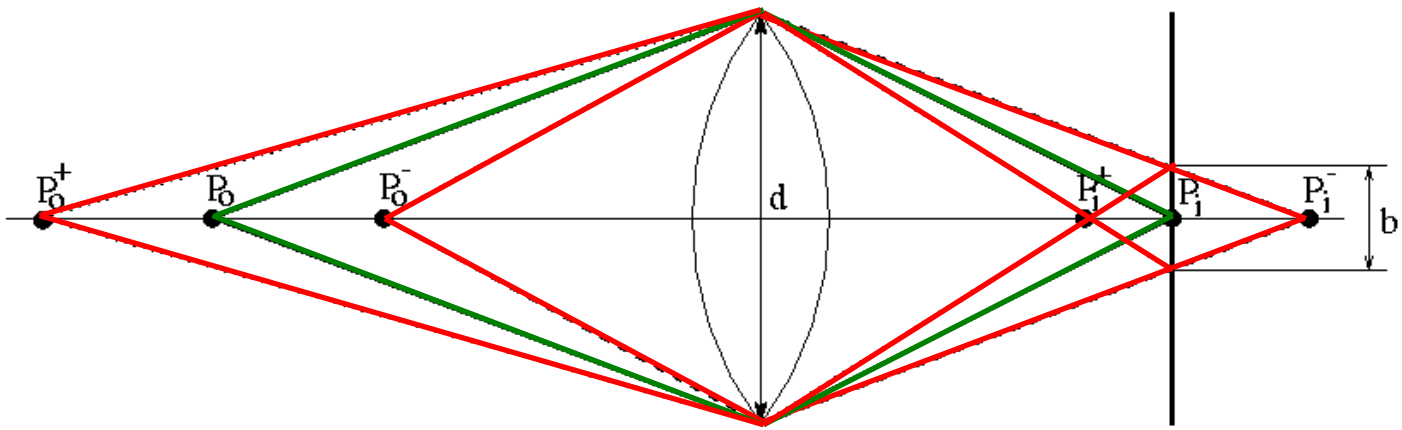
yields $$Z_o^- = f \frac{|Z_i^-|}{|Z_i^-| - f}$$

$$Z_o^- = f \frac{d\, Z_o}{b\, Z_0 + f\,(d - b)}$$

$$\Delta Z_o^- = Z_o - Z_o^- = \frac{Z_o\,(Z_o - f)}{Z_0 + f\, d / b - f}$$

Similar formula for $\Delta Z_o^+ = Z_o^+ - Z_o$

The depth-of-field



$$\Delta Z_0^- = Z_0 - Z_0^- = \frac{Z_0(Z_0 - f)}{Z_0 + f\ d/b - f}$$

decreases with d, increases with Z_0

strike a balance between incoming light and sharp depth range

Deviations from the lens model

3 assumptions :

1. all rays from a point are focused onto 1 image point

2. all image points in a single plane

3. magnification is constant

deviations from this ideal are *aberrations*

Aberrations

2 types :

1. geometrical

2. chromatic

geometrical : small for paraxial rays

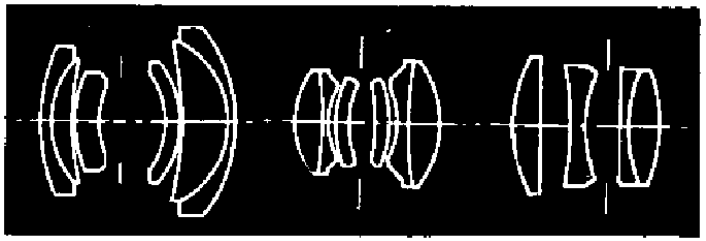
study through 3rd order optics $\sin(\theta) \approx \theta - \frac{\theta^3}{6}$

chromatic : refractive index function of wavelength

Geometrical aberrations

- spherical aberration
- astigmatism
- distortion
- coma

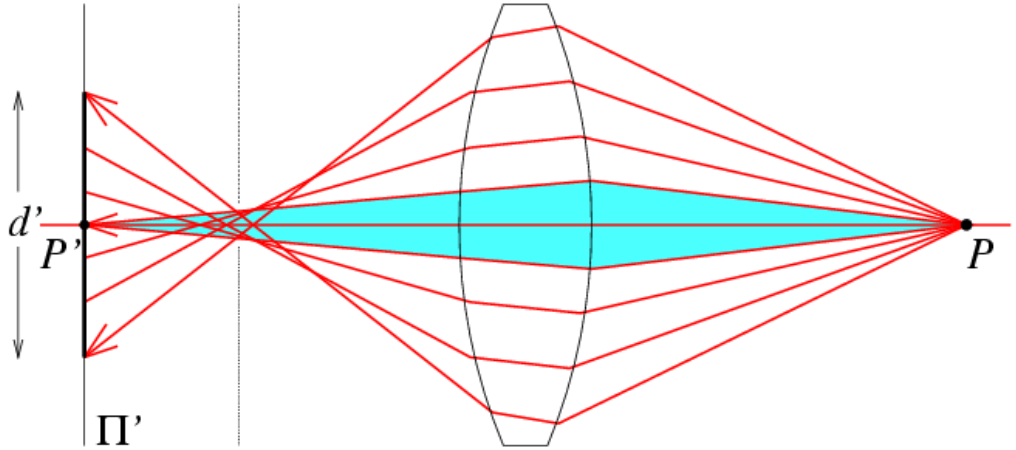
aberrations are reduced by combining lenses



Spherical aberration

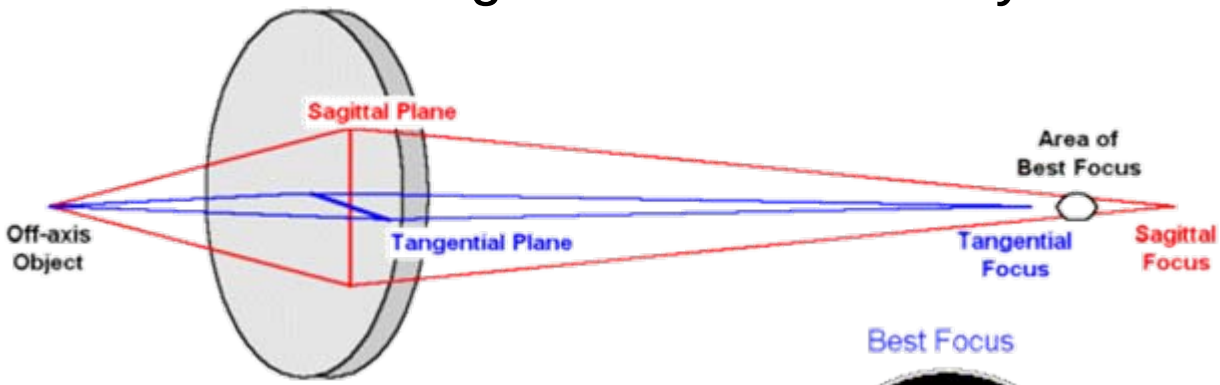
rays parallel to the axis do not converge

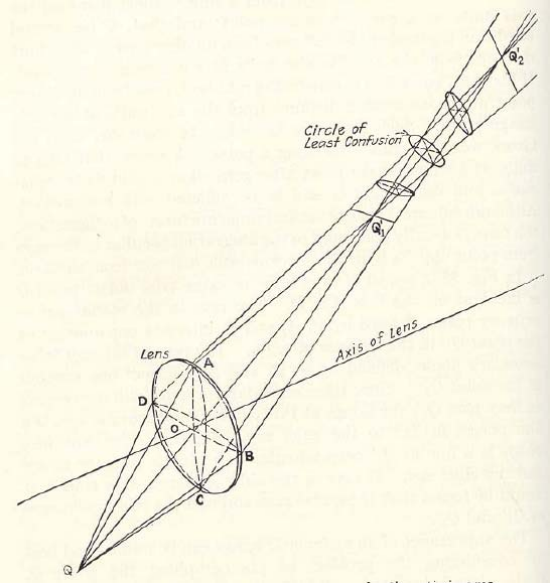
outer portions of the lens yield smaller focal lenghts

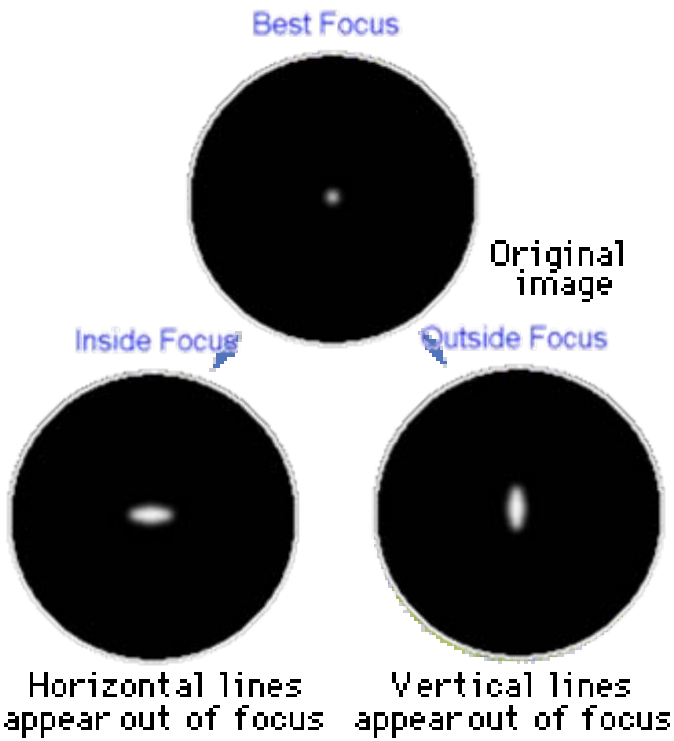


Astigmatism

Different focal length for inclined rays

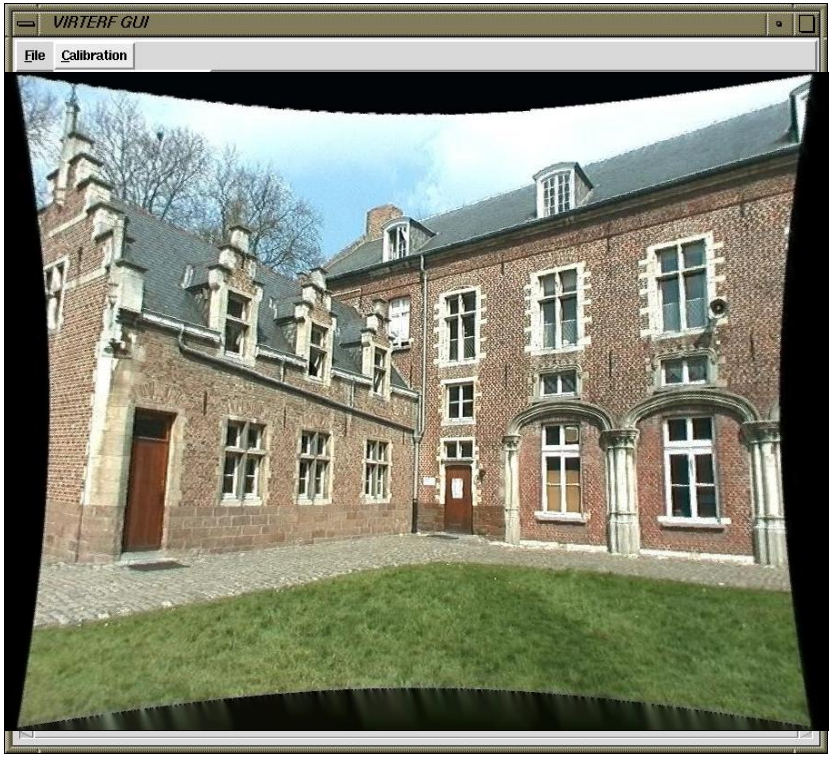






Distortion

magnification/focal length different for different angles of inclination



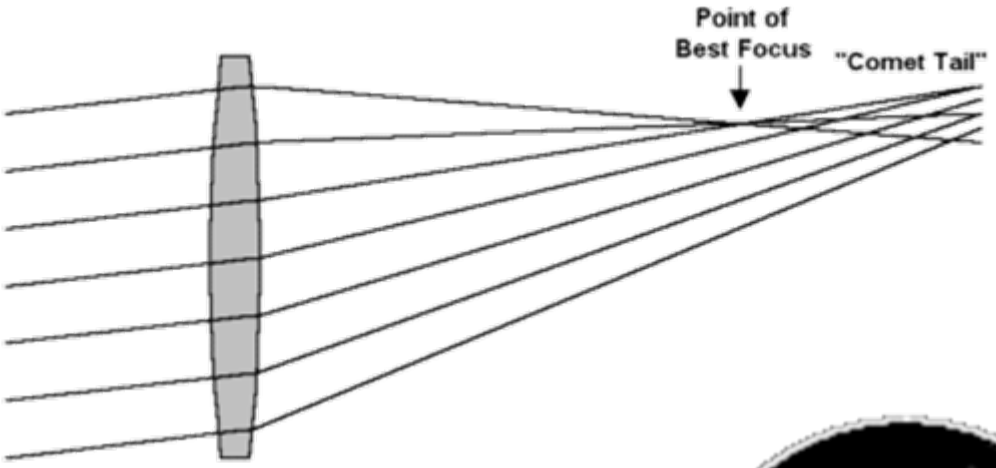
pincushion (tele-photo)

barrel (wide-angle)

Can be corrected! (if parameters are know)

Coma

point off the axis depicted as comet shaped blob



Chromatic aberration

rays of different wavelengths focused in different planes

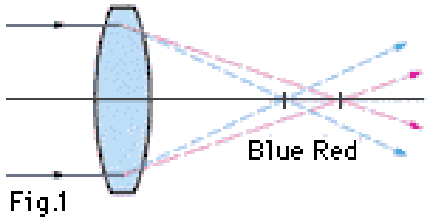


Fig.1
Axial chromatic aderration

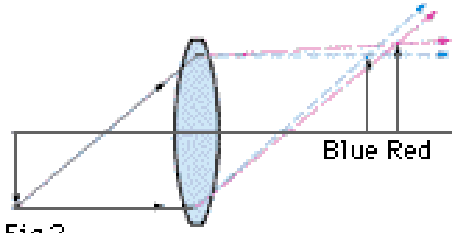
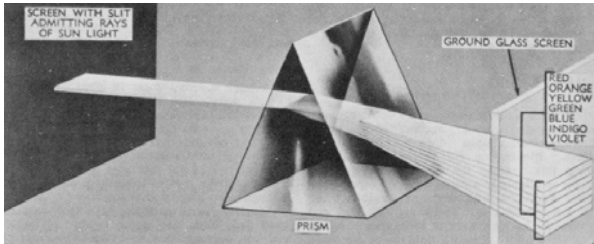


Fig.2
Magnification chromatic aderration

The image is blurred and appears colored at the fringe.

cannot be removed completely

sometimes *achromatization* is achieved for more than 2 wavelengths



Lens materials

reference wavelengths :

$\lambda_F = 486.13nm$
$\lambda_d = 587.56nm$
$\lambda_C = 656.28nm$

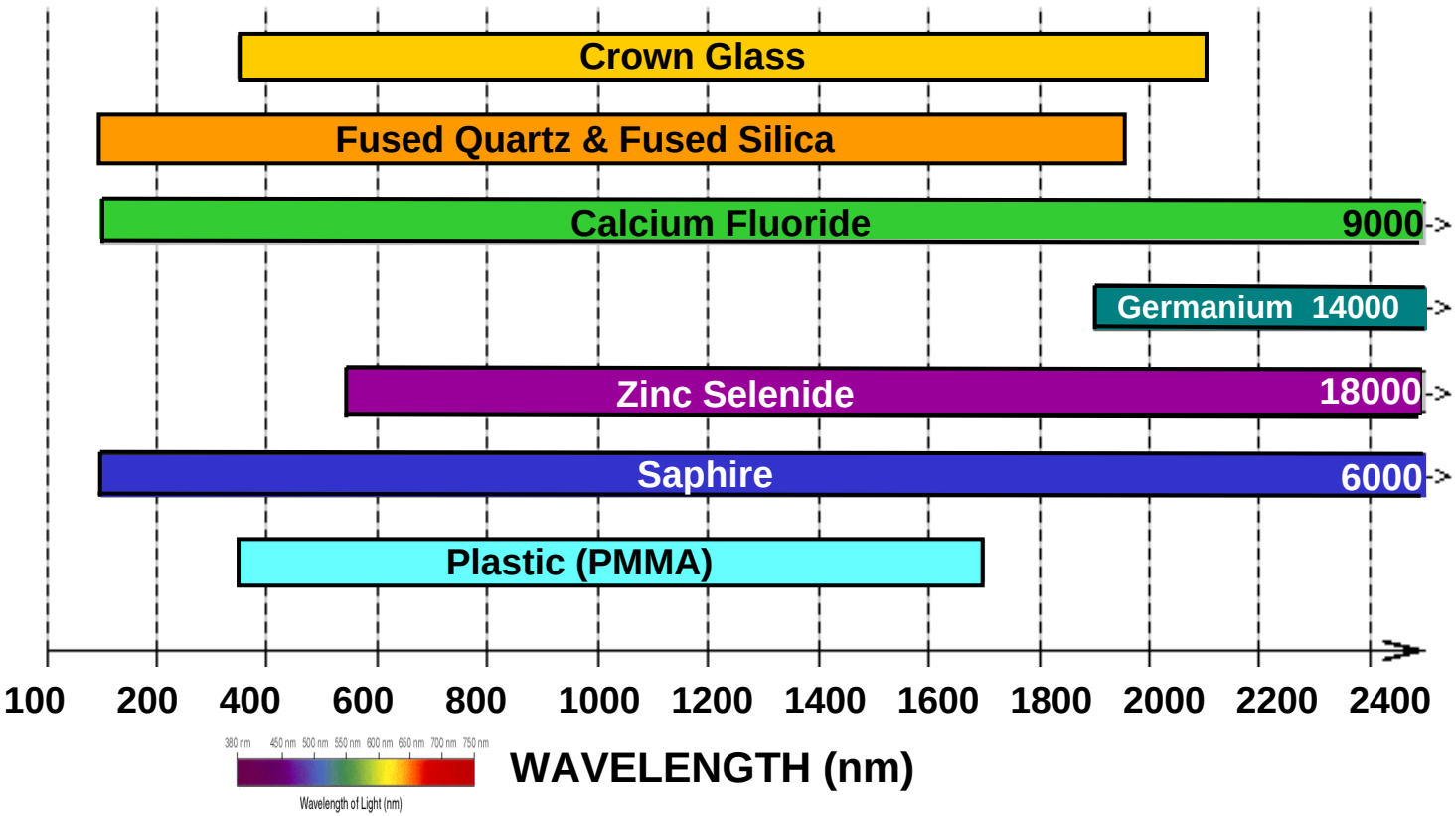
lens characteristics :

1. refractive index n_d
2. Abbe number $V_d = (n_d - 1) / (n_F - n_C)$

typically, both should be high
allows small components with sufficient refraction

notation : e.g. glass BK7(517642)
$n_d = 1.517$ and $V_d = 64.2$

Lens materials

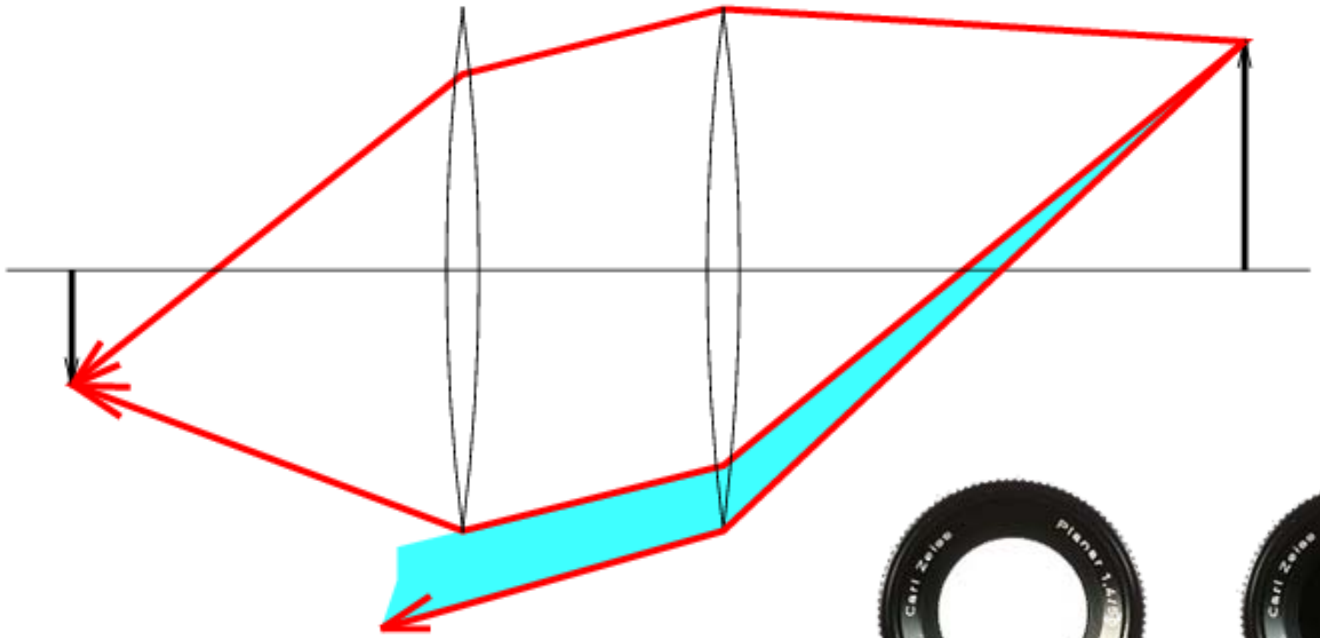


additional considerations :

humidity and temperature resistance, weight, price,...

Vignetting

Darkening of images near the corner



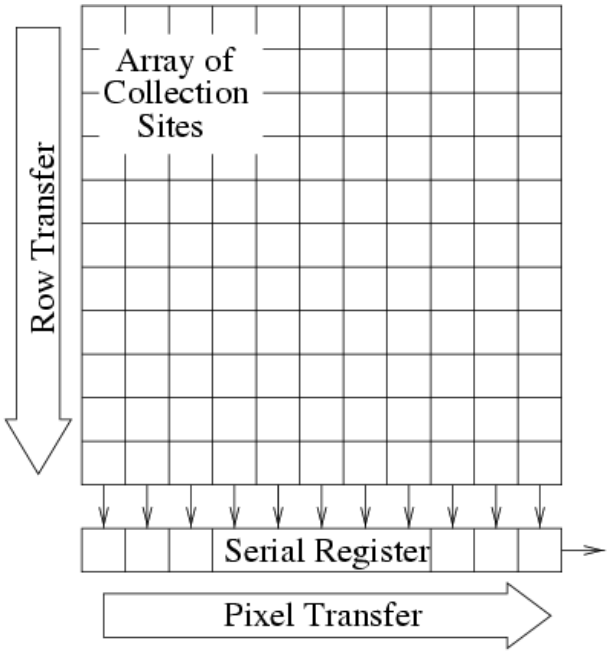
The white openings in the top illustrations denote the entrance pupil, which is the image of the aperture stop seen through all lens elements in front of it and from a position on the optical axis. The bottom illustrations show the lens from the semifield angle. Here, the white openings correspond to the clear aperture for light that is heading for the image corner.

Figure from http://www.vanwalree.com/optics/vignetting.html

Photographs
(Niepce,
“La Table Servie,” 1822)

Collection Harlingue-Viollet.

Milestones:
Daguerreotypes (1839)
Photographic Film (Eastman,1889)
Cinema (Lumière Brothers,1895)
Color Photography
(Lumière Brothers, 1908)
Television
(Baird, Farnsworth, Zworykin, 1920s)



CCD Devices (1970)
more recently CMOS

Cameras

we consider 2 types :

1. CCD
2. CMOS

CCD

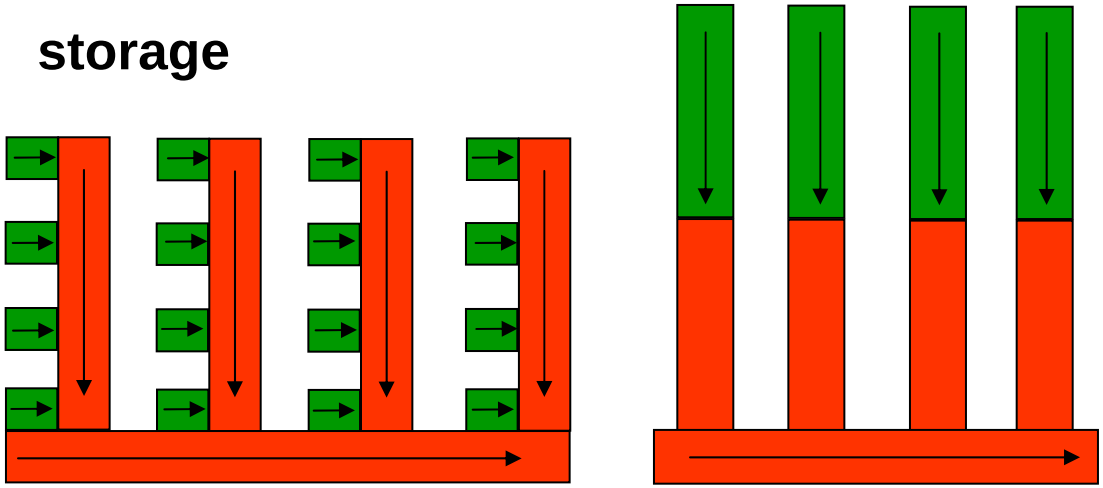
separate photo sensor at regular positions
no scanning

charge-coupled devices (CCDs)

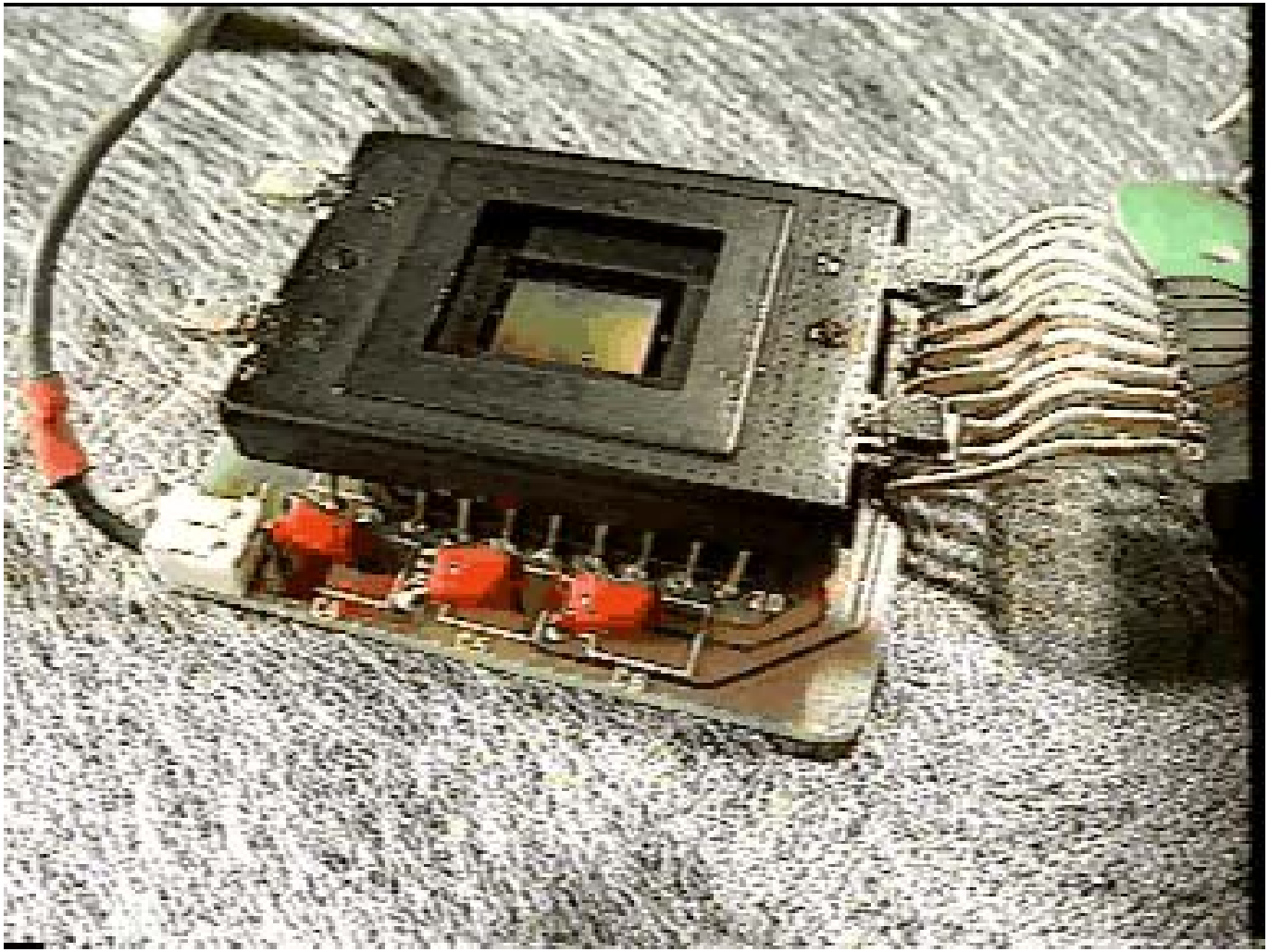
area CCDs and linear CCDs
2 area architectures :
interline transfer and *frame transfer*

photosensitive

storage



The CCD camera



CMOS

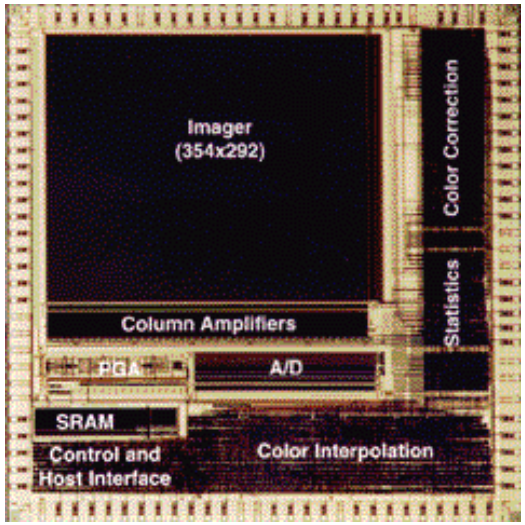
Same sensor elements as CCD

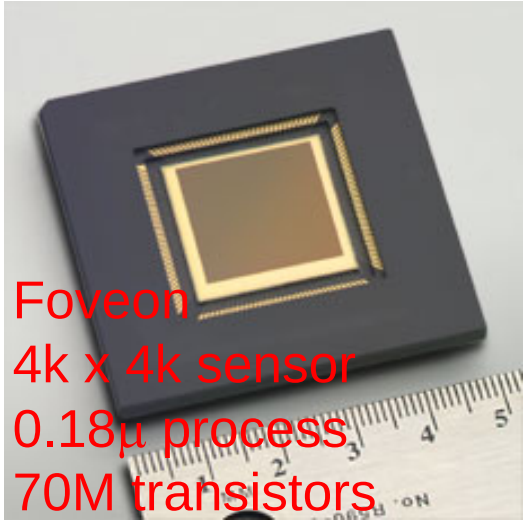
Each photo sensor has its own amplifier

- More noise (reduced by subtracting ‘black’ image)
- Lower sensitivity (lower fill rate)

Uses standard CMOS technology

- Allows to put other components on chip
- ‘Smart’ pixels





CCD vs. CMOS

- Mature technology
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

- Recent technology
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components

Color cameras

We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel

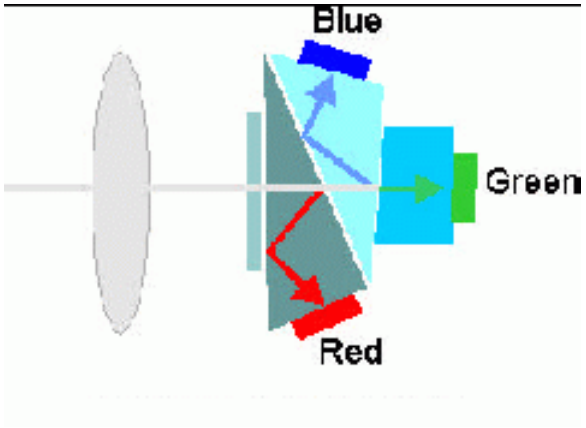
... and X3

Prism color camera

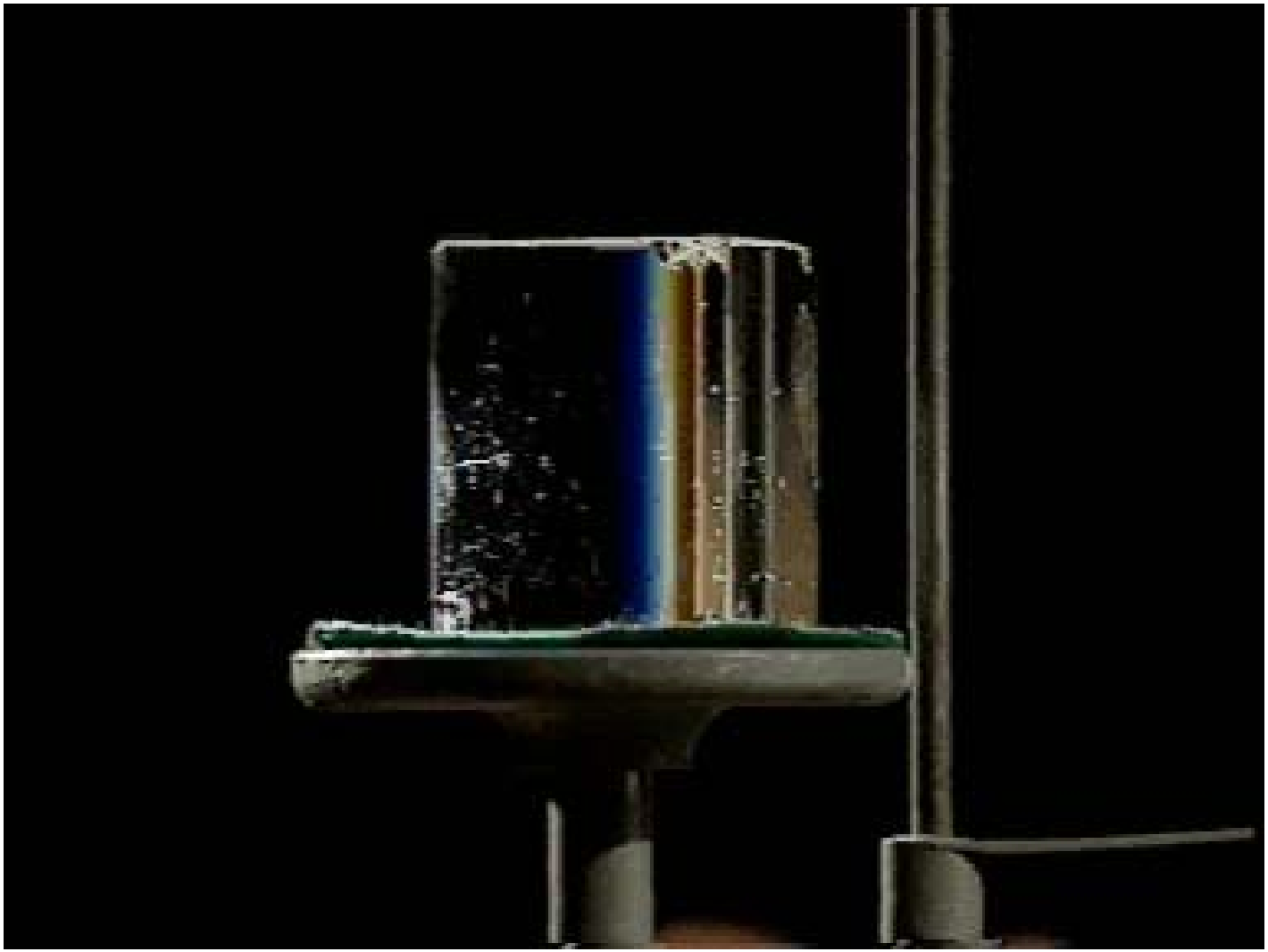
Separate light in 3 beams using dichroic prism

Requires 3 sensors & precise alignment

Good color separation

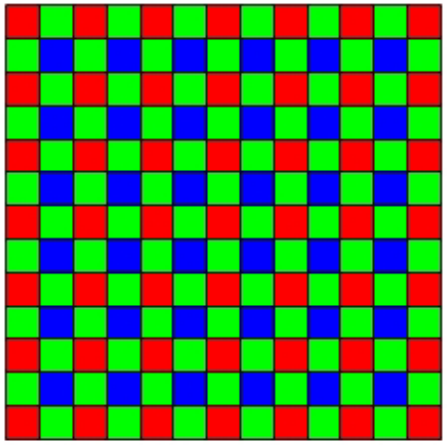


Prism color camera

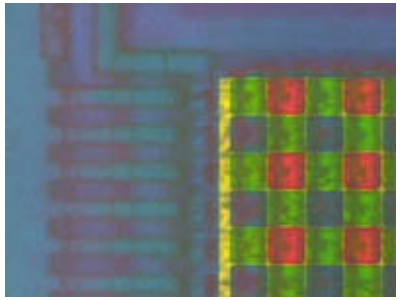


Filter mosaic

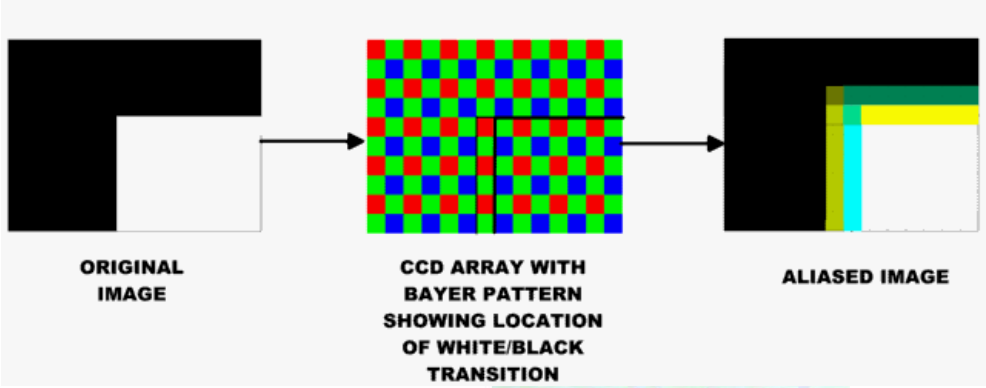
Coat filter directly on sensor



Bayer filter



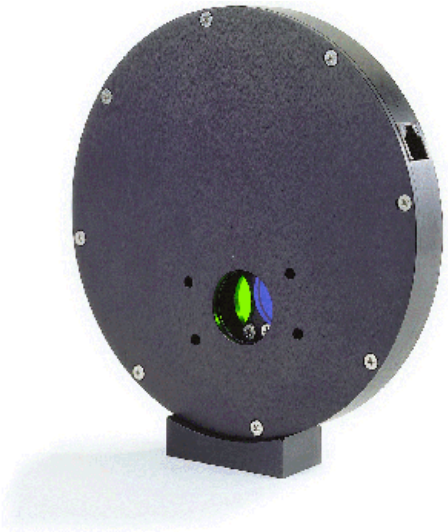
Demosaicing (obtain full colour & full resolution image)



Filter wheel

Rotate multiple filters in front of lens

Allows more than 3 colour bands

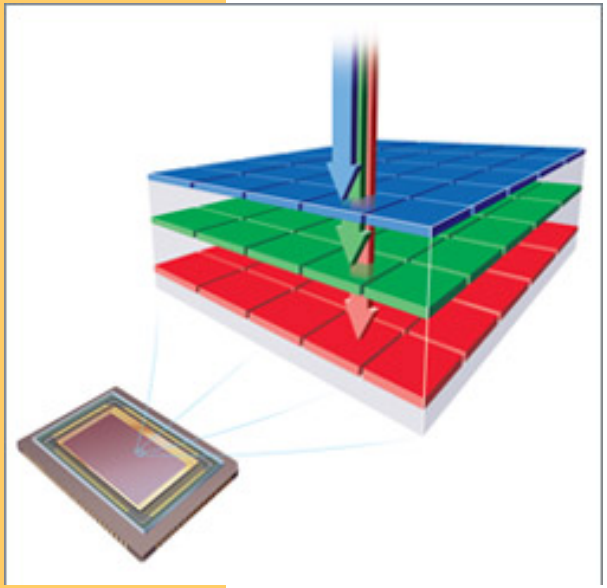


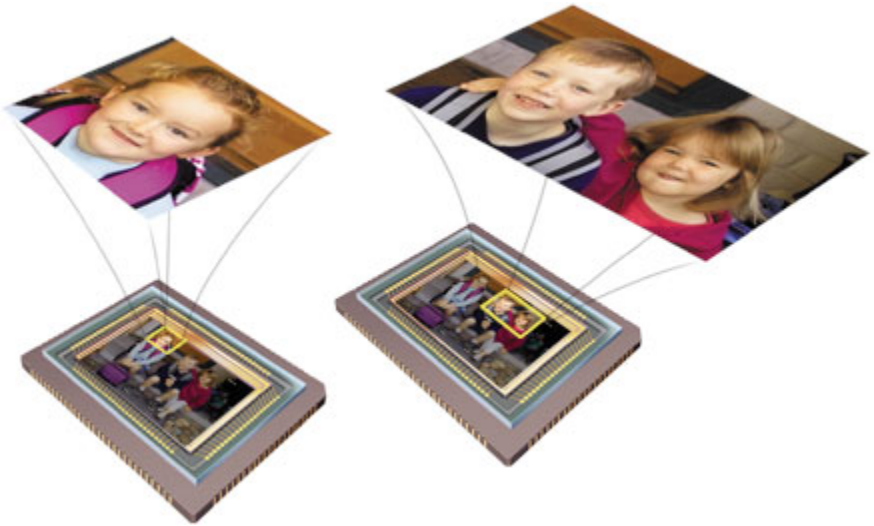
Only suitable for static scenes

Prism vs. mosaic vs. wheel

approach	Prism	Mosaic	Wheel
# sensors	3	1	1
Separation	High	Average	Good
Cost	High	Low	Average
Framerate	High	High	Low
Artefacts	Low	Aliasing	Motion
Bands	3	3	3 or more
	High-end cameras	Low-end cameras	Scientific applications

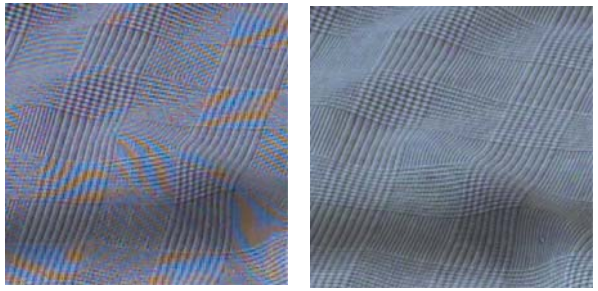
new color CMOS sensor Foveon's X3

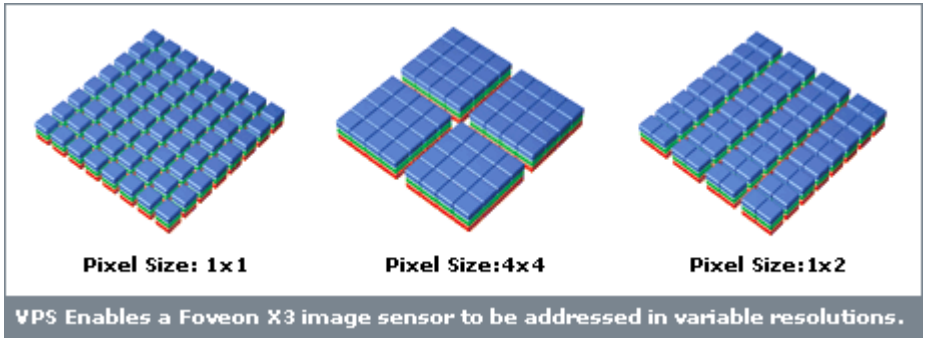




smarter pixels

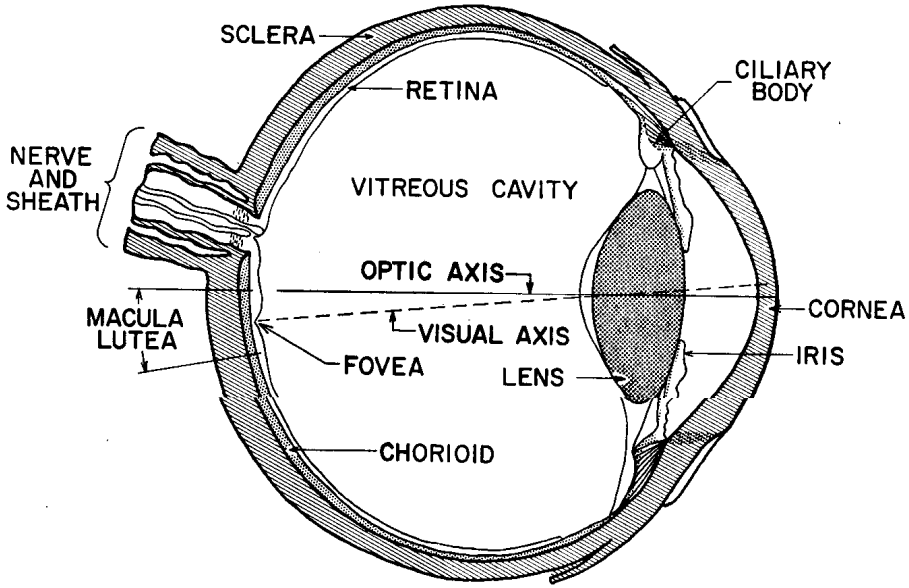
better image quality

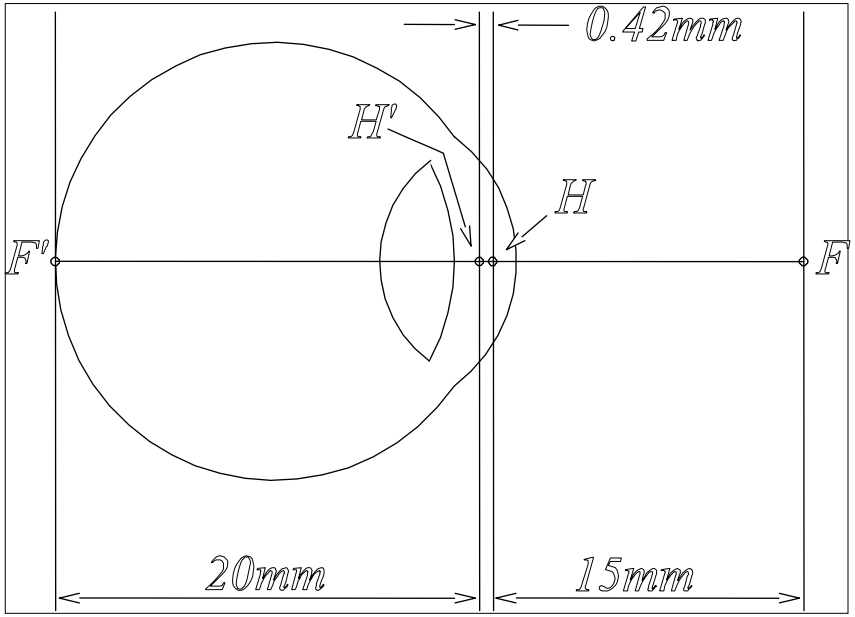




three layers of pixels. The layers of pixels are embedded in silicon to take advantage of the fact that red, green, and blue light penetrate silicon to different depths

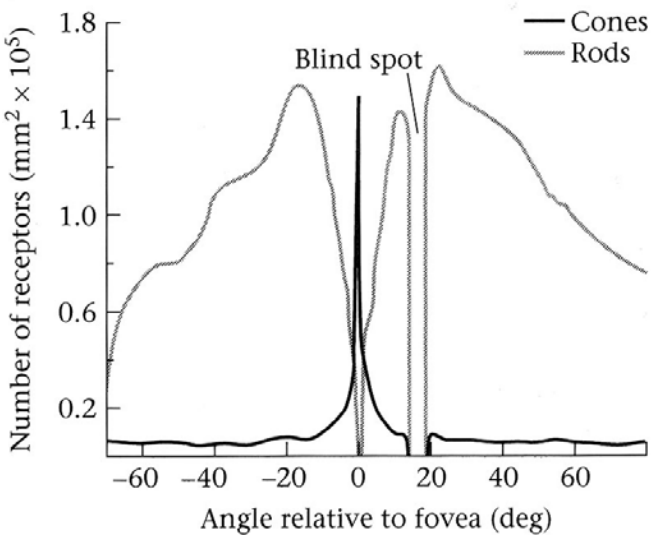
The Human Eye



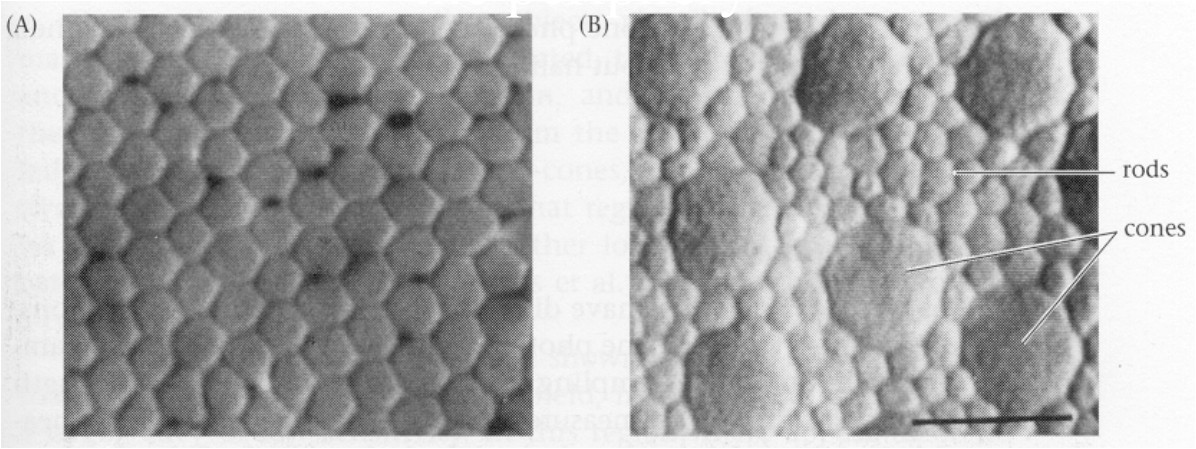


Helmoltz's Schematic Eye

The distribution of rods and cones across the retina



Cones in the fovea



Next class
Radiometry: lights and surfaces