Cameras, lenses and sensors

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COMP 256
Cameras, lenses and sensors

- Camera Models
  - Pinhole Perspective Projection
  - Affine Projection
- Camera with Lenses
- Sensing
- The Human Eye

Reading: Chapter 1.
Images are two-dimensional patterns of brightness values. They are formed by the projection of 3D objects.

Animal eye: a looonng time ago.

Photographic camera: Niepce, 1816.

Pinhole perspective projection: Brunelleschi, XVth Century.
Camera obscura: XVIth Century.
Distant objects appear smaller
Parallel lines meet

• vanishing point
Vanishing points

To different directions correspond different vanishing points.
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-plane
- Polygons go to polygons

Degenerate cases:
- line through focal point yields point
- plane through focal point yields line
Pinhole Perspective Equation

\[
\begin{align*}
  x' &= f' \frac{x}{z} \\
  y' &= f' \frac{y}{z}
\end{align*}
\]
Affine projection models: Weak perspective projection

\[
\begin{align*}
    x' &= -mx \\
    y' &= -my
\end{align*}
\]

where \( m = -\frac{f'}{z_0} \) is the magnification.

When the scene relief is small compared its distance from the Camera, \( m \) can be taken constant: weak perspective projection.
Affine projection models: Orthographic projection

When the camera is at a (roughly constant) distance from the scene, take $m=1$. 

\[
\begin{align*}
x' &= x \\
y' &= y
\end{align*}
\]
Planar pinhole perspective

Orthographic projection

Spherical pinhole perspective
Limits for pinhole cameras

2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.
Camera obscura + lens
Snell’s law

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]
Paraxial (or first-order) optics

Snell’s law: $n_1 \sin \alpha_1 = n_2 \sin \alpha_2$

Small angles: $n_1 \alpha_1 \approx n_2 \alpha_2$

$\alpha_1 = \beta_1 + \gamma = \frac{h}{d_1} + \frac{h}{R}$

$\alpha_1 = \gamma - \beta_2 = \frac{h-R}{d_2}$

$n_1 \left( \frac{h}{d_1} + \frac{h}{R} \right) = n_2 \left( \frac{h}{R} - \frac{h}{d_2} \right)$

$\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}$
Thin Lenses

spherical lens surfaces; incoming light ± parallel to axis; thickness << radii; same refractive index on both sides

\[
\frac{n_1 + n_2}{d_1} - \frac{n_2 - n_1}{d_2} = \frac{n_1 - n_2}{R}
\]

\[
\frac{1}{Z} + \frac{n}{Z'} = \frac{n-1}{R}
\]

\[
\frac{n}{Z'} + \frac{1}{Z'} = \frac{1-n}{R}
\]

\[
\frac{n}{Z'} = \frac{n-1}{R} - \frac{1}{Z}
\]

\[
\frac{n}{Z'} = \frac{1-n}{R} - \frac{1}{Z'}
\]

\[
\frac{n-1}{R} - \frac{1-n}{R} = \frac{1}{Z} - \frac{1}{Z'}
\]

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]

and

\[
f = \frac{R}{2(n-1)}
\]
Thin Lenses

\[
\begin{align*}
  x' &= z' \frac{x}{z} \\
  y' &= z' \frac{y}{z}
\end{align*}
\]

where

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]

and

\[
 f = \frac{R}{2(n-1)}
\]

http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html
Thick Lens
The depth-of-field
The depth-of-field

yields

\[ Z_o^- = f \frac{Z_i^-}{|Z_i^-| - f} \]

\[ Z_o^- = f \frac{d Z_o}{b Z_0 + f (d - b)} \]

\[ \Delta Z_o^- = Z_o - Z_o^- = \frac{Z_o (Z_o - f)}{Z_0 + f d / b - f} \]

Similar formula for \( \Delta Z_o^+ = Z_o^+ - Z_o \)
The depth-of-field

\[ \Delta Z_0^- = Z_0 - Z_0^- = \frac{Z_0(Z_0 - f)}{Z_0 + f \frac{d}{b} - f} \]

decreases with \( d \), increases with \( Z_0 \)

strike a balance between incoming light and sharp depth range
Deviations from the lens model

3 assumptions:

1. all rays from a point are focused onto 1 image point

2. all image points in a single plane

3. magnification is constant

deviations from this ideal are \textit{aberrations}
Aberrations

2 types:

1. geometrical

2. chromatic

**geometrical**: small for paraxial rays

study through 3\textsuperscript{rd} order optics \( \sin(\theta) \approx \theta - \frac{\theta^3}{6} \)

**chromatic**: refractive index function of wavelength
Geometrical aberrations

- spherical aberration
- astigmatism
- distortion
- coma

Aberrations are reduced by combining lenses
Spherical aberration

rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lengths
Astigmatism

Different focal length for inclined rays
Distortion

magnification/focal length different for different angles of inclination

Can be corrected! (if parameters are know)
Coma

point off the axis depicted as comet shaped blob
Chromatic aberration

rays of different wavelengths focused in different planes

cannot be removed completely

sometimes *achromatization* is achieved for more than 2 wavelengths
Lens materials

reference wavelengths:

\[ \lambda_F = 486.13 \text{nm} \]
\[ \lambda_d = 587.56 \text{nm} \]
\[ \lambda_C = 656.28 \text{nm} \]

lens characteristics:

1. refractive index \( n_d \)
2. Abbe number \( V_d = (n_d - 1) / (n_F - n_C) \)

typically, both should be high
allows small components with sufficient refraction

notation: e.g. glass BK7(517642)
\( n_d = 1.517 \) and \( V_d = 64.2 \)
Lens materials

- Crown Glass
- Fused Quartz & Fused Silica
- Calcium Fluoride
- Zinc Selenide
- Saphire
- Plastic (PMMA)

WAVELENGTH (nm)

Additional considerations:
- Humidity and temperature resistance
- Weight
- Price...
The white openings in the top illustrations denote the entrance pupil, which is the image of the aperture stop seen through all lens elements in front of it and from a position on the optical axis. The bottom illustrations show the lens from the semifield angle. Here, the white openings correspond to the clear aperture for light that is heading for the image corner.
Photographs (Niepce, “La Table Servie,” 1822)

Milestones:
Daguerreotypes (1839)
Photographic Film (Eastman, 1889)
Cinema (Lumière Brothers, 1895)
Color Photography (Lumière Brothers, 1908)
Television (Baird, Farnsworth, Zworykin, 1920s)

CCD Devices (1970)
more recently CMOS
Cameras

we consider 2 types:

1. CCD
2. CMOS
CCD

separate photo sensor at regular positions
no scanning
charge-coupled devices (CCDs)
area CCDs and linear CCDs
2 area architectures:
   * interline transfer*
   * frame transfer*

- photosensitive
- storage
The CCD camera
CMOS

Same sensor elements as CCD
Each photo sensor has its own amplifier
  More noise (reduced by subtracting ‘black’ image)
  Lower sensitivity (lower fill rate)
Uses standard CMOS technology
  Allows to put other components on chip
  ‘Smart’ pixels

Foveon
4k x 4k sensor
0.18μ process
70M transistors
CCD vs. CMOS

- Mature technology
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

- Recent technology
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components
Color cameras

We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel

... and X3
Prism color camera

Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation
Prism color camera
Filter mosaic

Coat filter directly on sensor

Demosaic (obtain full colour & full resolution image)
Filter wheel

Rotate multiple filters in front of lens
Allows more than 3 colour bands

Only suitable for static scenes
# Prism vs. mosaic vs. wheel

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<th>Approach</th>
<th>Prism</th>
<th>Mosaic</th>
<th>Wheel</th>
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<td># Sensors</td>
<td>3</td>
<td>1</td>
<td>1</td>
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<td>Separation</td>
<td>High</td>
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<td>Cost</td>
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<td>Bands</td>
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<td>3</td>
<td>3 or more</td>
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<tr>
<td></td>
<td>High-end cameras</td>
<td>Low-end cameras</td>
<td>Scientific applications</td>
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</table>
new color CMOS sensor  
Foveon’s X3

better image quality  
smarter pixels

three layers of pixels. The layers of pixels are embedded in silicon to take advantage of the fact that red, green, and blue light penetrate silicon to different depths.
The Human Eye


Helmoltz’s Schematic Eye
The distribution of rods and cones across the retina

Cones in the fovea

Next class
Radiometry: lights and surfaces