DPCM - Overview

- Principle of Differential Pulse Code Modulation (DPCM)
- Characteristics of DPCM quantization errors
- Predictive coding gain
- Adaptive intra-interframe DPCM
- Conditional Replenishment
Principle of DPCM

**coder**

**decoder**

Prediction error \[ e = s - \hat{s} \]  
Reconstruction \[ s' = e' + \hat{s} \]  
Reconstruction error = quantization error \[ s' - s = e' - e = q \]
Quantization error feedback in the DPCM coder

- Assuming a linear predictor, the DPCM coder is equivalent to the following structure:

  ![Diagram](image)

  - Transfer function of the prefilter:
    
    \[ \tilde{E}(\Omega) = [1 - P(\Omega)]S(\Omega) \]

  - Transfer function of quantization error feedback:
    
    \[ E'(\Omega) = \tilde{E}(\Omega) + [1 - P(\Omega)]Q(\Omega) \]
Power spectrum of the DPCM quantization error

- Power spectral density of the quantization error $q$ measured for intraframe DPCM with a 16 level quantizer
Signal distortions due to intraframe DPCM coding

- **Granular noise**: random noise in flat areas of the picture.
- **Edge busyness**: jittery appearance of edges (for video).
- **Slope overload**: blur of high-contrast edges, Moire patterns in periodic structures.
Example of intraframe DPCM coding

- 1 bit/pixel prediction error coding
- 2 bit/pixel edge busyness
- 3 bit/pixel slope overload
- 4 bit/pixel granular noise

- Linear predictor:
  \[
  0 \quad \frac{1}{4} \quad \frac{1}{4} \\
  \frac{1}{2}
  \]

- Lloyd-Max quantizers
- Fixed-length coding
Recall from EE398A: High-rate performance of scalar quantizers

- High-rate distortion-rate function

\[ d(R) \approx \varepsilon^2 \sigma_X^2 2^{-2R} \]

- Scaling factor \( \varepsilon^2 \)

<table>
<thead>
<tr>
<th></th>
<th>Shannon LowBd</th>
<th>Lloyd-Max</th>
<th>Entropy-coded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>( \frac{6}{\pi e} \approx 0.703 )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Laplacian</td>
<td>( \frac{e}{\pi} \approx 0.865 )</td>
<td>( \frac{9}{2} = 4.5 )</td>
<td>( \frac{e^2}{6} \approx 1.232 )</td>
</tr>
<tr>
<td>Gaussian</td>
<td>1</td>
<td>( \frac{\sqrt{3\pi}}{2} \approx 2.721 )</td>
<td>( \frac{\pi e}{6} \approx 1.423 )</td>
</tr>
</tbody>
</table>
Predictive coding gain

- Distortion-rate function with DPCM
  \[ d_{DPCM}(R) \approx \epsilon_e^2 \sigma_e^2 2^{-2R} \]

- Prediction gain
  \[ G_{DPCM} = \frac{\epsilon_s^2 \sigma_s^2}{\epsilon_e^2 \sigma_e^2} \]

- Smallest achievable prediction error variance for \( N \)-dimensional signal determined by *spectral flatness*
  \[ \sigma_e^2 = \exp \left( \frac{1}{(2\pi)^N} \int_{\Omega} \ln \left( \Phi_{xx}(\Omega) \right) d\Omega \right) \]
Predictive coding gain (cont.)

- Consider 1-D Gaussian Markov-1 process with correlation coefficient $\rho$

- Autocorrelation function
  \[ E\left[ S_n S_{n-k} \right] = \sigma_s^2 |\rho|^k \]

- Prediction gain
  \[ G_{DPCM} = \frac{1}{1 - \rho^2} \]
R-D curves for Gauss-Markov-1 source

\[
\text{SNR [dB]} = 10 \log_{10} \frac{\sigma^2}{D}
\]

- Linear predictor order \( N=1, a=0.9 \)
- Entropy-Constrained Scalar Quantizer with Huffman VLC
- Iterative design algorithm applied

![Graph showing R-D curves for Gauss-Markov-1 source with various methods compared: R(D*), ρ=0.9, DPCM & ECSQ, Panter & Dite App, Entropy-Constrained Opt.](image)
Prediction example: test pattern
Prediction example: Cameraman

original
Histograms: Cameraman

Image signal

Prediction error

Histogram of the image signal and the prediction error for the Cameraman image.
DPCM with entropy-constrained quantization

$K=511, H=4.79\text{ bpp}$  
$K=15, H=1.98\text{ bpp}$  
$K=3, H=0.88\text{ bpp}$

$K$...number of reconstruction levels, $H$...entropy

[J. R. Ohm]
Transmission errors in a DPCM system

• For a linear DPCM decoder, the transmission error response is superimposed to the reconstructed signal $S'$

• For a stable DPCM decoder, the transmission error response decays

• For variable length coding, loss of synchronization can lead to errors in many prediction error samples after a single bit-error
Transmission errors in a DPCM system (cont.)

Example: Lena, 3 \textit{bpp} (fixed code word length)

Error rate: $p=10^{-3}$

[J. R. Ohm]
Interframe coding of video signals

- Interframe coding exploits similarity of temporally successive pictures
- Important interframe coding methods:
  - Adaptive intra-interframe coding
  - Conditional replenishment
  - Motion-compensated prediction
“It has been customary in the past to transmit successive complete images of the transmitted picture.”

[...]

“In accordance with this invention, this difficulty is avoided by transmitting only the difference between successive images of the object.”
Principle of adaptive intra-interframe DPCM

- Predictor is switched between two states:

A: Intraframe prediction for moving or changed areas.

B: Interframe prediction (previous frame prediction) for still areas of the picture.

\[
\hat{S}_{\text{intra}} = a_1 S_1 + a_2 S_2 + a_3 S_3 + a_4 S_4
\]

\[
\hat{S}_{\text{inter}} = S_{20}
\]
Intra-interframe DPCM: feedback adaptation
Intra-interframe DPCM: feedforward adaptation

Coder

Decoder
Conditional replenishment

- Still areas: repeat from frame store
- Moving areas: encode and transmit address and waveform
Change detection

- **Example of a pixel-wise change detector**

  - Current frame
  - \[ \text{ABS} \rightarrow \text{Average of } 3\times3 \text{ window} \rightarrow \text{Threshold} \rightarrow \text{Eliminate isolated points or pairs of points} \]
  - Decision changed/unchanged

- **Example of a block-wise change detector**

  - Current frame
  - \[ \text{ABS} \rightarrow \text{Accumulate over } N\timesN \text{ blocks} \rightarrow \text{Threshold} \rightarrow \text{Decision changed/unchanged} \]
Rate-distortion optimized mode selection

- How to choose the decision threshold, if distortion $D$ shall be minimized for a given rate $R$?

- Assumptions
  - Blockwise mode selection, block index $i$
  - Additive overall distortion $D = \sum_i D_i$ and rate $R = \sum_i R_i$

- Lagrangian cost function
  \[ J = D + \lambda R = \sum_i D_i + \lambda R_i = \sum_i J_i \]

- Strategy: minimize $J_i$ for each block $i$ separately, using a common Lagrange multiplier $\lambda$
Rate-distortion optimized mode selection (cont.)

Consider 2 blocks with \( D(R) = D_1(R_1) + D_2(R_2) \)
The “Dirty Window” effect

- Conditional replenishment scheme with change detection threshold set too high leads to the subjective impression of looking through a dirty window.

![Diagram of moving areas picked up and missed by change detector]
Crawford noise reduction filter

\[
\delta f(\delta) \quad f(\delta)
\]

- noisy video signal
- non-linearity
- frame store
- clean video signal

\[
\delta
\]
DPCM - Summary

- DPCM: Prediction from previously coded/transmitted samples (known at transmitter and receiver)
- Typical signal distortions for intraframe DPCM: granular noise, edge busyness, slope overload
- Prediction gain depends on spectral flatness
- Adaptive Intra-Interframe-DPCM: forward adaptation vs. backward adaptation
- Conditional replenishment: only transmit frame-to-frame changes
- Temporal noise reduction by nonlinear, recursive frame differencing