Fractal Compression

- **Basic Idea:** fixed point transform.
- $X_0$ is a fixed point representation for $f$ if $f(X_0) = X_0$.
- **Transform:** $ax + b \rightarrow f$
- **Fixed point:** $X_0 = ax_0 + b$
  \[ X_0(1-a) = b \]
  \[ X_0 = \frac{b}{1-a} \]
- **Idea:** instead of transmitting $X_0$, transmit $a$ and $b$.
- **Then iterate:** $X_0^{(n+1)} = ax_0^{(n)} + b$
- **As $n \rightarrow \infty$$ X_0^{(n)} \rightarrow X_0$
- One can show this converges regardless of the initial guess.
Image Compression

- Image $I = \text{array of numbers}$
- Find a function $f$ such that $f(I) = I \Rightarrow I$ is a fixed point representation for $f$
- Approach: Divide the image into a domain block and a range block
  - Divide image into an $M \times M$ “Range block”
  - For each range block find another $2M \times 2M$ domain block from the same image such that for some transform $f_k$ we get $f_k(D_k) = R_k$
- Ref: Jacquin
Image Compression

- Given an $f_k$ and an $N$ range block, for each $f_k$, $k=1, \ldots, N$:

\[ f = \bigcup_k f_k \]

\[ I \approx f(I) \]

\[ \hat{I} \approx I \]

\[ \hat{I} = f(\hat{I}) \]

- Collage Theorem: Guarantees coverage to $\hat{I}$ using any arbitrary initial guess for the image.
Vector Quantization

Code Book

<table>
<thead>
<tr>
<th>B_1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_2</td>
<td>2</td>
</tr>
<tr>
<td>B_{19}</td>
<td>19</td>
</tr>
<tr>
<td>B_{52}</td>
<td>52</td>
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<tr>
<td>B_{200}</td>
<td>200</td>
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</tbody>
</table>

Codes

| 19 | B_{19} |
| 52 | B_{52} |
Vector Quantization

- Let $\mathbf{f}$ denote an $N$ dimensional vector consisting of $N$ real valued, continuous amplitude scalars.
- Map $\mathbf{f}$ into $L$ possible $N$ dimensional reconstruction vectors $\mathbf{r}_i$ for $1 \leq i \leq L$

$$\hat{\mathbf{f}} = VQ(\mathbf{f}) = \mathbf{r}_i, \quad f \in C_i$$

- Advantage of VQ: removes linear dependency of R.V.s
Codebook Design

- $L = 9$ cells, $C_i = i^{th}$ cell

Assume $r_1^{(0)}$, $r_2^{(0)}$, ..., $r_3^{(0)}$

- Classify $M$ training vectors into $L$ clusters
- Recompute $r_i$ based on classification of previous step
Complexity Design

- M = number of training vectors
- L = number of codewords = \(2^{NR}\)
- N = dimension of vectors
- R = bits/scalar
- There are MLN operations (adds and mults) per iteration.
  - N = 10, R = 2, M = 10L \(\Rightarrow\) 100 trillion operations
- Note that there are \(\frac{\log_2 L}{N}\) = bits/scalar
- Complexity at the receiver:
  - LN operations per vector \(\Rightarrow\) 10 million operations in this example