EE291E Homework 3: Control of Hybrid Systems.

Assigned April 22; Due May 12

We consider the motivational example given in Lecture Notes 7.

Problem 1: Two-aircraft collision avoidance, no mode switching.

Consider the case in which the aircraft follow straight paths only (mode 1 of the motivational example), and collision avoidance is achieved using linear velocity control only. Thus, the continuous inputs are the airspeeds of the aircraft \((u = v_1, d = v_2)\) and assume that the airspeeds are known to vary over specified ranges: \(u \in U = [\bar{v}_1, \bar{v}_1] \subset \mathbb{R}^+, \ d \in D = [\bar{v}_2, \bar{v}_2] \subset \mathbb{R}^+\), and model reduces to

\[
\begin{align*}
\dot{x}_r &= -u + d \cos \psi_r \\
\dot{y}_r &= d \sin \psi_r \\
\dot{\psi}_r &= 0
\end{align*}
\]

(1)

Design a MATLAB program which plots the subset of states which is doomed (whatever the controller does) to enter the 5-mile relative protected zone in \(T\) seconds. You can choose \(T\) to be anything you like; what happens as \(T \to -\infty\)?

For your code, use \([v_1, \bar{v}_1] = [2, 4], [v_2, \bar{v}_2] = [1, 5]\), and consider four different values of \(\psi_r\): \(\pi/2, 0, -\pi/4, \text{ and } -\pi/2\).

Problem 2: Two-aircraft collision avoidance, mode switching

Now consider the three mode example of Lectures Notes 7.

Assume that, in the straight modes, \(\omega_1 = \omega_2 = 0\), and in the circular arc mode, \(\omega_1 = \omega_2 = 1\); and assume that, in all modes, \(v_1 = v_2 = 5\). Assume that in all modes, \(\psi_r = 2\pi/3\).

Show that by increasing the radius of the circular arc in the “avoid” mode, the set of states which is doomed (whatever the controller does) to enter the 5-mile relative protected zone
Figure 2: In $q_1$ both aircraft follow a straight course, in $q_2$ a half circle, and in $q_3$ both aircraft return to a straight course.

decreases in size. You can use the code that you wrote for Problem 1 and the “overlapping set” argument presented in class, and answer this question using a set of illustrations.