EECS 291E: Hybrid & Intelligent Control
Homework 3: Games, Spring 2020

Assigned March 17th, Due April 2nd 11:59 pm.

Problem 1 Computing Mixed Saddle Points of Zero Sum Games

Find the minimax mixed solution (security level as well as policies for each player) of the zero sum game with payoff matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

Think about a way that you can systematically compute minimax mixed equilibria of general zero sum games, using convex programming in Matlab. You can consult Chapter 6 of "Noncooperative Game Theory : An Introduction for Scientists and Engineers", By J. P. Hespanha, Princeton University Press, 2017. No need to include code, but give it a try and discuss your analysis.

Problem 2 Nash equilibria for Non Zero Sum Games

Compute Nash equilibria for the two games of the Prisoner's Dilemma and the Couple's Quarrel. Both players seek to minimize their costs in A and B, with P1's choices indexing the rows and P2's choices indexing the columns.

Part 2A: Love is War

There is a deep fissure that divides lovers everywhere: Modern-era Classical versus Romantic-era Classical. Inevitably, one partner (P1) loves listening to Bach (action 1) and suffers through Stravinsky (action 2), while the second (P2) feels staunchly the opposite. Yet they both prefer to go to the concert together:

$$A = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$$

Part 2B: Prisoner's dilemma

Pitting two prisoners' against each other to rat the other out, Prisoner 1 must choose either to confess (action 1) or keep the secret (action 2) with rewards in A. Prisoner 2 makes the same choice with rewards in B.

$$A = \begin{bmatrix} 2 & 30 \\ 0 & 8 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 \\ 30 & 8 \end{bmatrix}$$

Part 2C: In General

Think about how you can systematically compute Nash equilibria of bimatrix games, using convex programming in Matlab. You can consult Chapter 10 of "Noncooperative Game Theory : An Introduction for Scientists and Engineers", By J. P. Hespanha, Princeton University Press, 2017. No need to include code, but give it a try and discuss your analysis.

Problem 3 Conflict Resolution for Aircraft

You may choose to do **either** Problem A or Problem B. These are both based on the motivational example given in Lecture Notes 7.

Problem A: No mode switching = Continuous Game

Consider the case in which the aircraft follow straight paths only (mode 1 of the motivational example), and collision avoidance is achieved using linear velocity control only. Thus, the continuous inputs are the airspeeds of the aircraft ($u = v_1, d = v_2$) and assume that the airspeeds are known to vary over specified ranges: $u \in U = [\underline{v}_1, \overline{v}_1] \subset \mathbb{R}^+$, $d \in D = [\underline{v}_2, \overline{v}_2] \subset \mathbb{R}^+$, and model reduces to

$$\begin{aligned} \dot{x}_r &= -u + d\cos\psi_r \\ \dot{y}_r &= d\sin\psi_r \\ \dot{\psi}_r &= 0 \end{aligned} \tag{1}$$

Compute and plot the subset of state space which is doomed (no matter what the controller u does) to enter the 5-mile relative protected zone in T seconds. Here, our plane controlled by u can not centrally coordinate with the plane controlled by d and so must work a security policy that plays robustly against even an adversarial d. You can choose T to be anything you like; what happens as $T \to \infty$?

For your code, use $[\underline{v}_1, \overline{v}_1] = [2, 4]$, $[\underline{v}_2, \overline{v}_2] = [1, 5]$, and consider four different values of ψ_r: $\pi/2$, 0, $-\pi/4$, and $-\pi/2$.

Consider using the Hybrid System Lab's "HelperOC" package with the "LSToolbox" for MATLAB. You can follow the tutorial here: `https://github.com/HJReachability/helperOC/blob/master/Intro%20to%20Reachability%20Code.pdf`. And then change the tutorial dynamics to relative Dubins and control speed rather than angle.

Problem B: One mode switch = Discrete Choice

Consider the three mode example of Lectures Notes 7 illustrated in Figures 1 and 2. All units are in miles.

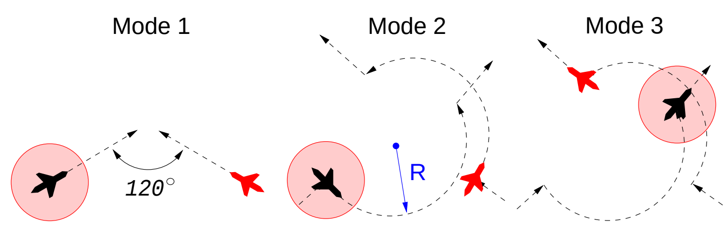

Figure 1: Two aircraft in three modes of operation: in modes 1 and 3 the aircraft follow a straight course and in mode 2 the aircraft follow a half circle. The initial relative heading (120°) is preserved throughout.

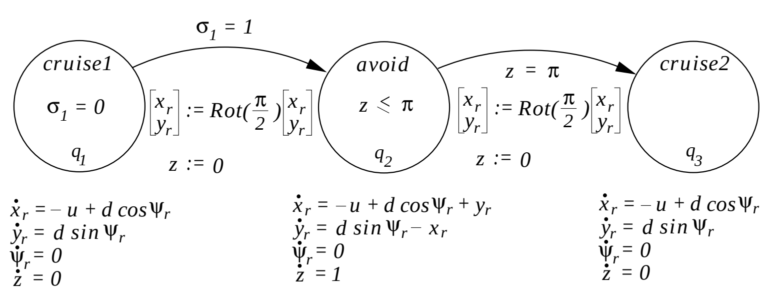

Figure 2: In q_1 both aircraft follow a straight course, in q_2 a half circle, and in q_3 both aircraft return to a straight course.

Assume that in all modes $v_1 = v_2 = 5$, in the straight "Cruise" modes $\omega_1 = \omega_2 = 0$ and in the circular arc "Avoid" mode $\omega_1 = \omega_2 = 5/r$ where r is a designed arc radius. Assume that in all modes, $\psi_r = 2\pi/3$. The constant velocities means that the controller only chooses when to transition to Mode 2 from Mode 1 to start the avoidance maneuver.

Show that increasing the radius r of the circular arc in the “avoid” mode will decrease the set of initial states which is doomed (no matter the choice of controller) to enter the 5-mile relative protected zone.