Name: Solutions

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EE 40
Midterm 2
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PLEASE WRITE YOUR NAME ON EACH ATTACHED PAGE

PLEASE SHOW YOUR WORK TO RECEIVE PARTIAL CREDIT

Problem 1: 10 Points Possible

Problem 2: 5 Points Possible

Problem 3: 15 Points Possible

Problem 4: 10 Points Possible

Problem 5: 10 Points Possible

Problem 6: 15 Points Possible

Problem 7: 15 Points Possible

Problem 8: 5 Points Possible

Problem 9: 15 Points Possible

Problem 10: 10 Points Possible

TOTAL: 110 Points Possible
Problem 1: 10 Points Possible

Perform nodal analysis on the circuit below. This means write a KCL equation for each node with unknown voltage. DO NOT SIMPLIFY the circuit. DO NOT SOLVE the KCL equations.

![Circuit Diagram]

**Required:** KVL @ \( V_1 \):

\[
\frac{V_1 - V_X}{R_1} - I_X + \frac{V_1 - V_Y}{R_5} = 0
\]

**Optional:** KVL @ \( V_2 \):

\[
\frac{V_2 - V_X}{R_3} + I_X = 0
\]

Problem 2: 5 Points Possible

In nodal analysis, when is a supernode needed? Why is a supernode needed?

A supernode is needed when there is a "floating" voltage source (neither terminal of the source is at ground). The source makes it difficult to write KCL equations at its terminal nodes because it has no V-I relationship (current is unknown). KCL is possible after enclosing the source in a supernode.
Problem 3: 15 Points Possible

Revisiting the circuit from Problem 1,

\[ V_{\text{T}} = V_{ab} = V_1 - V_Y \]

b) Deactivate independent sources.

\[ R_1 \text{ carries no current (series w/ open circuit)} \]

\[ R_2 \text{ carries no current (shorted out)} \]

Parallel w/ wire

\[ R_T = R_1 \parallel R_4 \parallel R_5 = \left( R_1^{-1} + R_4^{-1} + R_5^{-1} \right)^{-1} \]
**Problem 4: 10 Points Possible**

Suppose I have a black-box circuit (I can't see exactly what's inside) but I know it only contains resistors and linear dependent sources. The controlling voltages and currents for the dependent sources are also in the box.

I perform one experiment: When I attach a 5 V battery as shown, I measure a 200 mA current in the direction shown. The internal resistance of the battery is 4 Ω and the internal resistance of the DMM is 1 Ω.

![Circuit Diagram](image)

Can I find the Thevenin equivalent of the black-box circuit with this information? If yes, find the Thevenin equivalent. If no, explain why not.

Since there are only dependent sources and resistors, no independent sources,

\[ V_T = 0 \]

\[
\frac{5V}{200mA} = \frac{4R + 1R + R_T}{25R} = \frac{5R + R_T}{25R} \]

\[ R_T = 20Ω \]
**Problem 5:** 10 Points Possible

Find the Thevenin and Norton equivalents (if possible) for the following circuits:
(3 Points Possible for each Thevenin, 2 Points Possible for each Norton)

1. 7V source
   - **Thevenin Equivalent:**
     - $V_T = -7 \text{ V}$
     - $R_T = \infty$
   - **Norton Equivalent:**
     - No $y$-intercept $\Rightarrow I_N$ does not exist
     - Norton equivalent does not exist

2. 3Ω resistor
   - **Thevenin Equivalent:**
     - I-V graph same as 2A source alone without resistor
   - **Norton Equivalent:**
     - $V_N = 2 \text{ V}$
     - No $x$-intercept $\Rightarrow V_N$ does not exist
     - Thevenin equivalent does not exist
Problem 6: 15 Points Possible

For the ideal operational amplifier circuit below, find $V_o$ in terms of $V_1$ and $V_2$. Assume that the operational amplifier is operating linearly (ignore the rails).

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KVL on input loop:

A: $-V_2 + V_{R_1} + V_{R_2} = 0$  
but since same resistance and same current (no current into op-amp input),  
$-I_2 + 2V_{R_2} = 0$  
$V_{R_2} = V_2/2$

B: $-V_1 + V_{R_3} + V_{R_2} = 0$  
$V_{R_3} = V_1 - V_{R_2} = V_1 - V_2/2$

Feedback current:

$I_f = \frac{V_{R_3}}{5k} = \frac{V_1}{5k} - \frac{V_2}{10k}$

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KVL on output loop:

$-V_o - 15kI_f + V_{R_2} = 0$  
$V_o = V_{R_2} - 15kI_f$  
$= \frac{V_2}{2} - 15k\left(\frac{V_1}{5k} - \frac{V_2}{10k}\right)$  
$= \frac{V_2}{2} - 3V_1 + 1.5V_2$  
$= \sqrt{2V_2 - 3V_1}$
Problem 7: 15 Points Possible

Design an operational amplifier circuit that has an output voltage $V_o = 3V_2 - 5V_1$. The input voltage sources $V_1$ and $V_2$ cannot be detached from ground, and each have their negative terminals at ground. Assume that your amplifier is operating linearly.

You will lose 5 points if you use more than one differential amplifier. If you are desperate, the instructor will "sell" you a hint for points.

$3V_2 - 5V_1$ has same form as Problem 6
($V_2$ on non-inverting, $V_1$ on inverting terminal)

Use same circuit, change values:

$$V_o = V_2 \frac{R_2}{R_1 + R_2} - R_4 \left( V_1 - V_2 \frac{R_2}{R_1 + R_2} \right)$$

$$= -R_4 \frac{V_2}{R_3} V_1 + (1 + R_4) \left( \frac{R_2}{R_1 + R_2} \right) V_2$$

$$\frac{R_4}{R_3} = 5 \Rightarrow (1 + 5) \left( \frac{R_2}{R_1 + R_2} \right) = 3$$

$$\Rightarrow R_2 = \frac{V_2}{R_1 + R_2} = \frac{V_2}{1}$$

$$R_4 = 5 \, k\Omega$$

$R_1 = R_2 = R_3 = 1 \, k\Omega$

$$-V_0 - R_4 I_F + V_{R_2} = 0 \Rightarrow V_0 = V_{R_2} - R_4 I_F$$
Problem 8: 5 Points Possible

Suppose that we want to "clean up" a logic signal by transforming input voltages over 2.5 V (the threshold voltage) to 5 V (logic 1) as output and voltages under 2.5 V to logic 0. Design a differential amplifier circuit that will perform this function. You may use one ideal differential amplifier; $R_i = \infty$, $R_o = 0 \ \Omega$ and gain $A = \infty$. You must specify the rail voltages for this amplifier.
Problem 9: 15 Points Possible

Now suppose that your differential amplifier circuit from Problem 8 has a finite gain $A = 10,000$. For the input $v_i(t)$ defined below, determine the propagation delay $t_p$, where $t_p$ = time output reaches 50% of final value - time input reaches 50% of final value.

$$v_i(t) = \begin{cases} 
0 & \text{for } t < 0 \\
5 & \text{for } t > 5 \\
t & \text{for } 0 \leq t \leq 5
\end{cases}$$

$t$ in seconds, $v_i$ in volts

Time input reaches 50% of f.u. is $v_i(t) = 2.5$ V

$t = 2.5$ s (from above definition)

During transition, $V_{out}$ has equation

$$V_{out}(t) = A(v_p - v_n) = 10^4 (v_i(t) - 2.5 \text{ V})$$

$$= 10^4 v_i(t) - 2.5 \times 10^4$$

Want $V_{out}(t) = 2.5$

$$2.5 = 10^4 v_i(t) - 2.5 \times 10^4$$

$$10^4 v_i(t) = 2.5 + 2.5 \times 10^{-4}$$

$$v_i(t) = 2.5 + 2.5 \times 10^{-4}$$

This occurs when $t = 2.5 + 2.5 \times 10^{-4}$

Difference in times: $2.5 + 2.5 \times 10^{-4} - 2.5 = 250 \mu s$
Problem 10: 10 Points Possible

Find the time constant for the RC circuit below. DO NOT INCLUDE || symbol in final answer; write the full mathematical expression.

\[ T = R_{eq}C_{eq} \]
\[ = \left( R_1 + \frac{R_2R_3}{R_2+R_3} \right) \left( \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} \right) \]

\[ C_{a1||C_3} = C_2 + C_3 \]

\[ R_2||R_3 = \frac{R_2R_3}{R_2+R_3} \]