Problem 1: Circuit Analysis [15 points in total]

DO NOT SOLVE THE EQUATIONS!

a) 

KCL: The algebraic sum of all currents flowing into a node is zero.

Applying KCL to node a:

\[ \sum I = 0 \]

\[ R_1 I_a - V_a + R_2 I_b = 0 \]

b) 

Applying KCL to node b:

\[ \sum I = 0 \]

\[ \frac{V_a - V_b}{R_3} - \frac{V_b - V_d}{R_4} = 0 \]

Write your equations here.
a) Find $V_c$: [5 pts]

$$V_c = \frac{20 - V_o}{s}$$

$$i_c = 0.5 i_x - \frac{V_o}{10} = 0$$

b) In the circuit below, the independent source values and assignees are known. Use the node-voltage method to write a set of equations for $V_a$ and $V_b$. To receive credit, you MUST SOLVE THE EQUATIONS!

Applying KCL to supernode:

$$V_{HA} - V_a + \frac{0 - V_a}{R_1} + \frac{-I_{BB}}{R_2} = 0$$

Constraint imposed by floating voltage source:

$$V_b - V_a = V_{BB}$$

Applying KCL to node $A$:

$$i_x - 0.5 i_x - \frac{V_o}{10} = 0$$

$$i_x - \frac{V_o}{5} = 0$$

$$20 - V_o - V_b = 0$$

$$20 - 2V_o = 0$$

$$20 = 2V_o$$

$$10 = V_o$$
Problem 2: Equivalent Circuits [21 points in total]
a) Suppose you are given five resistors, each of value 10 kΩ.
   i) What is the maximum resistance which can be achieved by connecting these five resistors? Show how they should be connected in this case. [3 pts]
   Connect the resistors in series, in order to achieve the largest equivalent resistance. \( R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} = 50 \text{ k}\Omega \)

   Circuit diagram of resistors connected to give a maximum resistance value of 50 kΩ:

   \[
   10k\Omega \ || \ 10k\Omega \ || \ 10k\Omega \ || \ 10k\Omega \ || \ 10k\Omega
   \]

   ii) What is the minimum resistance which can be achieved by connecting these five resistors? Show how they should be connected in this case. [3 pts]
   Connect the resistors in parallel, in order to achieve the smallest equivalent resistance.
   \( R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} = \frac{1}{2k\Omega} \)

   Circuit diagram of resistors connected to give a minimum resistance value of 2 kΩ:

   \[
   10k\Omega \ || \ 10k\Omega \ || \ 10k\Omega \ || \ 10k\Omega \ || \ 10k\Omega
   \]

b) Find the Thévenin equivalent circuit for the circuit below. [6 pts]

   \[ 
   V_{Th} \text{ is just the open-circuit voltage } V_{oc}: 
   \]

   \[
   1A \rightarrow 1V \rightarrow 2A \rightarrow 3 \Omega \rightarrow 6A \rightarrow V_{oc} \rightarrow 1A \rightarrow 10V \rightarrow 2A \rightarrow 2\Omega \rightarrow V_{oc} 
   \]

   Using the voltage-divider formula: \( V_{oc} = \frac{2A}{2A + 2\Omega} (10V) \)
   \( V_{oc} = 5V \)

To find \( R_{Th} \), set all the independent sources to zero:
- current source \( \rightarrow \) open circuit
- voltage source \( \rightarrow \) short circuit

\[
V_{th} = 5 \text{ Volts} \\
R_{Th} = 1 \text{ \Omega}
\]

\[
2A \rightarrow 3\Omega \rightarrow 6A \rightarrow 2\Omega \rightarrow 2\Omega/2\Omega = 1 \text{ \Omega}
\]
ii) The I-V characteristic of a linear circuit is given below. Find the Thévenin equivalent of this circuit. [3 pts]

![Thévenin circuit diagram]

When $i = 0$, $v = V_{Th}$: From the I-V plot, $v = -2V$ when $i = 0$.
Therefore, $V_{Th} = -2V$

When $v = 0$, $i = -\frac{V_{Th}}{R_{Th}}$;

From the I-V plot, $i = 1mA$ when $v = 0$.
$1mA = -\frac{-2V}{R_{Th}}$

$R_{Th} = 2k\Omega$

iii) Suppose the circuit in part (c) is loaded with a resistor of resistance $R_L$ (connected between terminals $c$ and $d$). What is the maximum power that can be delivered to this load resistor? [3 pts] (Hint: You should choose the value of $R_L$ which results in the maximum power absorbed by the load resistor, and then calculate that power.)

From the maximum power transfer theorem, $R_L = R_{Th}$, for maximum power to be absorbed by $R_L$.

$P_{max} = 0.5 \text{ mW}$

Problem 3: Op Amp Circuit [14 points in total]
Consider the op amp circuit below:

*ideal op amp:*

$V_n = V_p = 0V$
$i_n = 0A$
$i_p = 0A$

![Op amp circuit diagram]

a) Assuming the op amp is operating in its linear region, find an expression for $v_{out}$ (as a function of $v_{in}$). [10 pts]

Applying KCL to inverting input node:

$\frac{v_{in} - 0}{1k\Omega} + \frac{0 - v_x - 0}{2k\Omega} = 0$

$\Rightarrow v_x = -2v_{in}$

Applying KCL to node $X$:

$0 - \frac{v_x}{2k\Omega} + \frac{0 - v_x}{2k\Omega} + \frac{v_{out} - v_x}{2k\Omega} = 0$

$\Rightarrow v_{out} = 3v_x = 3(-2v_{in})$

$v_{in} = -6v_{in}$

b) Sketch the voltage transfer characteristic for the op amp circuit, for $v_{in}$ ranging from -5 Volts to +5 Volts. Indicate the minimum and maximum values of $v_{out}$ [4 pts]

Due to the positive and negative power supply voltages, $-12V \leq v_{out} \leq 12V$