Problem 1

c) Start off with what we know:
Bulb rated for power = 20W and works with voltage supply = 12 V DC
Model our circuit as shown in the figure below:
- Voltage supply is modeled as an ideal independent voltage source supplying 12V
- Light bulb is modeled as a resistor

Then \( P = VI \) or \( P = \frac{V^2}{R} \) so solving for \( R \), we get \( R = \frac{V^2}{P} \) which implies:
\[
R = \frac{144}{20} \text{ [J/C]}^2\text{/[J/s]} = 7.2 \text{ [J/C]/(C/s)} = 7.2 \Omega
\]
Similarly, for a light bulb rated for 50W operation, \( R = \frac{144}{50} \text{ [V/A]} = 2.9 \Omega \)

b) Note that we have three resistors in series so \( R_{eq} = 3 \cdot R = 3 \times 7.2 \Omega = 21.6 \Omega \).
Using KVL for the loop shown we get: \( I \cdot R_{eq} - 12 \text{ [V]} = 0 \) which implies that:
\[
I = \frac{12}{21.6} \text{ [A]} = 0.56 \text{ A}
\]
Then \( V = IR = 0.56 \times 7.2 \text{ [A] [\Omega]} = 4 \text{ V} \) and the power dissipated in each bulb is given by:
\[
P = VI = 4 \times 0.56 \text{ [V] [A]} = 2.2 \text{ W}
\]

c) As in part b) we use KVL for the loop but replace the last resistance with a 50W-rated bulb as shown in the figure of the problem. We get the following equation:
\[
7.2I + 7.2I + 2.9I \ [\Omega \cdot A] - 12 \ [V] = 0 \] which implies \( I = 0.7 \text{ A} \), and
\[
V = IR50 = 0.7 \times 2.9 = 2 \text{ V} \]. The power dissipated in the 50W-bulb is:
\[
P = VI = 2 \times 0.7 \text{ A} = 1.4 \text{ W}
\]
Problem 2

To find the voltage $v_y$, we first need to find the current $i_1$ and then perform KVL in loop 2. To find $i_1$, we perform KCL at node 1 and then use KVL in loop 1.

**KCL at node 1:** $i_\beta + 29i_\beta = i_1$ which implies $i_1 = 30i_\beta$

**KVL at loop 1:** $i_\beta \times 10\, [k\Omega] - 0.8[V] + 30i_\beta \times 200[\Omega] - 15.2[V] = 0$ which gives $16\, 000\, [\Omega] \times i_\beta = 16\, [V]$ which implies $i_\beta = 1\, mA$

**KVL at loop 2:** $30 mA \times 200[\Omega] - 25[V] + 29 mA \times 500[\Omega] + v_y = 0$ which gives $6 - 25 + 14.5 + v_y = 0$ which implies $v_y = 4.5\, V$

**b)**

<table>
<thead>
<tr>
<th>Element</th>
<th>Power generated</th>
<th>Power absorbed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 k\Omega resistor</td>
<td>$P = I^2R: 1, [mA^2] \times 10, k\Omega = 0.01, W$</td>
<td></td>
</tr>
<tr>
<td>200 \Omega resistor</td>
<td>$P = I^2R: 30^2, [mA^2] \times 200, \Omega = 0.18, W$</td>
<td></td>
</tr>
<tr>
<td>500 \Omega resistor</td>
<td>$P = I^2R: 29^2, [mA^2] \times 500, \Omega = 0.42, W$</td>
<td></td>
</tr>
<tr>
<td>Dep. current source</td>
<td>$P = IV: 29, mA \times 4.5, V = 0.13, W$</td>
<td></td>
</tr>
<tr>
<td>15.2 V source</td>
<td>$P = IV: 1, mA \times 15.2, V = 0.015, W$</td>
<td></td>
</tr>
<tr>
<td>0.8 V source</td>
<td>$P = IV: 1, mA \times 0.8, V = 0.0008, W$</td>
<td></td>
</tr>
<tr>
<td>25 V source</td>
<td>$P = IV: 29, mA \times 25, V = 0.72, W$</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>0.74 W</strong></td>
<td><strong>0.74 W</strong></td>
</tr>
</tbody>
</table>

Problem 3

**c)** We know for any two resistances in parallel we have $R_1/R_2/R_1 + R_2$ (see equation 3.15 of textbook); if $R_1=R_2=R$ then the expression reduces to $R^2/2R$ which is equivalent to $R/2$.

For $n$ resistors of value $R$ in parallel, we obtain the following:

$1/Req = 1/R + 1/R + 1/R + \ldots + 1/R$ which implies that $1/Req = n/R$ or $Req = R/n$.

**c)** We need an equivalent resistance of 5.5 k\Omega using only 2 k\Omega resistors. To get 0.5 k\Omega we can put 4 2k\Omega resistances in parallel: $Req = 2/4 = 0.5\, k\Omega$. Now we need to construct 5 k\Omega circuit that we can put in series with the 0.5 k\Omega circuit. This can be obtained by putting 2 2k\Omega resistors in series to get 4 k\Omega and 2 2k\Omega in parallel which can be put in series with the rest of the network. Our network is shown below; all the resistors shown are 2k\Omega.
Problem 4

a)

KVL at loop 2: $30000 \times i_2 + 120000 \times i_2 - 75000 \times i_1 = 0$ implies that $i_1 = 2 \times i_2$

Then KCL at node a: $i = i_1 + i_2$ implies that $i = 3 \times i_2$

KVL at loop 1: $25000 \times i + 75000 \times i_1 - 240 = 0$ and substituting for $i_1$ and $i$ in terms of $i_2$, we obtain $225 \times i_2 = 240$ which implies $i_2 = 1.07$ mA.

Then $v_o = i_2 \times 120000 = 1.07 \text{mA} \times 120000 = 128$ V.

b)

Using the voltage divider formula, the voltage across the 75kΩ resistor is given by: $(75/100) \times 240 \text{V} = 180 \text{V}$. This implies that: $i = 180/75000 = 2.4$ mA.

Then the dependent voltage source is $75000 \times 2.4 = 180 \text{V}$. Using the voltage divider formula again for the second part of the circuit, $v_o$ is given by $(120/150) \times (75000 \times i) = 144$ V.

c) It has no effect since the current $i_1 = 0$. 
Problem 5

First reduce the circuit given to the one below using series and parallel resistance rules. 2R||2R is simply R. Then R in series with R is 2R. 3R||3R is 3R/2. R||4R is 4R/5. Then 3R/2 in series with 4R/5 is 23R/10. Then we obtain the $i_2$ in terms of $i$ using the current divider rule: $i_2 = i \times \frac{2R}{2R + 23R/10} = \frac{20i}{43}$.

Then we use the current divider rule again for the portion of the circuit on the right shown below

So we get $i* = i_2 \times \frac{4R}{5R} = (\frac{20i}{43}) \times (\frac{4}{5}) = \frac{16i}{43}$