Homework #4 Solutions

Problem 1

Op amp is ideal operating in linear region:

\( v_p = v_n \)
\( i_n = i_p = 0 \)
\( P = v_{16k\Omega}^2/R \)
\( v_o = v_{16k\Omega} \)

(a) KCL at +ve terminal:

\( i_p = i_{48k\Omega} = 0 \)
\((320\text{mV} - v_p)/48k\Omega = 0\)
\( v_p = 320\text{mV} \)
\( v_o = v_n = v_o \)
\( P = (320\text{mV})^2/16k\Omega \)
\( P = 6.4\mu\text{W} \)

(b)
This is a simple voltage divider circuit; \( v_o = (320\text{mV} \times 16k\Omega)/64k\Omega = 80\text{mV} \)
which means that \( P = (80\text{mV})^2/16k\Omega = 0.40\mu\text{W} \)

c) \( 6.4\mu\text{W} / 0.40\mu\text{W} = 16 \)

d) Yes. This is like the unity-gain voltage follower circuit; the weak voltage
source at the input can drive a heavy load at the output (like the 16k\(\Omega\)) with the
op amp providing the power.

**Problem 2**

\[ \text{[Diagram of Op Amp Circuit]} \]

a) Op amp is ideal and operating in linear region
\( v_s = v_p = v_n; i_n = i_p = 0 \)

KCL at –ve terminal:
\[ i_n = i_{2k\Omega} + i_{20k\Omega} = 0 \]
\[ (1 - v_n) / 2k\Omega + (v_{\text{out}} - v_n) / 20k\Omega = 0 \]
\[ (v_s - 1) / 2k\Omega = (v_{\text{out}} - v_s) / 20k\Omega \]
\[ 20v_s - 20 = 2v_{\text{out}} - 2v_s \]
\[ v_o = 11v_s - 10. \]

\[
\begin{array}{c|c}
  v_s & v_{\text{out}} = 11v_s - 10 \\
  \hline
  0 & 0 \\
  1 & 1 \\
  1.2 & 3.2 \\
  1.4 & 5.4 \\
  1.6 & 7.6 \\
  1.8 & 9.8 \\
  2 & 10 \\
  3 & 10 \\
  4 & 10 \\
\end{array}
\]
Problem 3

Op amp is ideal and operates in linear region.
\( v_n = v_p; \ i_n = i_o = 0. \)
\( R_a = 1k\Omega \)

KCL at -ve terminal:
\[ i_{10k} + i_{Rf} = i_n = 0 \]
\[ 0 = v_n/10k\Omega + (v_o - v_n)/R_f = 0 \]
\[ v_o = v_n(10k + R_f)/10k \]
\[ \Rightarrow v_n = v_o10k/(10k + R_f) \] [Note this is simple voltage division]

Let \( z = 10k/(10k + R_f) \)

Now use superposition to find contribution of each voltage source to \( v_p. \)
Note contribution by \( v_a \) to \( v_o \) is \( 4v_a: \)
\[ \Rightarrow v_n = 4v_a z \]

Contribution of \( v_a \) to \( v_o \) is given by:
\[ R_b || R_c \cdot v_a / (R_a + R_b || R_c) \] by voltage division
\[ \Rightarrow R_b || R_c / (R_a + R_b || R_c) = 4z \]

Similarly, for \( v_b \) which contributes \( 2v_b \) to \( v_o: \)
Contribution of $v_b$ to $v_p$ is given by:
$$R_a || R_c \cdot v_b / (R_b + R_a || R_c) \text{ by voltage division}$$
$$\Rightarrow R_a || R_c / (R_b + R_a || R_c) = 2z$$

And, for $v_c$ which contributes $v_c$ to $v_o$:
Contribution of $v_c$ to $v_p$ is given by:
$$R_a || R_b \cdot v_c / (R_c + R_a || R_b) \text{ by voltage division}$$
$$\Rightarrow R_a || R_b / (R_c + R_a || R_b) = z$$

Then using the above equations we can solve for $R_b$ and $R_c$:
$$\Rightarrow R_b || R_c / (R_a + R_b || R_c) = 4 \cdot R_a || R_b / (R_c + R_a || R_b)$$
$$\Rightarrow R_b R_c / (R_a R_b + R_a R_c + R_b R_c) = 4 R_a R_b / (R_a R_b + R_a R_c + R_b R_c) \cdot$$
$$\Rightarrow R_c = 4R_e$$
$$\Rightarrow R_e = 4k\Omega$$

$$R_a || R_c / (R_b + R_a || R_c) = 2 \cdot R_a || R_b / (R_c + R_a || R_b)$$
$$\Rightarrow R_a R_c / (R_a R_b + R_a R_c + R_b R_c) = 2 R_a R_b / (R_a R_b + R_a R_c + R_b R_c) \cdot$$
$$\Rightarrow R_c = 2R_b$$
$$\Rightarrow R_b = 2k\Omega$$

Then $z = 10k / (10k + R_f) = R_a || R_b / (R_c + R_a || R_b) = 1k || 2k / (4k + 1k || 2k) = 1/7$
$$\Rightarrow R_f = 60k\Omega$$

**Problem 4**

a) Redraw the circuit as follows:

```
\begin{center}
\includegraphics[width=0.5\textwidth]{circuit.png}
\end{center}
```

Note: $V_p = 0$; $V_n = V_a$; $V_b = V_o$
KCL at node a:
$$V_a / 400k + (V_a - V_b) / 320k = (V_g - V_a) / 8k$$
$$\Rightarrow V_a + 1.25V_a - 1.25V_b = 50V_g - 50V_a$$
$$\Rightarrow V_g = 1.045V_a - 0.025V_b$$
KCL at node b:
\[(Va - Vb)/320k + (A(-Vn) - Vb)/2k = Vb/1k\]
\[\Rightarrow (1 - 160A)Va - 161Vb = 320Vb\]
\[\Rightarrow Va = 481Vb/(1-160A)\]

\[Vg = 1.045[481Vb/(1-160A)] - 0.025Vb\]
\[Vg = -0.0250062Vo\]
\[Vo/Vg = -39.99\]
\[Vo/Vg = -40\]

Another solution is to use equation 5.49 in the textbook to determine Vo:
\[Vo/Vg = -A + (Ro/Rf) / (Rs/Rf)(1 + A + Ro/Ri + Ro/RL)+(1 +Ro/RL)(1+Rs/Ri) + Ro/Rf\]
Substituting the values for all the variables, we get:
\[Vo/Vg = -500 000 + (2k/320k) / (8k/320k)(1 + 500 000 + 2k/400k + 2k/1k)+(1 +2k/1k)(1+8k/400k) + 2k/320k\]
\[Vo/Vg = -39.99\]
\[Vo/Vg = -40\]

b) Ideal op amp in linear region; \(Vn = Vp; \text{In}=Ip=0\); we can directly apply the result for an inverting amplifier as derived in class:
\[Vo = -Rf \times Vs/Rs\]
\[Rf = 320k\Omega, Rs = 8k\Omega, Vs = Vg\]
\[Vo/Vg = -320/8 = -40\]