Lecture #6

ANNOUNCEMENT

- Check the Master Schedule posted online for updated information on TA sections & office hours

OUTLINE

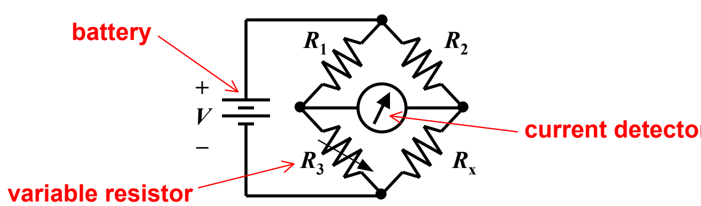
- The Wheatstone bridge circuit
- Delta-to-Wye equivalent circuits
- Node-voltage circuit analysis method

Reading

Finish Chapter 3, Chapter 4.1-4.2

The Wheatstone Bridge

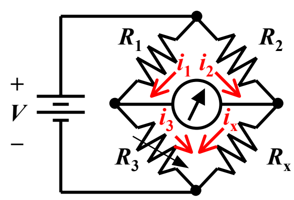
- Circuit used to precisely measure resistances in the range from 1 Ω to 1 MΩ, with ±0.1% accuracy
  - R_1 and R_2 are resistors with known values
  - R_3 is a variable resistor (typically 1 to 11,000Ω)
  - R_x is the resistor whose value is to be measured



Finding the value of R_x

- Adjust R_3 until there is no current in the detector

Then, $\boxed{R_x = \frac{R_2}{R_1} R_3}$



Typically, R_2 / R_1 can be varied from 0.001 to 1000 in decimal steps

Derivation:

KCL => $i_1 = i_3$ and $i_2 = i_x$

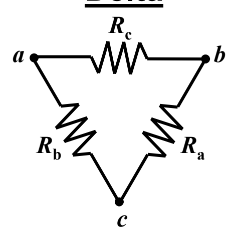
KVL => $i_3 R_3 = i_x R_x$ and $i_1 R_1 = i_2 R_2$

$i_1 R_3 = i_2 R_x$

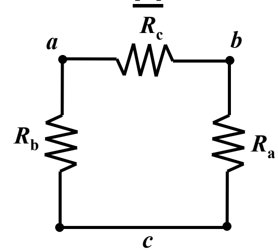
$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

Delta (Pi) and Wye (Tee) Interconnections

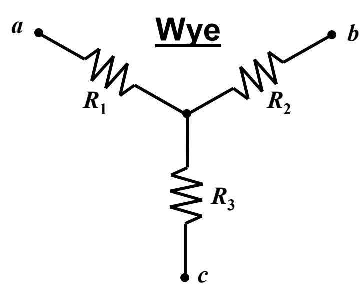
Delta



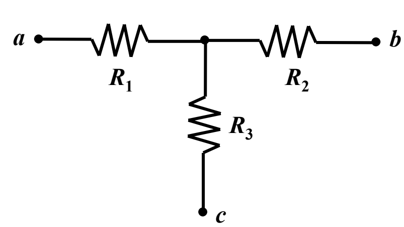
Pi



Wye

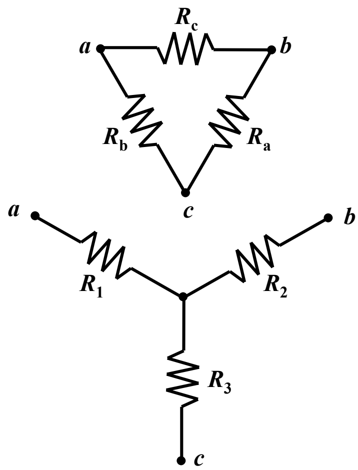


Tee



Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

- In order for the Delta interconnection to be equivalent to the Wye interconnection, the resistance between corresponding terminal pairs must be the same



$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

$$R_{bc} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3$$

$$R_{ca} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} = R_1 + R_3$$

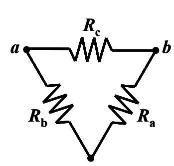
Δ-Y and Y-Δ Conversion Formulas

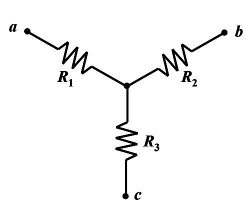
Delta-to-Wye conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$





Wye-to-Delta conversion

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Resistive Circuits: Summary

- Equivalent resistance of ***k*** **resistors in series**:

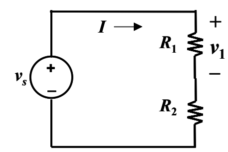
$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k$$

- Equivalent resistance of ***k*** **resistors in parallel**:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}$$

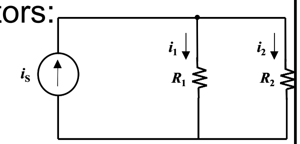
- Voltage divided between 2 series resistors:

$$v_1 = \frac{R_1}{R_1 + R_2} v_s$$



- Current divided between 2 parallel resistors:

$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$



Node-Voltage Circuit Analysis Method

1. **Choose a reference node** ("ground")

   Look for the one with the most connections!

2. **Define unknown node voltages**

   those which are not fixed by voltage sources

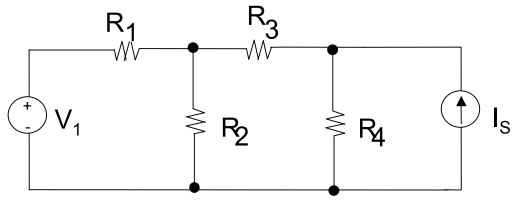
3. **Write KCL at each unknown node**, expressing current in terms of the node voltages (using the *I*-*V* relationships of branch elements)

   Special cases: floating voltage sources

4. **Solve the set of independent equations**

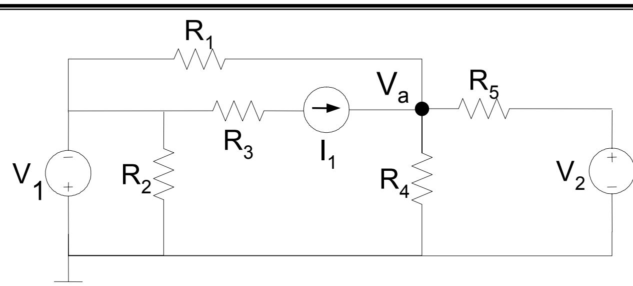
   N equations for *N* unknown node voltages

Nodal Analysis: Example #1



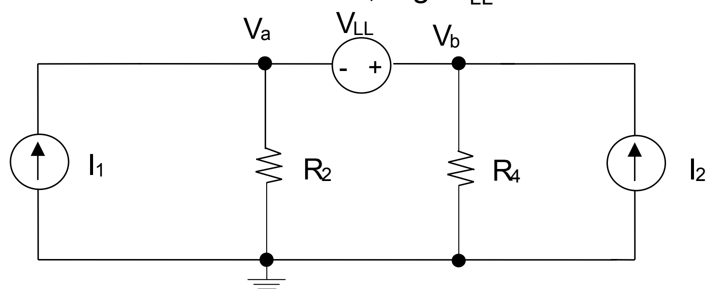
1. Choose a reference node.
2. Define the node voltages (except reference node and the one set by the voltage source).
3. Apply KCL at the nodes with unknown voltage.
4. Solve for unknown node voltages.

Nodal Analysis: Example #2



Nodal Analysis w/ "Floating Voltage Source"

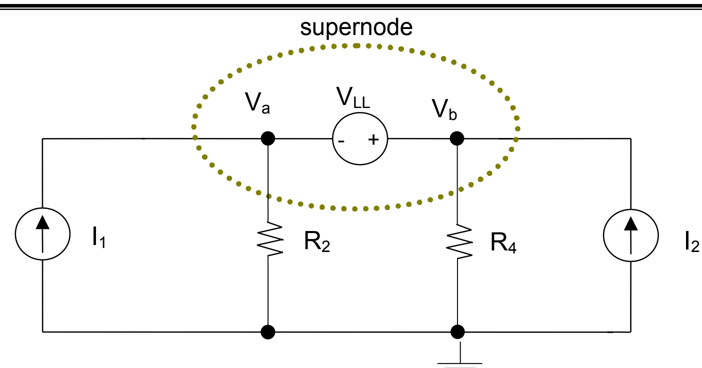
A "floating" voltage source is one for which neither side is connected to the reference node, e.g. V_{LL} in the circuit below:



Problem: We cannot write KCL at nodes a or b because there is no way to express the current through the voltage source in terms of V_a-V_b.

Solution: Define a "supernode" – that chunk of the circuit containing nodes a and b. Express KCL for this supernode.

Nodal Analysis: Example #3



Eq'n 1: KCL at supernode

Eq'n 2: Property of voltage source: