## Lecture \#8

## ANNOUNCEMENTS

- HW\#2 solutions, HW\#3 are posted online
- Change in Farhana's O.H. : Th 5-6 instead of Mo 3-4
- Prof. King will be away next Monday \& Wednesday
- Guest lecturer: Prof. Neureuther
- Prof. King's office hour on Wed. 9/17 cancelled


## OUTLINE

- Thévenin and Norton equivalent circuits
- Maximum power transfer
- Superposition


## Reading

Chapter 4.10-4.13

## Thévenin Equivalent Circuit

- Any network of voltage sources, current sources, and resistors can be replaced by an equivalent circuit consisting of an independent voltage source in series with a resistor without affecting the operation of the rest of the circuit.



## $I-V$ Characteristic of Thévenin Equivalent

- The I-V characteristic for the series combination of elements is obtained by adding their voltage drops:

For a given current $i$, the voltage drop
$v_{a b}$ is equal to the sum of the voltages dropped across the source ( $\boldsymbol{V}_{\mathbf{T h}}$ ) and the across the resistor ( $\boldsymbol{i R}_{\mathbf{T h}}$ )

$I-V$ characteristic of resistor: $\boldsymbol{v}=\boldsymbol{i} \boldsymbol{R}$
$I-V$ characteristic of voltage source: $v=V_{\mathrm{Th}}$

## Finding $V_{\text {Th }}$ and $R_{\text {Th }}$

Only two points are needed to define a line. Choose two convenient points:

1. Open circuit across terminals $\mathrm{a}, \mathrm{b}$

$$
i=0, v_{\mathrm{ab}} \equiv v_{\mathrm{oc}}
$$


2. Short circuit across terminals $\mathrm{a}, \mathrm{b}$

$$
\boldsymbol{v}_{\mathrm{ab}}=\mathbf{0}, i \equiv-i_{\mathrm{sc}}=-V_{\mathrm{Th}} / \boldsymbol{R}_{\mathrm{Th}}
$$




## Calculating a Thévenin Equivalent

1. Calculate the open-circuit voltage, $v_{o c}$

2. Calculate the short-circuit current, $\boldsymbol{i}_{\mathrm{sc}}$

- Note that $i_{\mathrm{sc}}$ is in the direction of the open-circuit voltage drop across the terminals a,b!


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## Thévenin Equivalent Example

Find the Thevenin equivalent with respect to the terminals $\mathrm{a}, \mathrm{b}$ :


## Alternative Method of Calculating $\boldsymbol{R}_{\mathrm{Th}}$

For a network containing only independent sources and resistors:

1. Set all independent sources to zero voltage source $\rightarrow$ short circuit current source $\rightarrow$ open circuit
network of
independent
sources and
resistors, with
each source
set to zero
2. Find equivalent resistance $\boldsymbol{R}_{\mathrm{eq}}$ between the terminals

$$
R_{\mathrm{Th}}=R_{e q}
$$

For a network containing dependent sources: $I_{\text {TEst }}$

1. Set all independent sources to zero
2. Apply a test voltage source $\mathrm{V}_{\text {TEST }}$
3. Calculate $\mathrm{I}_{\text {TEST }}$
$R_{\mathrm{Th}}=\frac{V_{\mathrm{TEST}}}{I_{\mathrm{TEST}}}$


## $R_{\text {Th }}$ Calculation Example \#1



Set all independent sources to 0 :

## Comments on Dependent Sources

A dependent source establishes a voltage or current whose value depends on the value of a voltage or current at a specified location in the circuit.
(imaginary device, used to model behavior of transistors \& amplifiers)
To specify a dependent source, we must identify:

1. the controlling voltage or current (must be calculated, in general)
2. the relationship between the controlling voltage or current and the supplied voltage or current
3. the reference direction for the supplied voltage or current

The relationship between the dependent source and its reference cannot be broken!

- Dependent sources cannot be turned off for various purposes (e.g. to find the Thévenin resistance).


## $\boldsymbol{R}_{\text {Th }}$ Calculation Example \#2

Find the Thevenin equivalent with respect to the terminals $\mathrm{a}, \mathrm{b}$ :


## Networks Containing Time-Varying Sources

Care must be taken in summing time-varying sources!
Example:

$V_{\mathrm{Th}}=\frac{1 \mathrm{k} \Omega}{1 \mathrm{k} \Omega+1 \mathrm{k} \Omega}[20 \cos (100 t)]+10 \sin (100 t)=10 \sqrt{2} \sin \left(100 t+90^{\circ}\right)$
$R_{\mathrm{Th}}=1 \mathrm{k} \Omega \| 1 \mathrm{k} \Omega=500 \Omega$

## Norton Equivalent Circuit

- Any network of voltage sources, current sources, and resistors can be replaced by an equivalent circuit consisting of an independent current source in parallel with a resistor without affecting the operation of the rest of the circuit.



## Finding $I_{\mathrm{N}}$ and $\boldsymbol{R}_{\mathrm{N}}$

- We can derive the Norton equivalent circuit from a Thévenin equivalent circuit simply by making a source transformation:

$i_{\mathrm{L}}=\frac{v_{\mathrm{Th}}}{R_{\mathrm{Th}}+R_{\mathrm{L}}}$

$$
R_{\mathrm{N}}=R_{\mathrm{Th}}=\frac{v_{\mathrm{oc}}}{i_{\mathrm{sc}}} ; \quad i_{\mathrm{N}}=\frac{v_{\mathrm{Th}}}{R_{\mathrm{Th}}}=i_{\mathrm{sc}}
$$

## Maximum Power Transfer Theorem

Thévenin equivalent circuit


Power absorbed by load resistor:

$$
p=i_{\mathrm{L}}^{2} R_{\mathrm{L}}=\left(\frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}}+R_{\mathrm{L}}}\right)^{2} R_{\mathrm{L}}
$$

To find the value of $R_{\mathrm{L}}$ for which $p$ is maximum, set $\frac{d p}{d R_{L}}$ to 0 :

$$
\begin{aligned}
\frac{d p}{d R_{L}} & =V_{\mathrm{Th}}^{2}\left[\frac{\left(R_{\mathrm{Th}}+R_{\mathrm{L}}\right)^{2}-R_{\mathrm{L}} \times 2\left(R_{\mathrm{Th}}+R_{\mathrm{L}}\right)}{\left(R_{\mathrm{Th}}+R_{\mathrm{L}}\right)^{4}}\right]=0 \\
& \Rightarrow\left(R_{\mathrm{Th}}+R_{\mathrm{L}}\right)^{2}-R_{\mathrm{L}} \times 2\left(R_{\mathrm{Th}}+R_{\mathrm{L}}\right)=0 \\
& \Rightarrow R_{\mathrm{Th}}=R_{\mathrm{L}} \quad \begin{array}{l}
\text { A resistive load receives maximum power from a circuit if the } \\
\text { load resistance equals the Thévenin resistance of the circuit. }
\end{array}
\end{aligned}
$$

## Superposition

A linear circuit is constructed only of linear elements

- can be described by a linear differential equation


## Principle of Superposition:

- In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.
Note: Superposition cannot be used to find power!
- This principle is useful for analysis of op-amp circuits.


## Procedure:

1. Determine contribution due to an independent source

- Set all other sources to 0

2. Repeat for each independent source
3. Sum individual contributions to obtain desired voltage or current

## Superposition Example

- Find $V_{0}$


