Lecture #11

ANNOUNCEMENTS

• Homework Assignment #4 will be posted today
• Midterm #1: Monday Sept. 29th (11:10AM-12:00PM)
  – closed book; one page (8.5”x11”) of notes & calculator allowed
  – covers Chapters 1-5 in textbook (HW#1-4)
• Midterm Review Session: Friday 9/26 7-9PM, 277 Cory
• Extra office hours:
  – Steve: 9/26 from 12-2PM
  – Farhana: 9/27 from 1-3PM, 9/28 from 9-11AM
• Practice problems and old exam are posted online

OUTLINE

– Review: op amp circuit analysis
– The capacitor (Chapter 6.2 in text)

Review: Op Amp Circuit Analysis

Procedure:
1. Assume that the op amp is ideal
   a) Apply KCL at (+) and (–) terminals, noting \( i_p = 0 \) & \( i_n = 0 \)
   b) Note that \( v_n = v_p \)
   c) Write an expression for \( v_o \)
2. Calculate \( v_o \)
3. Check: Is the op-amp operating in its linear region?
   If \( V^- \leq v_o \leq V^+ \), then the assumption is valid.
   If calculated \( v_o > V^+ \), then \( v_o \) is saturated at \( V^+ \)
   If calculated \( v_o < V^- \), then \( v_o \) is saturated at \( V^- \)
Op Amp Circuit Analysis Example

Consider the following circuit:
Assume the op amp is ideal.

a) Calculate $v_o$ if $v_s = 100$ mV
b) What is the **voltage gain** $v_o/v_s$ of this amplifier?
c) Specify the range of values of $v_s$ for which the op amp operates in a linear mode

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Op Amp Circuit Analysis Example cont’d.

What if the op amp is not ideal?

- $R_i = 10$ kΩ
- $R_o = 1$ kΩ
- $A = 10^3$
Re-draw the circuit & analyze:

KCL @ node a:

KCL @ node b:

\[ -v_o \approx 9.87 < 10 \]
\[ v_s \]

Effect of Load Resistance \( R_L \)

KCL @ node b:

\[ -\frac{v_o}{v_s} \approx 9.75 < 9.87 \]

- For an ideal op amp (\( R_o = 0 \Omega \)), \( v_o \) does not depend on the “load”. However, for a realistic op amp, it does.
The Capacitor

Two conductors (a,b) separated by an insulator:
- difference in potential = $V_{ab}$
- $\Rightarrow$ equal & opposite charge $Q$ on conductors

$$Q = CV_{ab}$$  
(stored charge in terms of voltage)

where $C$ is the capacitance of the structure,
- positive (+) charge is on the conductor at higher potential

**Parallel-plate capacitor:**
- area of the plates = $A$
- separation between plates = $d$
- dielectric permittivity of insulator = $\varepsilon$

$\Rightarrow$ capacitance

$$C = \frac{A\varepsilon}{d}$$

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Symbol: 

| C | or | C |

**Units:** Farads (Coulombs/Volt)

(typical range of values: 1 pF to 1 µF)

**Current-Voltage relationship:**

$$i_c = \frac{dQ}{dt} = C\frac{dv_c}{dt} + v_c\frac{dC}{dt}$$

**Note:** $v_c$ must be a continuous function of time
Voltage in Terms of Current

\[ Q(t) = \int_{0}^{t} i_c(t) \, dt + Q(0) \]
\[ v_c(t) = \frac{1}{C} \int_{0}^{t} i_c(t) \, dt + \frac{Q(0)}{C} = \frac{1}{C} \int_{0}^{t} i_c(t) \, dt + v_c(0) \]

Stored Energy

You might think the energy stored on a capacitor is \( QV \), which has the dimension of Joules. But during charging, the average voltage across the capacitor was only half the final value of \( V \).

Thus, energy is

\[ \frac{1}{2} QV = \frac{1}{2} CV^2 \]

**Example:** A 1 pF capacitance charged to 5 Volts has \( \frac{1}{2}(5V)^2 \ (1pF) = 12.5 \ pJ \)
A more rigorous derivation

\[ i_c \]

\[ v_c \]

\[ t = t_{\text{Final}} \]
\[ v = V_{\text{Final}} \]
\[ t = t_{\text{Initial}} \]
\[ v = V_{\text{Initial}} \]

\[ w = \int v_c \cdot i_c \, dt = \int v_c \, dQ = \int v_c \, dt = V_{\text{Final}} - V_{\text{Initial}} \]

\[ v = \frac{1}{2} CV_{\text{Final}}^2 - \frac{1}{2} CV_{\text{Initial}}^2 \]

Integrating Amplifier

\[ v_o(t) = -\frac{1}{RC} \int_0^t v_{IN}(t) \, dt + v_C(0) \]