Lecture #12

ANNOUNCEMENTS
• Graduate school workshop tomorrow (9/23)
  – 3-4 PM, Wozniak Lounge (Soda Hall)

OUTLINE
– Capacitors in series and in parallel
– Practical capacitors
– The inductor
– Inductors in series and in parallel

Reading
Chapter 6.1-6.3

Correction to Lecture 9, Slide 3
• If there are no independent sources in a circuit, \( V_{Th} = 0 \).
  – If there are dependent sources in the circuit, we need to apply an external voltage in order to determine \( R_{Th} \).
  
Example: Circuit used in \( R_{Th} \) Calculation Example #2, Lecture 8:

\[
\begin{align*}
\text{Applying KCL to node } x: & \quad V_x - 40i_A + V_x - V_{\text{TEST}} = 0 \\
\text{Definition of } i_A: & \quad i_A = -\frac{V_x}{80} \\
\text{\Rightarrow } & \quad V_x = \frac{8}{25}V_{\text{TEST}}
\end{align*}
\]

\[
R_{Th} = \frac{V_{\text{TEST}}}{I_{\text{TEST}}} = \frac{75}{4} \Omega
\]
Example: Current, Power & Energy for a Capacitor

\[ v(t) = \frac{1}{C} \int_{0}^{t} i(\tau) d\tau + v(0) \]

\[ i = C \frac{dv}{dt} \]

\[ v_c \text{ must be a continuous function of time; however, } i_c \text{ can be discontinuous.} \]

Note: In “steady state” (dc operation), time derivatives are zero \( \rightarrow C \text{ is an open circuit} \)

\[ p = vi \]

\[ w = \int_{0}^{t} pd\tau = \frac{1}{2} Cv^2 \]
Capacitors in Parallel

\[ i = i_1 + i_2 = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} \]
\[ i = (C_1 + C_2) \frac{dv}{dt} = C_{eq} \frac{dv}{dt} \]

Equivalent capacitance of capacitors in parallel is the sum

\[ C_{eq} = C_1 + C_2 \]

Capacitors in Series

\[ i = C_1 \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt} \]
\[ \Rightarrow \frac{dv_1}{dt} = \frac{i}{C_1} \quad \text{and} \quad \frac{dv_2}{dt} = \frac{i}{C_2} \]
\[ i = C_{eq} \frac{dv_1}{dt} + C_{eq} \frac{dv_2}{dt} \]
\[ i = C_{eq} \frac{dv_1}{dt} + C_{eq} \frac{dv_2}{dt} \]
\[ i = C_{eq} \frac{i}{C_1} + C_{eq} \frac{i}{C_2} \Rightarrow \]
\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \]
Capacitive Voltage Divider

Q: Suppose the voltage applied across a series combination of capacitors is changed by $\Delta v$. How will this affect the voltage across each individual capacitor?

$$\Delta v = \Delta v_1 + \Delta v_2$$

Note that no net charge can be introduced to this node. Therefore, $-\Delta Q_1 + \Delta Q_2 = 0$

$$\Rightarrow C_1 \Delta v_1 = C_2 \Delta v_2$$

$$\Delta v_2 = \frac{C_1}{C_1 + C_2} \Delta v$$

Note: Capacitors in series have the same charge.

Application Example: MEMS Accelerometer

- Capacitive position sensor used to measure acceleration (by measuring force on a proof mass)

$$F = kx = ma$$

EECS40, Fall 2003 Lecture 12, Slide 7 Prof. King
Sensing the Differential Capacitance

- Fixed electrodes are biased at $+V_s$ and $-V_s$
- Movable electrode (proof mass) is biased at $V_o$

Circuit model

$V_a = -V_s + \frac{C_1}{C_1 + C_2} (2V_s) = \frac{C_1 - C_2}{C_1 + C_2} V_s$

$V_a = \frac{\varepsilon A}{g_1} - \frac{\varepsilon A}{g_2}$

$V_o = \frac{g_1}{g_2} - \frac{g_1}{g_2} = \frac{g_2 - g_1}{g_2 + g_1} = \frac{g_2 - g_1}{\text{const}}$

Practical Capacitors

- A capacitor can be constructed by interleaving the plates with two dielectric layers and rolling them up, to achieve a compact size.

- To achieve a small volume, a very thin dielectric with a high dielectric constant is desirable. However, dielectric materials break down and become conductors when the electric field (units: V/cm) is too high.
  - Real capacitors have maximum voltage ratings
  - An engineering trade-off exists between compact size and high voltage rating
The Inductor

- An inductor is constructed by coiling a wire around some type of form.

- Current flowing through the coil creates a magnetic field or flux that links the coil: \( Li_L \)

- When the current changes, the magnetic flux changes \( \rightarrow \) a voltage across the coil is induced:

\[
v_L(t) = L \frac{di_L}{dt}
\]

**Note:** In “steady state” (dc operation), time derivatives are zero \( \rightarrow L \) is a short circuit

**Symbol:**

\[
L
\]

**Units:** Henrys (Volts \( \cdot \) second / Ampere)

(typical range of values: \( \mu \text{H} \) to 10 H)

**Current in terms of voltage:**

\[
di_L = \frac{1}{L} v_L(t) dt
\]

\[
i_L(t) = \frac{1}{L} \int_{t_0}^{t} v_L(\tau) d\tau + i(t_0)
\]

**Note:** \( i_L \) must be a continuous function of time
### Stored Energy

Consider an inductor having an initial current $i(t_0) = i_0$

\[ p(t) = v(t)i(t) = Li(t) \frac{di}{dt} \]

\[ w(t) = \int_{t_0}^{t} p(\tau) d\tau = \int_{i_0}^{i} Li \frac{dt}{d\tau} d\tau = \int_{i_0}^{i} L i dt \]

\[ w(t) = \frac{1}{2} Li^2 - \frac{1}{2} Li_0^2 \]

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### Inductors in Series

The equivalent inductance of inductors in series is the sum

\[ v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} = (L_1 + L_2) \frac{di}{dt} = L_{eq} \frac{di}{dt} \]

\[ L_{eq} = L_1 + L_2 \]
**Inductors in Parallel**

\[ i(t) = i_1(t) + i_2(t) = \frac{1}{L_1} \int_{t_0}^{t} v(t') + i_1(t_0) dt' + \frac{1}{L_2} \int_{t_0}^{t} v(t') + i_2(t_0) dt' \]

\[ i = \left[ \frac{1}{L_1} + \frac{1}{L_2} \right] \int_{t_0}^{t} v(t') + \left[ i_1(t_0) + i_2(t_0) \right] dt' \]

\[ \Rightarrow \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \]

with \( i(t_0) = i_1(t_0) + i_2(t_0) \)

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**Summary**

### Capacitor

\[ i = C \frac{dv}{dt} \]

\[ w = \frac{1}{2} Cv^2 \]

\( v \) cannot change instantaneously

\( i \) can change instantaneously

Do not short-circuit a charged capacitor (\( \rightarrow \) infinite current!)

\[ n \text{ cap.'s in series: } \frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i} \]

\[ n \text{ cap.'s in parallel: } C_{eq} = \sum_{i=1}^{n} C_i \]

### Inductor

\[ v = L \frac{di}{dt} \]

\[ w = \frac{1}{2} Li^2 \]

\( i \) cannot change instantaneously

\( v \) can change instantaneously

Do not open-circuit an inductor with current (\( \rightarrow \) infinite voltage!)

\[ n \text{ ind.'s in series: } L_{eq} = \sum_{i=1}^{n} L_i \]

\[ n \text{ ind.'s in parallel: } \frac{1}{L_{eq}} = \sum_{i=1}^{n} \frac{1}{L_i} \]

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