Lecture #14

ANNOUNCEMENTS

• Midterm Exam #1:
  – Monday 9/29, 11:10AM-12noon
  – Closed book; one page of notes & calculator allowed
  – Students with last name beginning L through Z should go directly to Sibley Auditorium (Bechtel Engr. Center)
  – Pizza & drinks afterwards, on the Bechtel Terrace
    • Your constructive feedback is solicited!

OUTLINE

– Natural response of an RC circuit

Reading

Chapter 7.2

Review (Conceptual)

• Any first-order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.

\[ V_{Th} \quad R_{Th} \quad I_{Th} \quad L \quad R_{Th} \quad V_{Th} \quad + \quad C \]

– In steady state, an inductor behaves like a short circuit
– In steady state, a capacitor behaves like an open circuit
Natural Response of an RC Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:

  ![Circuit Diagram]

  **Notation:**
  - $0^-$ is used to denote the time just prior to switching
  - $0^+$ is used to denote the time immediately after switching

- The voltage on the capacitor at $t = 0^-$ is $V_o$

Solving for the Voltage ($t \geq 0$)

- For $t > 0$, the circuit reduces to

  ![Simplified Circuit Diagram]

  - Applying KCL to the RC circuit:

  - Solution: $v(t) = v(0)e^{-t/RC}$
Solving for the Current \((t > 0)\)

- Note that the voltage changes abruptly:
  \[ i(0^-) = 0 \]

For \( t > 0 \), \( i(t) = \frac{v}{R} = \frac{V_o}{R} e^{-t/RC} \)

\[ \Rightarrow i(0^+) = \frac{V_o}{R} \]

Solving for Power and Energy Delivered \((t > 0)\)

\[ p = \frac{v^2}{R} = \frac{V_o^2}{R} e^{-2t/RC} \]

\[ w = \int_0^t p(x)dx = \int_0^t \frac{V_o^2}{R} e^{-2x/RC} dx \]

\[ = \frac{1}{2} CV_o^2 \left(1 - e^{-2t/RC}\right) \]
Time Constant $\tau$

- In the example, we found that
  \[ v(t) = V_o e^{-t/RC} \quad \text{and} \quad i(t) = \frac{V_o}{R} e^{-t/RC} \]

- Define the time constant $\tau = RC$
  - At $t = \tau$, the voltage has reduced to $1/e \sim 0.37$ of its initial value.
  - At $t = 5\tau$, the voltage has reduced to less than 1% of its initial value.

Natural Response Summary

RL Circuit

- Inductor current cannot change instantaneously
  \[ i(0^-) = i(0^+) \]
  \[ i(t) = i(0)e^{-t/\tau} \]
- Time constant $\tau = \frac{L}{R}$

RC Circuit

- Capacitor voltage cannot change instantaneously
  \[ v(0^-) = v(0^+) \]
  \[ v(t) = v(0)e^{-t/\tau} \]
- Time constant $\tau = RC$