Lecture #14

ANNOUNCEMENTS

- **Midterm Exam #1:**
  - Monday 9/29, 11:10AM-12noon
  - Closed book; one page of notes & calculator allowed
  - **Students with last name beginning L through Z should go directly to Sibley Auditorium (Bechtel Engr. Center)**
  - Pizza & drinks afterwards, on the Bechtel Terrace
    - Your constructive feedback is solicited!

OUTLINE

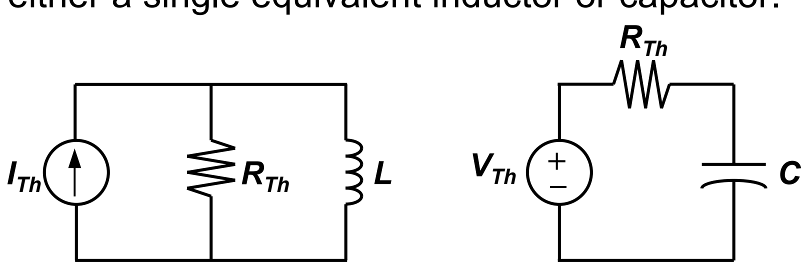
– Natural response of an RC circuit

Reading

Chapter 7.2

Review (Conceptual)

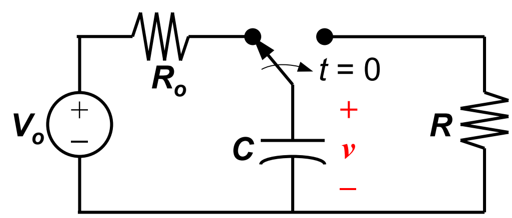
- Any first-order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.



  - In steady state, an inductor behaves like a short circuit
  - In steady state, a capacitor behaves like an open circuit

Natural Response of an RC Circuit

- Consider the following circuit, for which the switch is closed for $t < 0$, and then opened at $t = 0$:



Notation:

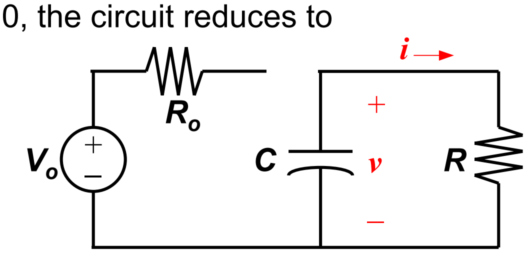
0^- is used to denote the time just prior to switching

0^+ is used to denote the time immediately after switching

- The voltage on the capacitor at $t = 0^-$ is V_o

Solving for the Voltage ($t \geq 0$)

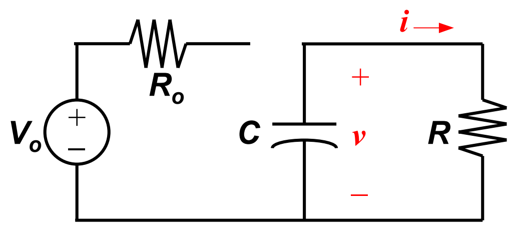
- For t > 0, the circuit reduces to



- Applying KCL to the RC circuit:

- Solution: $v(t) = v(0)e^{-t/RC}$

Solving for the Current (t > 0)



$$v(t) = V_o e^{-t/RC}$$

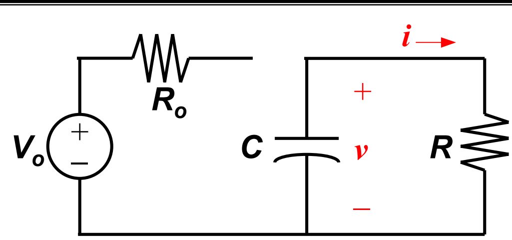
- Note that the voltage changes abruptly:

$$i(0^-) = 0$$

$$\text{for } \; t > 0, \; i(t) = \frac{v}{R} = \frac{V_o}{R} e^{-t/RC}$$

$$\Rightarrow \; i(0^+) = \frac{V_o}{R}$$

Solving for Power and Energy Delivered (t > 0)



$$v(t) = V_o e^{-t/RC}$$

$$p = \frac{v^2}{R} = \frac{V_o^2}{R} e^{-2t/RC}$$

$$w = \int_0^t p(x)dx = \int_0^t \frac{V_o^2}{R} e^{-2x/RC} dx$$

$$= \frac{1}{2} C V_o^2 \left(1 - e^{-2t/RC}\right)$$

Time Constant τ

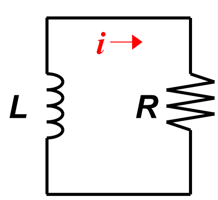
- In the example, we found that

$$v(t) = V_o e^{-t/RC} \quad \text{and} \quad i(t) = \frac{V_o}{R} e^{-t/RC}$$

- Define the ***time constant*** $\boxed{\tau = RC}$
  - At $t = \tau$, the voltage has reduced to $1/e$ (~0.37) of its initial value.
  - At $t = 5\tau$, the voltage has reduced to less than 1% of its initial value.

Natural Response Summary

RL Circuit



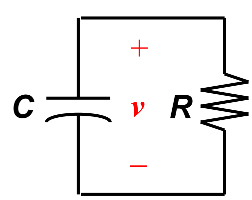
- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \dfrac{L}{R}$

RC Circuit



- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$