Lecture #18

OUTLINE
– Generation and recombination
– Charge-carrier transport in silicon
– Resistivity as a function of doping

Reference Texts on reserve in Engr. Library
• Howe & Sodini
  Chapter 2.1: Pure semiconductors
  Chapter 2.2: Generation, recombination, thermal equilibrium
  Chapter 2.3: Doping
  Chapter 2.4: Carrier Transport
  Chapter 2.6: IC Resistors
• Schwarz and Oldham
  Chapter 13: Semiconductor Devices

Generation
• We have seen that conduction (mobile) electrons and holes can be created in pure (intrinsic) silicon by thermal generation.
  – Thermal generation rate increases exponentially with temperature $T$

• Another type of generation process which can occur is optical generation
  – The energy absorbed from a photon frees an electron from covalent bond
    • In Si, the minimum energy required is $1.1\text{eV}$, which corresponds to $\sim1\ \mu\text{m}$ wavelength (infrared region)

• Note that conduction electrons and holes are continuously generated, if $T > 0$
Recombination

- When a conduction electron and hole meet, each one is eliminated. The energy lost by the conduction electron (when it “falls” back into the covalent bond) can be released in 2 ways:
  1. to the semiconductor lattice (vibrations) "thermal recombination" $\rightarrow$ semiconductor is heated
  2. to photon emission "optical recombination" $\rightarrow$ light is emitted

  - Optical recombination is negligible in Si. It is significant in compound semiconductor materials, and is the basis for light-emitting diodes and laser diodes.

Generation and Recombination Rates

- The generation rate is dependent on temperature $T$, but it is independent of $n$ and $p$: $G = G_{\text{thermal}}(T) + G_{\text{optical}}$

- The recombination rate is proportional to both $n$ and $p$:
  $$R \propto np$$

- In steady state, a balance exists between the generation and recombination rates.
  $$G = R \quad \Rightarrow \quad np = f(T)$$

- A special case of the steady-state condition is thermal equilibrium: no optical or electrical sources
  $$np = n_i^2(T)$$
Carrier Scattering

- Mobile electrons and atoms in the Si lattice are always in random thermal motion.
  - Average velocity of thermal motion for electrons in Si:
    \(~10^7\) cm/s @ 300K
  - Electrons make frequent collisions with the vibrating atoms
    • “lattice scattering” or “phonon scattering”
  - Other scattering mechanisms:
    • deflection by ionized impurity atoms
    • deflection due to Coulombic force between carriers

- The average current in any direction is zero, if no electric field is applied.

Carrier Drift

- When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:

- Electrons drift in the direction opposite to the \( E \)-field
  \( \rightarrow \) Current flows

- Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as classical particles moving at a constant average drift velocity.
Drift Velocity and Carrier Mobility

Mobile charge-carrier drift velocity is proportional to applied $E$-field:

$$|v| = \mu E$$

$\mu$ is the mobility (Units: cm$^2$/V•s)

Note: Carrier mobility depends on total dopant concentration ($N_D + N_A$)!

Current Density

The current density $J$ is the current per unit area ($J = I / A$; $A$ is the cross-sectional area of the conductor)

If we have $N$ positive charges per unit volume moving with average speed $v$ in the $+x$ direction, then the current density in the $+x$ direction is just $J = qNv$

Example:

- 2 x $10^{16}$ holes/cm$^3$ moving to the right at 2 x $10^4$ cm/sec
- $J = 1.6 \times 10^{-19} \times 2 \times 10^{16} \times 2 \times 10^4 = 64$ A/cm$^2$

Suppose this occurs in a conductor 2 µm wide and 1 µm thick:

$$I = J \times A = 64 \times (2 \times 10^{-4} \times 1 \times 10^{-4}) = 1.28 \mu A$$
Electrical Conductivity $\sigma$

When an electric field is applied, current flows due to drift of mobile electrons and holes:

- **Electron current density**: $J_n = (-q)nv_n = qn\mu_n E$
- **Hole current density**: $J_p = (+q)pv_p = qp\mu_p E$
- **Total current density**: $J = J_n + J_p = (qn\mu_n + qp\mu_p)E$

$$J = \sigma E$$

**Conductivity** $\sigma \equiv qn\mu_n + qp\mu_p$

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Electrical Resistivity $\rho$

- $\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$
- $\rho \equiv \frac{1}{qn\mu_n}$ for n-type mat'l
- $\rho \equiv \frac{1}{qp\mu_p}$ for p-type mat'l

(Units: ohm-cm)

*Note: This plot does not apply for compensated material (doped with both donors and acceptors)*
Example

Consider a Si sample doped with $10^{16}$/cm$^3$ Boron. What is its resistivity?

Answer:

$N_A = 10^{16}$/cm$^3$, $N_D = 0$  \( (N_A \gg N_D \rightarrow p\text{-type}) \)

$\rightarrow p \approx 10^{16}$/cm$^3$ and $n \approx 10^4$/cm$^3$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \approx \frac{1}{qp\mu_p}$$

$$= \left[ (1.6 \times 10^{-19})(10^{16})(450) \right]^{-1} = 1.4 \, \Omega \, \text{cm}$$

From $\mu$ vs. $(N_A + N_D)$ plot

Example (cont'd)

Consider the same Si sample, doped additionally with $10^{17}$/cm$^3$ Arsenic. What is its resistivity?

Answer:

$N_A = 10^{16}$/cm$^3$, $N_D = 10^{17}$/cm$^3$  \( (N_D \gg N_A \rightarrow n\text{-type}) \)

$\rightarrow n \approx 9 \times 10^{16}$/cm$^3$ and $p \approx 1.1 \times 10^3$/cm$^3$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \approx \frac{1}{qn\mu_n}$$

$$= \left[ (1.6 \times 10^{-19})(9 \times 10^{16})(700) \right]^{-1} = 0.10 \, \Omega \, \text{cm}$$

The sample is converted to n-type material by adding more donors than acceptors, and is said to be “compensated”. 
Sheet Resistance $R_s$

$R = \rho \frac{L}{Wt} = R_s \frac{L}{W} \Rightarrow R_s \equiv \frac{\rho}{t}$ (Unit: ohms/square)

$R_s$ is the resistance when $W = L$

- The $R_s$ value for a given layer in an IC technology is used
  - for design and layout of resistors
  - for estimating values of parasitic resistance in a circuit

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<tr>
<th>$R$</th>
<th>$R = R_s$</th>
<th>$R = R_s/2$</th>
<th>$R = 2R_s$</th>
<th>$R = 3R_s$</th>
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Velocity Saturation

At high electric fields, the average velocity of carriers is NOT proportional to the field; it saturates at $\sim 10^7$ cm/sec for both electrons and holes: