

## Lecture #18

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### OUTLINE

- Generation and recombination
- Charge-carrier transport in silicon
- Resistivity as a function of doping

### Reference Texts on reserve in Engr. Library

- **Howe & Sodini**
  - Chapter 2.1: Pure semiconductors
  - Chapter 2.2: Generation, recombination, thermal equilibrium
  - Chapter 2.3: Doping
  - Chapter 2.4: Carrier Transport
  - Chapter 2.6: IC Resistors
- **Schwarz and Oldham**
  - Chapter 13: Semiconductor Devices

## Generation

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- We have seen that conduction (mobile) electrons and holes can be created in pure (intrinsic) silicon by ***thermal generation***.
  - Thermal generation rate increases exponentially with temperature  $T$
- Another type of generation process which can occur is ***optical generation***
  - The energy absorbed from a photon frees an electron from covalent bond
    - In Si, the minimum energy required is  $1.1\text{eV}$ , which corresponds to  $\sim 1\text{ }\mu\text{m}$  wavelength (infrared region)
- Note that conduction electrons and holes are continuously generated, if  $T > 0$

## Recombination

- When a conduction electron and hole meet, each one is eliminated. The energy lost by the conduction electron (when it “falls” back into the covalent bond) can be released in 2 ways:
  1. to the semiconductor lattice (vibrations)  
“thermal recombination” → semiconductor is heated
  2. to photon emission  
“optical recombination” → light is emitted
    - Optical recombination is negligible in Si. It is significant in compound semiconductor materials, and is the basis for light-emitting diodes and laser diodes.

## Generation and Recombination Rates

- The generation rate is dependent on temperature  $T$ , but it is independent of  $n$  and  $p$ :  $G = G_{thermal}(T) + G_{optical}$
- The recombination rate is proportional to both  $n$  and  $p$ :

$$R \propto np$$

- In steady state, a balance exists between the generation and recombination rates.

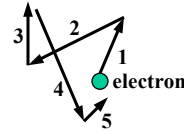
$$G = R \quad \Rightarrow \quad np = f(T)$$

- A special case of the steady-state condition is **thermal equilibrium**: no optical or electrical sources

$$np = n_i^2(T)$$

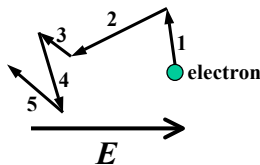
## Carrier Scattering

- **Mobile electrons and atoms in the Si lattice are always in random thermal motion.**
  - Average velocity of thermal motion for electrons in Si:  
 $\sim 10^7$  cm/s @ 300K
  - Electrons make frequent collisions with the vibrating atoms
    - “lattice scattering” or “phonon scattering”
  - Other scattering mechanisms:
    - deflection by ionized impurity atoms
    - deflection due to Coulombic force between carriers
- **The average current in any direction is zero, if no electric field is applied.**



## Carrier Drift

- **When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge-carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:**



- **Electrons *drift* in the direction opposite to the  $E$ -field**  
**→ Current flows**
- ❖ **Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as classical particles moving at a constant average *drift velocity*.**

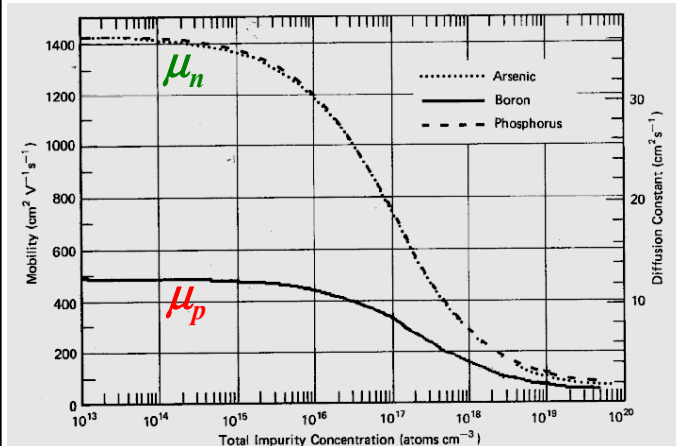
# Drift Velocity and Carrier Mobility

Mobile charge-carrier drift velocity is proportional to applied  $E$ -field:

$$|v| = \mu E$$

$\mu$  is the **mobility**

(Units:  $\text{cm}^2/\text{V}\cdot\text{s}$ )

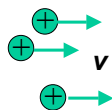


**Note:** Carrier mobility depends on **total** dopant concentration ( $N_D + N_A$ ) !

# Current Density

The current density  $J$  is the current per unit area ( $J = I / A$  ;  $A$  is the cross-sectional area of the conductor)

If we have  $N$  positive charges per unit volume moving with average speed  $v$  in the  $+x$  direction, then the current density in the  $+x$  direction is just  $J = qNv$

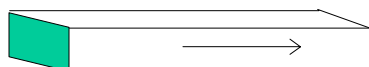


Example:

$2 \times 10^{16}$  holes/ $\text{cm}^3$  moving to the right at  $2 \times 10^4$  cm/sec

$$J = 1.6 \times 10^{-19} \times 2 \times 10^{16} \times 2 \times 10^4 = 64 \text{ A/cm}^2$$

Suppose this occurs in a conductor  $2 \mu\text{m}$  wide and  $1 \mu\text{m}$  thick:



$$I = J \times A = 64 \times (2 \times 10^{-4} \times 1 \times 10^{-4}) = 1.28 \mu\text{A}$$

## Electrical Conductivity $\sigma$

When an electric field is applied, current flows due to drift of mobile electrons and holes:

electron current density:  $J_n = (-q)nv_n = qn\mu_n E$

hole current density:  $J_p = (+q)pv_p = qp\mu_p E$

total current density:  $J = J_n + J_p = (qn\mu_n + qp\mu_p)E$

$$J = \sigma E$$

conductivity  $\sigma \equiv qn\mu_n + qp\mu_p$

## Electrical Resistivity $\rho$

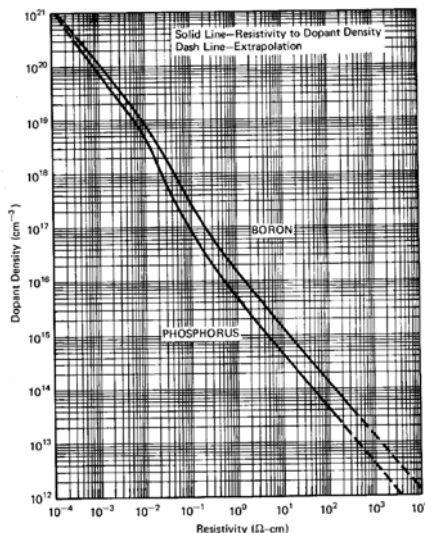


Figure 1.14 Dopant density versus resistivity at 23°C (296 K) for silicon doped with phosphorus and with boron. The curves can be used with little error to represent conditions at 300 K. [W. R.

$$\rho \equiv \frac{1}{\sigma} = \frac{1}{qn\mu_n + qp\mu_p}$$

$$\rho \approx \frac{1}{qn\mu_n} \quad \text{for n-type mat'l}$$

$$\rho \approx \frac{1}{qp\mu_p} \quad \text{for p-type mat'l}$$

(Units: ohm-cm)

Note: This plot does not apply for compensated material (doped with both donors and acceptors)

## Example


Consider a Si sample doped with  $10^{16}/\text{cm}^3$  Boron. What is its resistivity?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 0 \quad (N_A \gg N_D \rightarrow \text{p-type})$$

$$\rightarrow p \approx 10^{16}/\text{cm}^3 \quad \text{and} \quad n \approx 10^4/\text{cm}^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qp\mu_p}$$
$$= \left[ (1.6 \times 10^{-19})(10^{16})(450) \right]^{-1} = 1.4 \, \Omega - \text{cm}$$

From  $\mu$  vs.  $(N_A + N_D)$  plot 

## Example (cont'd)

Consider the same Si sample, doped *additionally* with  $10^{17}/\text{cm}^3$  Arsenic. What is its resistivity?

Answer:

$$N_A = 10^{16}/\text{cm}^3, N_D = 10^{17}/\text{cm}^3 \quad (N_D \gg N_A \rightarrow \text{n-type})$$

$$\rightarrow n \approx 9 \times 10^{16}/\text{cm}^3 \quad \text{and} \quad p \approx 1.1 \times 10^3/\text{cm}^3$$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \cong \frac{1}{qn\mu_n}$$
$$= \left[ (1.6 \times 10^{-19})(9 \times 10^{16})(700) \right]^{-1} = 0.10 \, \Omega - \text{cm}$$

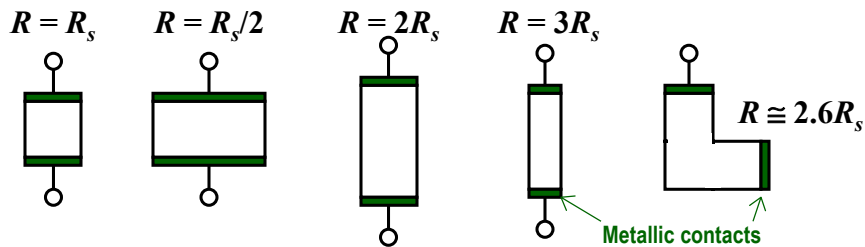
The sample is converted to n-type material by adding more donors than acceptors, and is said to be "compensated".

## Sheet Resistance $R_s$

$$R = \rho \frac{L}{Wt} = R_s \frac{L}{W} \Rightarrow R_s \equiv \frac{\rho}{t} \quad (\text{Unit: ohms/square})$$

$R_s$  is the resistance when  $W = L$

- The  $R_s$  value for a given layer in an IC technology is used
  - for design and layout of resistors
  - for estimating values of parasitic resistance in a circuit



## Velocity Saturation

**At high electric fields, the average velocity of carriers is NOT proportional to the field; it saturates at  $\sim 10^7$  cm/sec for both electrons and holes:**