
![Circuit Diagram]

(a) Solve for currents $i_1$ and $i_2$.
Assume $D_1$ and $D_2$ are ideal diodes, i.e., their $I-V$ characteristic looks like

```
I  
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/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
/  
V
```

Soln. 4 possible configurations for diodes:

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>On</td>
<td>On</td>
<td>On</td>
</tr>
<tr>
<td>Off</td>
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Let's make a guess and solve, then we can check if our guess was correct.

Say D₁ on and D₂ on.

The circuit now looks like:

We shorted the diodes! Why because ideal diodes behave like short circuits when on.

KCL at node 1:
\[ i_1 = i_2 + i_3 \]

KVL in upper mesh gives:
\[-10 + 6000 i_1 = 0 \]
\[\Rightarrow i_1 = \frac{10}{6000} A = 1.67 \text{ mA} \]

KVL in lower mesh gives:
\[-10 + 5000 i_2 = 0 \]
\[\Rightarrow i_2 = \frac{10}{5000} A = 2 \text{ mA} \]
\[\Rightarrow i_2 = i_1 - i_2 = 0.33 \text{ mA} \]

\( i_2 \) is < 0 \Rightarrow the flow of current through D₁ is

\[ \text{which violates the} \]

assumption of an ideal diode

\[ \text{Our assumption was wrong!} \]
Let's make another guess:

$D_1$ off, $D_2$ on.

The circuit now looks like:

![Circuit Diagram]

(We replaced $D_1$ by an open circuit because an ideal diode is an open circuit when off.)

**KCL at node 1**

$i_1 = i_2$

**KVL in the mesh gives us:**

$-10 + 6000i_1 + 5000i_2 - 10 = 0$

$\Rightarrow i_1 = \frac{20}{11000} A = 1.82 \text{ mA}$

Now let's check our assumption:

$V_{D_1} = 10 - 6000i_1 = 10 - (6000)\left(\frac{1.82}{11000}\right)$

$= -0.92 \text{ V}$

...The voltages at both the ends of $D_1$ are:

$-0.92 \text{ V}$

$\Rightarrow D_1$ is reverse-biased and therefore should be off, which is consistent.

Also, $i_2 = \text{Current through } D_2 \text{ is 0}$ which matches our assumption that $D_2$ is on.
(b) Solve (a) assuming $V_T = 2$ V for all the diodes.

As usual, we guess!

Now that we have already done (a), we can make smarter guesses.

4 possible diode configurations

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Let’s guess $D_1$ off and $D_2$ on.

The equivalent circuit looks like

Note that we replaced $D_2$ by a voltage drop at $V_T = 2$ V

$V_T = 10$ V, $V_{DD} = 2$ V

Using KVL in the mesh, we get:

$-10 + 6k \cdot i_1 + 2 + 5k \cdot i_1 - 10 = 0$

or:

$i_1 = \frac{20 - 2}{11k} \quad A = 1.63 \text{ mA}$
Let's check our assumption:
Calculate the voltages at both ends of each diode.

\[ V_0 = 10 - (6000)(1.63 \, \frac{1}{1000}) \, V \]

\[ = 0.22 \, V \]

\[ \therefore \text{ The terminals of } D_1 \text{ look like:} \]

\[ \text{The forward bias voltage is} \]
\[ 0.22 \, V < V_T \]

\[ \therefore D_1 \text{ should be off} \]

which is consistent with our guess.

Questions to think about:
1. How do we make intelligent guesses to avoid having to look at all cases?
Load Line Analysis in Diode Circuits

Problem 2:

Suppose we are given a Zener diode with the following I-V characteristic:

Now consider the following circuit:

\[ V_{SS} \quad 1k\Omega \quad D_1 \quad R_L \rightarrow \text{Load} = 1.5k\Omega \]

\[ V_{SS} \text{ varies between 20 V and 25 V} \]

Find the range of voltage on the load resistor.
Soln: First, reorder the circuit to bring the linear elements together.

Next, replace the linear portion by an equivalent Thévenin circuit. (Why do we need to do this? To simplify the circuit with respect to the diode! Then load line analysis is fairly easy.)

\[ R_{Th} = R \parallel R_L = \frac{R \cdot R_L}{R + R_L} = 600 \Omega \]

\[ V_{Th} = \frac{R_L \cdot V_{ss}}{R + R_L} \]

Now the equivalent circuit is:

\[ R_{Th} = 600 \Omega \]

Next, we draw the load line for \( V \) vs. \( I \)
Suppose we know \( V \), what is \( I \)?

Use KVL:

\[
V + R_{Th} \cdot I + V_{Th} = 0
\]

\[\Rightarrow I = -\frac{V_{Th} - V}{R_{Th}}\]

Let's find \( V \) when \( V_{SS} = 20 \) V.

\[\Rightarrow V_{Th} = \frac{(20)/(1.5)}{2.5} = 12 \text{ V}\]

\[I = -\frac{12 - V}{600 \Omega}\]

Similarly, the load line when \( V_{SS} = 2.5 \) is

\[I = -\frac{15 - V}{600 \Omega}\]

Does it stabilize?
More complicated question:

1. \[ R_1 \quad \text{??} \quad R_2 \quad \text{??} \quad R_3 \]

\[ i_1 \quad i_2 \]

How would you find \( i_1 \) and \( i_2 \) just by doing load line analysis?