1.1 Digital Basics
   a) Convert the decimal number 71 to binary, hexadecimal, and base 3.
      Using a process of successive divisions by the target base, taking the remainder as
      the least significant digit yet to be determined and repeating the process with the
      quotient, we obtain:

      1) Binary representation of $71_{10}$
         i. $71 / 2 = 35 \text{ R } 1$
         ii. $35 / 2 = 17 \text{ R } 1$
         iii. $17 / 2 = 8 \text{ R } 1$
         iv. $8 / 2 = 4 \text{ R } 0$
         v. $4 / 2 = 2 \text{ R } 0$
         vi. $2 / 2 = 1 \text{ R } 0$
         vii. $1 / 2 = 0 \text{ R } 1$
      The end result is that $71_{10} = 1000111_2$

      2) Hexadecimal representation of $71_{10}$
         i. $71 / 16 = 4 \text{ R } 7$
         ii. $4 / 16 = 0 \text{ R } 4$
      The end result is that $71_{10} = 47_{16}$

      3) Base 3 representation of $71_{10}$
         i. $71 / 3 = 23 \text{ R } 2$
         ii. $23 / 3 = 7 \text{ R } 2$
         iii. $7 / 3 = 2 \text{ R } 1$
         iv. $2 / 3 = 0 \text{ R } 2$
      The end result is that $71_{10} = 2122_3$

   b) Sketch an implementation of the Boolean expression for the Hot Tub Controller.
      The Hot Tub Controller gives a high voltage output only when the following are
      satisfied: Either T is closed (true), or A, B, and C are simultaneously closed.

      Therefore, the following is a logic expression exactly expressing the behavior of
      the Hot Tub Controller:
      \[ \text{Out} = (A \cdot B \cdot C) + T \]
Using NANDs, note the following:
Feeding the same input into both lines of a 2-input NAND, we obtain the following truth table:

<table>
<thead>
<tr>
<th>A</th>
<th>A</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

In other words, a NAND can be used to invert.
Two NANDs can thus be used to perform an AND operation using the following circuit:

Also, NAND can be used to synthesize OR using the following structure:

This is resultant from one of DeMorgan’s Laws, specifically that

\[ A + B = (\overline{A} \cdot \overline{B}) \]

and from the fact that NAND can be used to invert.

So, a logically equivalent circuit of only NANDs takes the following form:
2.2 Two Element Networks

a) Plot $I_{\text{out}}$ vs. $V_{\text{out}}$ for the circuit shown for:
For this problem, Ohm’s Law will be used for the voltage across the resistor. Suppose we set up $V_R$ such that $V_{\text{OUT}} = V_S + V_R$. This setup satisfies the equation $I_R = V_R/R$ where $I_R$ goes leftward through the resistor. Since the resistance is in series with the output, $I_{\text{OUT}} = -I_R$. With some algebra we come upon the following descriptive equation:

$$V_{\text{OUT}} = V_S - I_{\text{OUT}} R \Rightarrow I_{\text{OUT}} = -V_{\text{OUT}} R + V_S / R$$

Note that these equations mirror the standard form $y = mx + b$ for graphing of lines. This provides the following graphs (we use the 2nd equation above because we are plotting $I$ vs. $V$ (standard is “y-axis vs. x-axis”):

a. $V_s = 3V$, $R = 10k\Omega$

b. $V_s = 3V$, $R = 20k\Omega$

c. $V_s = -2V$, $R = 10k\Omega$
b) When $V_{\text{out}} = 5V$, determine the power out, the power into $R$ and the power from the source. Check with conservation of power. Is everything consistent?

a. $V_S = 3V$, $R = 10k\Omega$

If $V_{\text{out}} = 5V$, then positive current is going through the resistor in the direction from the output to the positive side of $V_S$. From the power equation, we obtain a power of:

$$P_R = \frac{V^2}{R} = \frac{(2V)^2}{10k\Omega} = 0.4mW$$

For the output and voltage source, we must calculate the current through the resistor using Ohm’s Law:

$$I_R = \frac{V}{R} = \frac{2V}{10k\Omega} = 0.2mA$$

(where $I_R = -I_{\text{out}}$)

Then, we obtain the following keeping reference directions in mind:

$$P_S = IV = 0.2mA*3V = 0.6mW$$
$$P_{\text{OUT}} = -IV = -(0.2mA*5V) = -1.0mW$$

By conservation of power, the summation of power into all elements of the circuit should yield 0, which does happen here:

$$\Sigma(\text{Power}) = 0.4mW + 0.6mW - 1.0mW = 0mW$$

b. $V_S = 3V$, $R = 20k\Omega$

Repeating the calculations,

$$I_R = \frac{V}{R} = \frac{2V}{20k\Omega} = 0.1mA$$
$$P_R = IV = 0.1mA*2V = 0.2mW$$
$$P_S = IV = 0.1mA*3V = 0.3mW$$
$$P_{\text{OUT}} = -IV = -(0.1mA*5V) = -0.5mW$$

$$\Sigma(\text{Power}) = 0.2mW + 0.3mW - 0.5mW = 0mW$$

c. $V_S = -2V$, $R = 10k\Omega$

Repeating the calculations,

$$I_R = \frac{V}{R} = \frac{7V}{10k\Omega} = 0.7mA$$
$$P_R = IV = 0.7mA*(7V) = 4.9mW$$
$$P_S = IV = 0.7mA*(-2V) = -1.4mW$$
$$P_{\text{OUT}} = -IV = -(0.7mA*5V) = -3.5mW$$

$$\Sigma(\text{Power}) = 4.9mW - 1.4mW - 3.5mW = 0mW$$