12.1 Latency and Throughput of Lumped and Pipelined Logic

Answer Problem 11.3 in Problem Set #11.

12.2 Sinusoidal Forcing Functions

A sinusoidal current source \( I_s(t) = 10\text{mA}\cos(2\pi \times 2\text{GHz}\times t) \) is connected to a parallel RLC circuit by a switch that closes at \( t = 0 \), as shown in Figure 1. The initial conditions are \( i_C(0) = 0 \) and \( v_o(t) = 0 \). Assume component values as follows: \( R = 500\Omega \), \( L = 3\text{nH} \), \( C = 1\text{pF} \).

a. Find the forced (steady-state) response for \( v_o(t) \) for \( t > 0 \), using phasors.
b. Find the difference between the initial values \( i_C(0) \) and \( v_o(0) \) of the forced response and the given initial conditions.
c. Find the natural (transient) response for \( v_o(t) \) for \( t > 0 \). Is the circuit overdamped, underdamped, or critically damped? Which circuit elements determine the time it takes for the transient response to decay?
d. Find the total response for \( v_o(t) \) for \( t > 0 \). Qualitatively sketch the forced response, the natural response, and the total response as a function of time.

Figure 1 – Parallel RLC Circuit
12.3 2nd Order Transient

A man driving a tractor (let’s call him Kent) has a mass of 100kg. The seat of the tractor that Kent is sitting on can be modeled as a mass-spring system as shown in Figure 2. As Kent is driving his tractor he hits a bump which causes the seat to move 30cm below the resting position at $x = 0$ (i.e., the initial condition is $x(t=0) = -30cm$; assume the initial velocity is 0). The system will restore the driver seat position to $x(t=\infty) = 0$. Assume component values as follows: $k = 6 \times 10^4$ N/m, $b = 7 \times 10^3$ N*s/m.

a. Find the differential equation describing $x(t)$, the position of the mass as a function of time, for $t > 0$ (a free-body diagram may help here). Recall that the force due to the spring is given by $F_{spring} = -kx(t)$ and the force due to the damper is given by $F_{damper} = -b \frac{dx(t)}{dt}$, and neglect the force of gravity.

b. Find a circuit analog to the mechanical system by finding an equivalent series RLC circuit which gives the same differential equation found in part (a). Let electrical charge be analogous to the position of the mass. Give the values for $R$, $L$, $C$, and the initial conditions.

c. Use a circuit simulation program (e.g., PSPICE or MultiSIM) to plot the position of the mass as a function of time, by running a transient analysis on the circuit found in part (b). **Don’t forget to set the initial conditions in your schematic; there should be two of them.** In PSPICE, you can set the initial voltage between two nodes using the “IC2” part in the “SPECIAL” library. Attach a copy of your schematic and the plot of the transient response. What is the damping condition of the system?

d. Find the damping factor $b$ required for critical damping, and determine the time it takes for the mass to get 3cm away from $x = 0$.

e. Find the damping factor $b$ required for a damping ratio of 0.2, and determine the time it takes for the mass to settle to within $\pm 3$cm of $x = 0$.

f. Find the maximum force Kent experiences in parts (d) and (e) and compare to the force of gravity.