1. This is a closed book exam. However, you are allowed to bring one page (8.5” x 11”), single-sided notes PLUS your 1-page notes from midterm 1, 2 and 3.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. Slide rules are allowed.
4. SHOW all the steps on the exam. **Answers without steps will be given only a small percentage of credits.** Partial credits will be given if you have proper steps but no final answers.
5. **Remember to put down units.** Points will be taken off for answers without units.

Last (Family) Name:_____________________________________________________

First Name:___________________________________________________________

Student ID:___________________________Discussion Session:_______________

Signature:_____________________________________________________________

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1. [30pts] Thevenin Equivalent Circuit

![Diagram of circuit](image)

a) What is the current $i_x$ through the 10Ω resistor?

\[
KVL: 20V - (i_x \cdot 10Ω) - (0.5i_x \cdot 20Ω) - (0.5i_x \cdot 40Ω) = 0
\]

\[
20V = 10Ω \cdot i_x + 30Ω \cdot i_x
\]

\[
i_x = \frac{20V}{40Ω} = 0.5A
\]

b) Find $V_1$ when there is no load connected to the ports a and b (as depicted by the figure above).

Voltage drop across the 2 resistors:

\[
V_1 = (0.5i_x)(20Ω + 40Ω)
\]

\[
= (0.25A)(60Ω)
\]

\[
= 15V
\]

c) What is the open circuit voltage across a-b, $V_{oc}$?

Voltage across the 20Ω resistor:

\[
V_{oc} = (0.5i_x)(20Ω)
\]

\[
= (0.25A)(20Ω)
\]

\[
= 5V
\]

d) What is the short circuit current across a-b, $I_{sc}$?

0.5$i_x$ flows directly from $V_1$ to 40Ω resistor, KVL now reads:

\[
20V - (i_x \cdot 10Ω) - (0.5i_x \cdot 40Ω) = 0
\]

\[
20V = 10Ω \cdot i_x + 20Ω \cdot i_x
\]

\[
i_x = \frac{20V}{30Ω} = \frac{2}{3}A
\]

\[
i_{sc} = 0.5i_x = \frac{1}{3}A
\]

e) What is $R_m$? Draw the Thevenin equivalent circuit.
$R_{th} = \frac{V_{oc}}{i_{sc}}$

$= \frac{5V}{1/3A}$

$= 15\Omega$

2. [30pts] Equivalent Impedances

a) What is $R_{eq}$ across the indicated nodes?

Break down 2 resistors at a time:
\[ R_{eq} = 25 \Omega \]

b) What is \( Z_{eq} \) across the indicated nodes?

Break down 2 capacitors at a time:
\( C_{eq} = 10 \text{mF} \)

\[
Z_{eq} = \frac{1}{j \omega C_{eq}} = \frac{100}{j \omega} \Omega \quad \text{or} \quad \frac{1}{10 j \omega} k\Omega
\]
3. [50 pts] Semiconductor Device Physics
Suppose we have the charge density distribution given by the figure. The permittivity of the material is $\varepsilon$.

![Charge density diagram](image)

a) (5 pts) Which side is p side and which side n side?

Left side ($x<0$): p  
Right side ($x>0$): n

b) (5 pts) What is the direction of the built-in electric field?

The built-in electric field is in $-x$ direction

c) (10 pts) What is the value for the electric field $E(x)$ for $x \leq -x_2$ and $x_2 \leq x$?

$E(x)=0$ for both regions.

d) (10 pts) What is $E(x)$ for $-x_2 \leq x \leq -x_1$?

From the figure, we can write down the expression for charge density in this region:

$$\rho(x) = \frac{qN}{x_1 - x_2} (x + x_2)$$

According to Gauss’s law

$$\frac{d}{dx} E(x) = \frac{\rho(x)}{\varepsilon}$$

we can arrive at the following result by carrying out the integration

$$E(x) = \frac{qN}{\varepsilon(x_1 - x_2)} \frac{(x + x_2)^2}{2}$$
e) (10 pts) What is $E_{\text{max}}$? Here $E_{\text{max}}$ represents the maximum absolute value.

The electric field reaches its maximum value at the $x=0$. The magnitude can be calculated by evaluating the area enveloped by the curve on the left hand side, and divide the area by $\varepsilon$. Therefore,

$$E_{\text{max}} = \frac{qN(x_2 - x_1)}{2\varepsilon} + \frac{qNx_1}{\varepsilon}$$

$$= \frac{qN(x_2 + x_1)}{2\varepsilon}$$

f) (10 pts) Sketch $E(x)$ vs. X. Label your plot. Also describe the curves.

$E(x)$ is symmetrical with respect to $x$. It is a quadratic function when $x \in [-x_2,-x_1] \cup [x_1,x_2]$, a linear function when $x \in [-x_1,0] \cup [0,x_1]$, and zero anywhere else.
4. [40 pts] Diodes/Op-Amps
Assume the op-amp is ideal.
Use the large-signal model for the diode with turn-on voltage of 0.6V.

(a) (10 pts) When the diode is on, what is $V_{\text{out}}$ as a function of $V_{\text{in}}$?

When the diode is on, it acts like a 0.6V source.
By the summing-point constraint, $V_{-} = V_{+} = 0V$.
By applying KCL at $V_{-}$, we get $V_{\text{in}}/2k + V_{\text{out}}/6k + (V_{\text{out}}-0.6V)/3k = 0V$.
Solving yields $V_{\text{out}} = 0.4V - V_{\text{in}}$.

(b) (10 pts) When the diode is off, what is $V_{\text{out}}$ as a function of $V_{\text{in}}$?

When the diode is off, it acts as an open circuit.
Now KCL at $V_{-}$ gives $V_{\text{in}}/2k + V_{\text{out}}/6k = 0V$.
Solving yields $V_{\text{out}} = -3V_{\text{in}}$.
Also notice that this is just a regular inverting amplifier.

(c) (5 pts) For what range of $V_{\text{in}}$ will the diode be on?

For the diode to be on, there must be positive current through the diode or, alternatively, through the 3k resistor (since they are in series).
This means there must be a voltage drop across the resistor must be positive.
So that gives us $V_{\text{out}} - 0.6V > 0V$.
Substituting in our expression from (a), we get $0.4V - V_{\text{in}} - 0.6V > 0V$.
Solving yields $V_{\text{in}} < -0.2V$.

Alternatively, we can see that for the diode to be off, we need the diode voltage to be less than the threshold of 0.6V, which gives us $V_{\text{out}} - 0 < 0.6V$.
Plugging in our expression from (b) gives us $-3V_{\text{in}} < 0.6V$.
Solving yields $V_{\text{in}} > -0.2V$.
This means the diode will be on for $V_{\text{in}} < -0.2V$.

(d) (15 pts) Complete the following plot for $V_{\text{out}}$. Indicate which regions the diode is on. Make sure to label your axes.
The key points are that the diode switches when $V_{in} = -0.2V$ and $V_{out} = 0.6V$. The negative peak is $-3V$, and the positive peak is $1.4V$. Also notice that when $V_{in} = 0V$, $V_{out} = 0V$. 
5. [60 pts] Second Order Circuits
Suppose the following circuit. Assume that the switch has been in position A for an infinitely long time before $t = 0$. \( v_{in} = 3.0 \sin(1000t) \)

![Circuit Diagram](image)

a) (10 pts) Write the input voltage in the Phasor format, \( V_{in} \)?
Noting the input is a sin, we apply the identity:
\[
\cos(\omega t - \pi / 2) = \sin(\omega t)
\]
Thus, \( V_{in} = 3.0 \angle -\pi / 2 \)
This angle is equivalent to -90 degrees.

b) (10 pts) Assuming an AC steady state condition and the switch is in the A position. What is the output voltage in Phasor format, \( V_{out} \)?
\[
V_{out} = \frac{Z_c}{Z_c + R} V_{in} = \frac{1}{1 + j \omega RC} V_{in} = \frac{1}{1 + j} 3.0 \angle -\pi / 2 = \frac{3.0 \angle -\pi / 2}{\sqrt{2} \angle \pi / 4} = \frac{3}{\sqrt{2}} \angle -\frac{3\pi}{4}
\]

c) (10 pts) What is the output voltage in time domain, \( v_{out}(t) \)?
By inspection, \( V_{out} = \frac{3}{\sqrt{2}} \cos\left(1000t - \frac{3\pi}{4}\right) \)
This angle is equivalent to -135 degrees or +225 degrees.

d) (20 pts) At time \( t = \pi \) ms, the switch flips from position A to B, disconnecting the AC source from the output. Provide a differential equation describing \( v_{out} \) with respect to time. The solution should be a second-order differential equation of \( V_{out} \).
\[
\frac{V_{out} - 5V}{R} + C \frac{dV_{out}}{dt} + \frac{1}{L} \int V_{out} dt = 0
\]
Taking the derivative,
\[
\frac{d^2V_{out}}{dt^2} + \frac{1}{RC} \frac{dV_{out}}{dt} + \frac{1}{LC} V_{out} = 0
\]
Plugging in values,
\[
\frac{d^2V_{out}}{dt^2} + 10^3 \frac{dV_{out}}{dt} + 10^{14} V_{out} = 0
\]
e) (10 pts) What is the $V_{\text{out}}$ at very large $t$?
In steady state, the inductor becomes a short circuit (while it is in parallel with the capacitor).
So, $V_{\text{out}} \to 0V$. 
6. [140 pts] MOSFET
In the circuit below, $V_{DD}=5V$, and both transistors are identical with $V_{to}=1V$, $K=0.1$ mA/V^2.

(A) (50 pts) DC analysis
a) (5 pts) What is $V_1$? Given $R_1 = R_2 = 5k\Omega$

$$V_1 = V_{dd} \frac{R_2}{R_1 + R_2} = 5 \frac{5k\Omega}{5k\Omega + 5k\Omega} = 2.5V$$

b) (10 pts) What is $I_{D1}$? Given $R_3 = R_4 = 5k\Omega$. Write down the necessary equation(s) and draw load line in the following graph.

$$V_1 - I_D R_4 = V_{GS}$$
$$2.5 - I_D * 5k = V_{GS}$$
$$I_D = \frac{2.5 - V_{GS}}{5k} = \frac{2.5 - V_{GS}}{5} mA$$

From the Load Line
$I_D = 0.1$ mA
$V_{gs} = 2V$
c) (5 pts) Which mode is transistor M1 in?

Saturation

d) (5 pts) What is $V_{DS1}$?

$$V_{DS1} = Vdd - I_{D1}R3 - I_{D1}R4 = 5 - .1m*5k - .1m*5k = 5 - 0.5 - 0.5 = 4V$$

e) (5 pts) What is $V_2$?

$$V_{DS1} = Vdd - I_{D1}R3 = 5 - .1m*5k = 4.5V$$

f) (5 pts) For transistor M2, write down the equations for $V_{DS2}$ and $V_{GS2}$. What mode is this transistor in?

$$V_{DS1} = Vdd - I_{D2}R5$$

$$V_{GS2} = 4.5 - I_{D2}R5$$

The transistor is also in saturation.

g) (10 pts) What is $I_{D2}$? Given $R_5 = 3.75k\Omega$. Write down the necessary equation(s) and draw load line in the following graph.

$$V_{GS2} = V_2 - I_{D2}R5$$

$$I_{D2} = \frac{4.5 - V_{GS2}}{3.75k} = \frac{4.5 - V_{GS2}}{3.75} \text{mA}$$

$$I_{D2} = 0.4mA @ V_{GS} = 3V$$
h) (5 pts) What is $V_{DS2}$?

\[
V_{DS1} = Vdd - I_{D1} R5 = 5 - 0.4m \times 3.75k = 3.5V
\]
(B) (90 pts) AC small-signal analysis:

a) (20 pts) For transistor M1, the small signal gain $A_{v1} = \frac{V_2}{V_{in}}$. Draw the small signal model for M1. Consider $r_d \to \infty$ for both transistors.

b) (15 pts) What is $A_{v1}$?

To find $A_v$, we first find $G_M$ and then find $R_{out}$.

To find $G_M$, we short the output ($V_2$) and apply a test voltage at the input.

Applying KCL at $V_2$ and $V_S$ (source of M1):
\[
\frac{V_s}{R_4} - \text{gm}(V_t - V_s) = 0
\]

\[
V_s = \frac{\text{gm}R_4V_t}{1 + \text{gm}R_4}
\]

\(I_{out} + \text{gm}(V_t - V_s) = 0\)

\[
I_{out} = -\text{gm}(V_t - \frac{\text{gm}R_4V_t}{1 + \text{gm}R_4}) = -\text{gm}V_t(1 - \frac{\text{gm}R_4V_t}{1 + \text{gm}R_4}) = -\text{gm}V_t\left(\frac{1 + \text{gm}R_4}{1 + \text{gm}R_4} - \frac{\text{gm}R_4}{1 + \text{gm}R_4}\right)
\]

\[
I_{out} = \frac{-\text{gm}V_t}{1 + \text{gm}R_4}
\]

\[
GM = \frac{I_{out}}{V_t} = \frac{-\text{gm}}{1 + \text{gm}R_4}
\]

Ro is simply R3, because rd is infinity.
Therefore the gain \(A_{v1}\):

\[
A_{v1} = GM\cdot Ro = \frac{-\text{gm}R_3}{1 + \text{gm}R_4}
\]

c) (20 pts) For transistor M2, the small signal gain \(A_{v2} = V_{out}/V_2\). Draw the small signal model for M2.

\[
\begin{align*}
V_{in} & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \qa
d) (15 pts) What is \(A_{v2}\)?

To find the gain, we find GM and Ro. If we short the output VS (the source of M2), and apply a test voltage at the gate (V2), we see that GM = \(I_{out} / V_t = \text{gm}\).

\[ I_{out} = \text{gm}V2 \]

\[ GM2 = \frac{I_{out}}{V2} = \text{gm} \]

To find Ro, we short the input (V2) and apply a test voltage at the output VS. Then we measure the current flowing into the output node. First, combine R5 and RL together to form RL'.

Using KCL:

\[-I_{out} + \frac{V_t}{RL'} - \text{gm}(0 - V_t) = 0\]

\[ I_{out} = V_t\left(\frac{1}{RL'} + \text{gm}\right) \]

\[ RO2 = \frac{V_t}{I_{out}} = \frac{1}{\left(\frac{1}{RL'} + \text{gm}\right)} = \frac{RL'}{1 + \text{gm}RL'} \]

\[ A_{v2} = GM2 * RO2 \]

\[ A_{v2} = \frac{\text{gm}RL'}{1 + \text{gm}RL'} \]

e) (5 pts) What is the overall gain \(A_{v2} = V_{out}/V_{in}\)?

\[ A_{v} = A_{v1} * A_{v2} = -\frac{gmR3}{1 + gmR4} * \frac{gmRL'}{1 + gmRL'} \]

f) (5 pts) What is the input impedance of the overall circuit?

The input impedance is simply \(R1 \| R2\) because the input impedance of a NMOS is infinity.

\[ R_{in} = R1 \| R2 = 2.5k\Omega \]

g) (10 pts) What is the output impedance of the overall circuit?

As found above in part d), the output impedance of the overall circuit is the same as the output impedance of the second stage (consisting of M2).

\[ Z_o = RL' \| (1/\text{gm}) = \frac{RL'}{1 + \text{gm}RL'} \]