

# EECS 40, Fall 2007, Prof. Chang-Hasnain

## Homework 2 Solutions

### 1 2.50

First, replace the top two resistors in series ( $2\Omega$  and  $3\Omega$ ) by a single resistor ( $5\Omega$ ).

Next, use KCL to write any 3 of the following 4 equations (all set up as “incoming flow = 0”):

$$\begin{aligned} \text{At reference node: } & V_1/10 + V_2/5 + V_3/5 = 0 \\ \text{At node 1} & (0 - V_1)/10 + (V_2 - V_1)/20 - 2 = 0 \\ \text{At node 2} & (V_3 - V_2)/4 + (V_1 - V_2)/20 + (0 - V_2)/5 = 0 \\ \text{At node 3} & (0 - V_3)/5 + (V_2 - V_3)/4 + 2 = 0 \end{aligned} \tag{1}$$

We can rewrite the equations in matrix form

$$\begin{pmatrix} 1/10 & 1/5 & 1/5 \\ -3/20 & 1/20 & 0 \\ 1/20 & -1/2 & 1/4 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \tag{2}$$

(1 point for getting standard form correct, or putting it in matrix form.)

and solve for  $V_1 = -12.9032$ ,  $V_2 = 1.2903$ , and  $V_3 = 5.1613$ .

### 2 2.60

Form a super node around the dependent voltage source.

We obtain one equation using KVL around the loop containing dependent voltage source:

$$5i_x = 10i_x + V_2. \tag{3}$$

We obtain two more equations by writing the KCL equations for all but one node (we chose to skip the supernode):

$$\begin{aligned} \text{At node 1} & 2 + (V_1 - 5i_x)/5 + 1 = 0 \\ \text{At node 2} & -i_x + V_2/20 - 1 = 0. \end{aligned} \tag{4}$$

These equations can be written as

$$\begin{pmatrix} 1/5 & 0 & -1 \\ 0 & -1/20 & 1 \\ 0 & 1 & 5 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ i_x \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix}, \quad (5)$$

and solved to obtain  $V_1 = -19$ ,  $V_2 = 4$ , and  $i_x = -4/5$ .

### 3 2.62

Use KVL:

$$\begin{aligned} \text{Loop 1} \quad & 7(i_1 - i_3) + 100 + 5i_1 = 0 \\ \text{Loop 2} \quad & 13i_2 - 100 + 11(i_2 - i_3) = 0 \\ \text{Loop 3} \quad & 9i_3 + 11(i_x - i_2) + 7(i_x - i_1) = 0. \end{aligned} \quad (6)$$

Rewriting:

$$\begin{pmatrix} -7 & -11 & 27 \\ 12 & 0 & -7 \\ 0 & 24 & -11 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -100 \\ 100 \end{pmatrix}. \quad (7)$$

Solving:  $i_1 = -8.74$ ,  $i_2 = 3.85$ ,  $i_3 = -0.70$ .

The power delivered by the source is  $P = iV = 100(i_1 - i_2) = 1259W$ .

We could also check that the sum of the power delivered to each element is the same:  $P = \sum_{\text{all elements}} i^2 R = 7(i_1 - i_3)^2 + 5(i_1)^2 + 13(i_2)^2 + 11(i_2 - i_3)^2 + 9(i_3)^2 = 1259W$ .

### 4 -

a) Keeping only the left voltage source, (replacing the current source by an open circuit, and the other voltage source by a short circuit) gives a single loop with two resistors in series, so  $i = V/R = 2V/(1\Omega + 1\Omega)$ . Keeping only the top voltage source results in no current  $i$ , because no closed loop with a voltage or current source is formed.

Keeping only the 1A current source gives puts the current source in series with two parallel 1Ω resistors.

The current flow is equally split between the two 1Ω resistors, so  $i = \frac{1}{2} * (1A)$ .

Adding the currents induced by each independent source all together (by linearity) gives  $i = \frac{3}{2}A$ .

$$\text{b) } P = i^2 R = (9/4A^2)(1\Omega) = \frac{9}{4}W$$

### 5 2.79

Zeroing out the sources gives a 12Ω resistor in parallel with a 24Ω resistor, so that  $R_{\text{eff}} = (12)(24)/(12 + 24) = 8\Omega$ .

We compute the Norton equivalent current by short-circuiting  $ab$ . Short-circuiting  $ab$  means that the 12Ω resistor forms a loop in series with the 12V source, so the current through that resistor (to the left) is  $i = V/R = 1A$ . The current through  $ab$  is the sum of this with and the 1A current source, so the Norton equivalent current is given by  $i_{ba} = 2A$ , or  $i_{\text{Norton}} = -2A$ . The Norton equivalent circuit is given by the current source  $i_{\text{Norton}}$  in parallel with  $R_{\text{eff}} = 8\Omega$ .

The Thevenin voltage is given by  $V_{th} = i_{ab}R_{eff} = -2 * 8 = -16$ . The Thevenin equivalent circuit is given by the voltage source  $V_{th}$  in series with  $R_{eff} = 8\Omega$ .

The  $10\Omega$  resistor has no effect because it is in parallel to a constant voltage source. One could replace the resistor and voltage  $12V$  source with a Thevenin equivalent circuit that was just the voltage source.

## 6 2.83

Because there is a dependent voltage source, one cannot simply short  $ab$  to determine resistance.

First we compute the voltage if  $ab$  is open. Denote the current through the  $10\Omega$  resistor by  $i_2$ . If  $ab$  is open, KVL gives  $20V - 5i_x - 10\Omega i_2 = 0$ . KCL gives  $i_x - 0.5i_x - i_2 = 0$ . Together these yield  $i_x = 2A$  and  $i_2 = 1A$ . Therefore  $V_{th} = V_{ab} = V_2 = 10\Omega i_2 = 10V$ .

Next we compute the current if  $ab$  is shorted. If  $ab$  is shorted, KVL around the outermost loop gives  $20A = 5i_x$ , so  $i_x = 4A$ . KCL gives  $i_x - 0.5i_x - i_{ab}$ , so  $i_{Norton} = i_{ab} = 2A$ .

$R_{eff}$  can now be computed as  $R_{eff} = V_{ab}/i_{ab} = 5$ .

The Thevenin equivalent circuit is given by  $R_{eff}$  in series with  $V_{th}$ . The Norton equivalent circuit is given by  $i_{Norton}$  in parallel with  $R_{eff}$ .

## 7 2.103

If  $R_1$  and  $R_3$  are too small, large currents are drawn from the source. If the source were a battery, it would need to be replaced frequently. Large power dissipation could occur, leading to heating of the components and inaccuracy due to changes in resistance values with temperature.

If  $R_1$  and  $R_3$  are too large, we would have very small detector current when the bridge is not balanced, and it would be difficult to balance the bridge accurately.

## 8 2.104

The Thevenin resistance is obtained by short-circuiting  $V_s$  to obtain  $R_2$  and  $R_x$  in parallel with one another, and then in series with the parallel pair  $R_3$  and  $R_1$ . Hence,  $R_{th} = R_2R_x/(R_2 + R_x) + R_1R_3/(R_1 + R_3)$ .

We determine Thevenin voltage by opening  $ab$ . Using voltage division across two of the loops, we see that  $V_x = V_sR_x/(R_2 + R_x)$  and  $V_3 = V_sR_3/(R_1 + R_3)$ .  $V_{ab} = V_3 - V_x = V_s[R_3/(R_1 + R_3) - R_x/(R_2 + R_x)]$ .

When the bridge is balanced then  $R_3/R_1 = R_x/R_2$ , so the above expression for Thevenin voltage becomes 0. This is precisely what we'd expect.

## 9 1.24

$$p(t) = v(t)i(t) = 20e^{-t}W$$

$$\text{Energy} = \int_0^{\infty} p(t)dt = -20e^{-t}|_0^{\infty} = 20\text{joules.}$$

The element absorbs the energy (because the direction of current flow is the same as that of voltage drop).

## 10 -

$$\text{a) } P = i^2R = (1\mu A)^2(10\Omega) = 10^{-11}W$$

b)  $P = iV = (4\mu A)(5V) = 2 * 10^{-5}W$

c) The amperemeter does not measure the precise current flowing through R, because some current flows through  $R_V$ .

d) You can correct for the discrepancy.  $P_{\text{measured}} = iV$  where  $i$  is the current through  $R$  and  $R_V$  and  $V$  is the voltage across them (1 point).  $P_{\text{actual}} = i_R V$  where  $i_R$  is the current through resistor  $R$  alone. We use current division to discover that  $i_R = i * R_V / (R_V + R)$ , so  $P_{\text{actual}} = R_V / (R_V + R) * P_{\text{measured}}$ .

e) One can fix the amperemeter's reading by first putting it in series with  $R$  and then measuring the voltage across the sum of these. However, now the voltmeter provides an inaccurate reading due to the fact it is also measuring the voltage drop across  $R_a$ .