UNIVERSITY OF CALIFORNIA AT BERKELEY   EECS Department

EECS 40, Fall 2007
Prof. Chang-Hasnain

Homework #4

Due at 5 pm in 240 Cory on Thursday, 10/04/07
Total Points: 100

- **Put (1) your name and (2) discussion section number on your homework.**
- **You need to put down all the derivation steps to obtain full credits of the problems. Numerical answers alone will at best receive low percentage partial credits.**
- **No late submission will be accepted expect those with prior approval from Prof. Chang-Hasnain.**
- **Problems of this HW are from Hambley 4th Edition**

P4.57 (Second-Order Circuits) (5 points)
The sketch should look like the □ = 0.1 curve in Fig 4.29. Severely underdamped (□ << 1) second-order circuits display a lot of overshoot and ringing.

P4.61 (Second-Order Circuits) (20 points)
This circuit is considered on pages 185-186.
a) Writing eq. 4.101 using KCL and comparing with 4.106 gives □ = 1/(2RC) = 2*10^7, □0 = 1/(LC)^.5 = 10^7, and □ = □/□0 = 2. (3 pts)
Therefore our circuit is overdamped. (2 pts)

b) Using equation 4.100 (before differentiating) and evaluating at t=0+, we get
$C\, v'(0+) + 1/R\, v(0+) + 1/L \int_0^t v(t)\, dt + i_L(0+) = i_n(0+)$
Plugging in v(0+) = 0 and $i_L(0+) = 0$ and v(t) =0 for t<0, and $i_n(0+)$=1A, we get V'(0+) =

1/C = 10^9.

c) Under steady-state conditions, the inductor acts as a short circuit. So $i_L(\infty)$=0, so the particular solution $v_p(t)$ = 0. (5 pts)

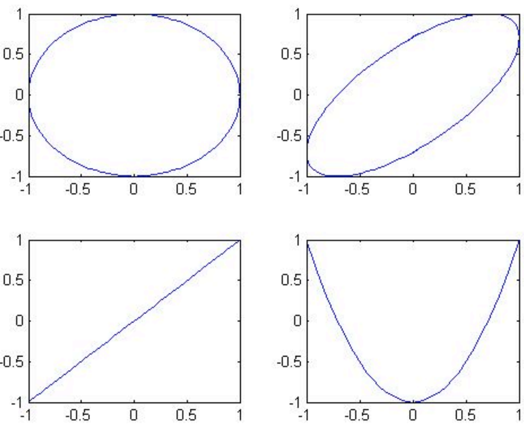
d) The complementary solution (that which solves the homogenous equation) is of the form K1 exp(s1 t) + K2 exp(s2 t). The particular solution was 0 by part (c). So the complete solution is v(t) = K1 exp(s1 t) + K2 exp(s2 t). Plugging this into the differential equation in 4.106 gives us the s1 = -□ + √(□^2 + □0^2) and s2 = -□ - √(□^2 + □0^2). Evaluating these gives -2.679*10^6 and -37.32*10^6. Next, use the initial conditions v(0+) = 0 an v'(0+) = 10^9:
v(0+) = 0 = K1 + K2
v'(0+) = 10^9 = s1*K1+s2*K2.
Solving, one finds K1 = 28.87 and K2 = -28.87. Therefore the solution is
v(t) = 28.87 exp(s1*t) – 28.87 exp(s2 *t).

P5.10 (Sinusoidal Currents and Voltages) You may also sketch the Lissajous figures by hand. (10 points, 2.5 for each plot)

You can do this in MATLAB as follows:

```
Theta_vect = [ 90 45 0 0]*pi/180;
for i = 1:4
Wx = 2*pi;
Wy = Wy_vect(i)
Theta = Theta_vect(i);
t = 0:0.01:20;
x = cos(Wx*t);
y = cos(Wy*t + Theta);
```

```
Subplot(2,2,i)
plot(x,y)
end
```



P5.16 (RMS-value) (5 points)
Vrms = √ (1/T $\int_0^T$ v^2(t) dt) = √ ($\int_0^1$ [3 exp(-t)]^2 dt) = √ ($\int_0^1$ [9 exp(-2t)] dt) = √ ((9/2) $[\exp(-2t)]_{t=0}^{t=1}$) = √ (4.5*(1-exp(-2)) = 1.973 V

Elementary operations (Complex Arithmetic) (12 points, 2 each)
Perform the following operations:

a) $(5+j3)+(3-j7)=$ (5+3) +j(3-7) = 8 -j4

b) $(2+j5)-(9+j4)=$ *(2-9) +j(5-4) = -7+j*

c) $(7+j8)(4-j2)=$ 4*7 + j(8*4-2*7) – $j^2$2*8 = 28 +16 + j(18) = 44 + j18

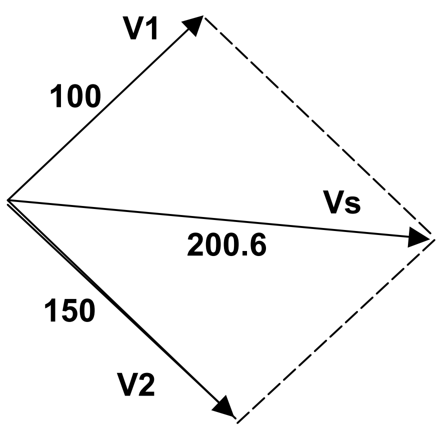
d) $\frac{(1+j3)}{(4+j9)}$ = (4-j9)(1+j3)/(16+81) = (4 + 27 + 3j)/(97) = (31 + 3j)/97 = 0.3196 + j 0.0309

e) Convert $(1+j3)$ **into polar form.** = (1+9)^.5 * e^(j arctan(3/1)) = 10^.5 * exp(j arctan(3)) = 3.16 exp(j 1.25*180/pi) = 3.16 exp(j 72º)

f) Convert $2e^{j35°}$ **into rectangular form =** 2cos(35º)+j sin(35º) = 1.64 + j 1.15

P5.25 (Phasors) (10 points)

V1(t) = 100 cos(wt + 45º)
V2(t) = 150 sin(wt + 60º)
Rewriting as phasors, we get:
V1 = 100∠45 = 70.71 + j 70.71
V2 = 150∠-30 = 129.9 - j 75
Vs = V1 + V2 = 200.6 – j4.29 = 200.6∠ -1.23º
Vs(t) = 200.6 cos(wt - 1.23º)

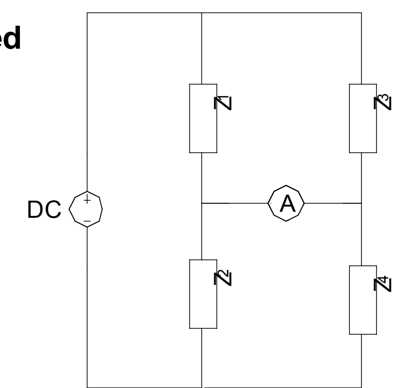


V2 lags V1 by 75 º. Vs lags V1 by 46.23 º. Vs leads V2 by 28.77 º.

P5.24 (Phasors) (8 points)
The magnitude of the phasors are 8*√(2) and 3*√(2). If the phase angle is the same then you get the maximal value by just adding the phasor magnitudes to get 11*√(2). The minimal value is achieved when they are 180º out of phase, so that you just subtract them to get 5*√(2) .

Wheatstone Bridge (Complex Impedances) (? points)

The circuit on the right hand side shows a generalized version of the Wheatstone Bridge. Z_1, Z_2, Z_3, Z_4 are complex impedances.
Derive the condition for the current through the amperemeter to be zero. (3 points)



Z1/(Z1 + Z2) = Z3/(Z3+Z4) □ Z4/Z3 = Z2/Z1

Let $f = 60Hz$, Z_1 consists of a 60Ω resistor and a 0.2H inductance, Z_2 is a 100Ω resistor. Z_3 consists of a 200Ω resistor and a 10 □F capacitor. Calculate the complex impedances of Z_1, Z_2, Z_3. (9 points)

Z1 = R + □L60 □90 º = 60 + (2pi*60) * 0.2 □90 º = 60 + j 75.4 = (60^2 + 75^2)^.5 □ arctan(75/60) = 96.4 □ 51.5 º
Z2 = 100
Z3 = 200- j/(□C) = 200 –j /(2*pi*60*50*10^-6) = 200 – j 53 ~ 200 = 206.9 □ -14.8 º

(3 points each)

Calculate Z_4 such that there is no current flowing through the amperemeter.
With which circuit elements can you construct Z_4? (6 points)
Z4 = Z3*Z2/Z1 = (206.9 □ -14.8 º)*100 / (96.4 □ 51.5 º)=214.6 □ -66.3 º = 86.3 -j196.5
We can construct Z4 out of a 86.3□ resistor and a capacitor satisfying 1/(□C) = j196.5, so C = 1/(□*196.5) = 1/(2pi*60*196.5) = 13.5 □F.
(2 point writing right equations, 2 point identifying correct element based on sign, 2 point computations)

P5.38 (Complex Impedances) (10 points)

a) From the plot, we see that T = 4ms, so f = 1/T = 250 Hz and □ = 2□f = **500□ (1 point)**. Current lags **(1 point)** voltage by 1ms = T/4 = 90º, so we have an inductance **(1 point for correct, based on previous part).** □L = V_m/I_m = 5□, so L = 3.18 mH **(2 points)**.

b) We see that T = 8ms, so □ = 2□f = 250□ **(1 point)**. Current leads **(1 point)** voltage by 2ms, or 90º, so we have a capacitance **(1 point)**. 1/□C = V_m/I_m = (10V)/(4mA) = 2.5*10^3 = 2500, so C = .5093 □F **(2 points)**.